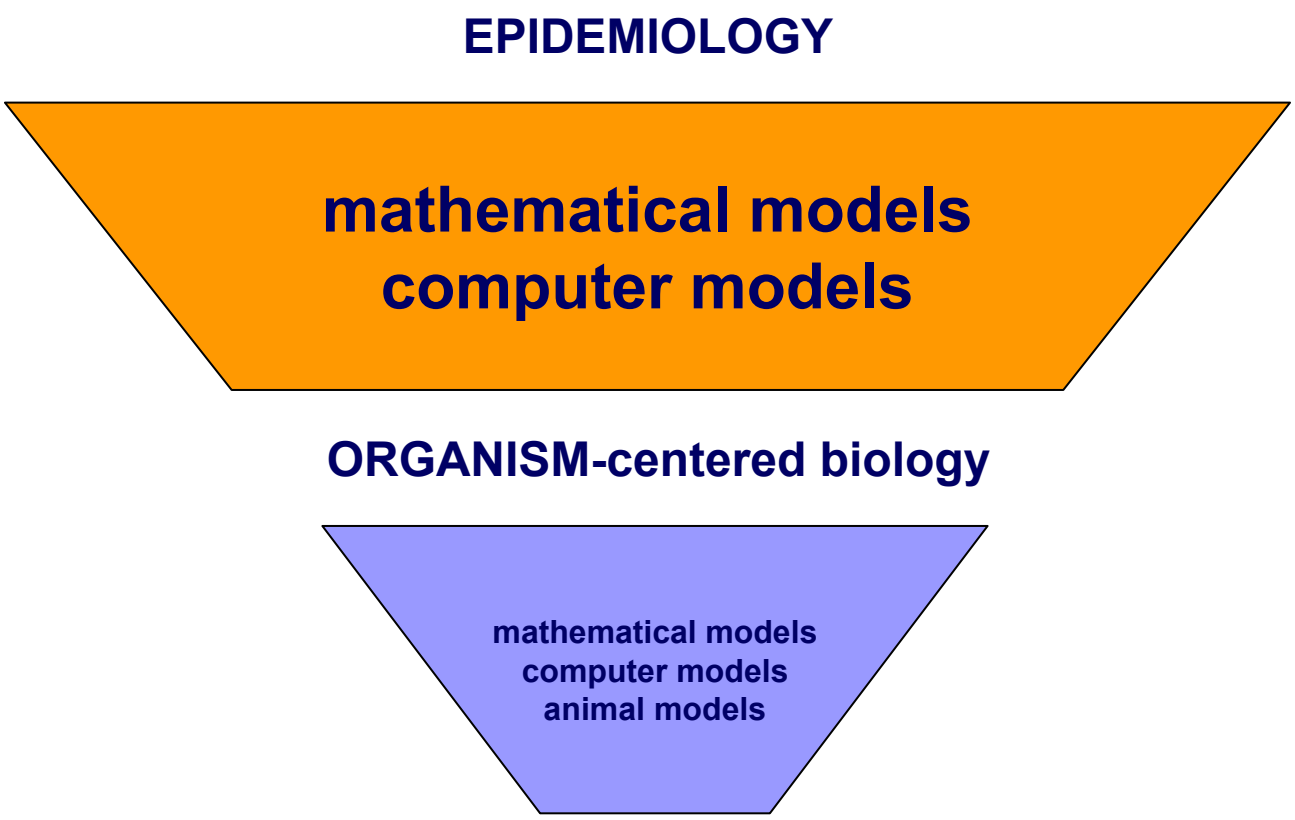
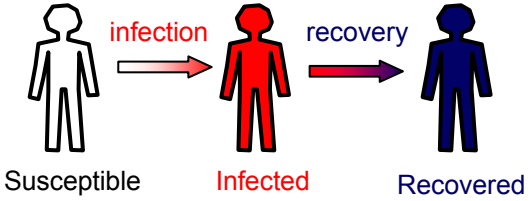
The SIRI Model in Epidemiology: Pertussis and Malaria

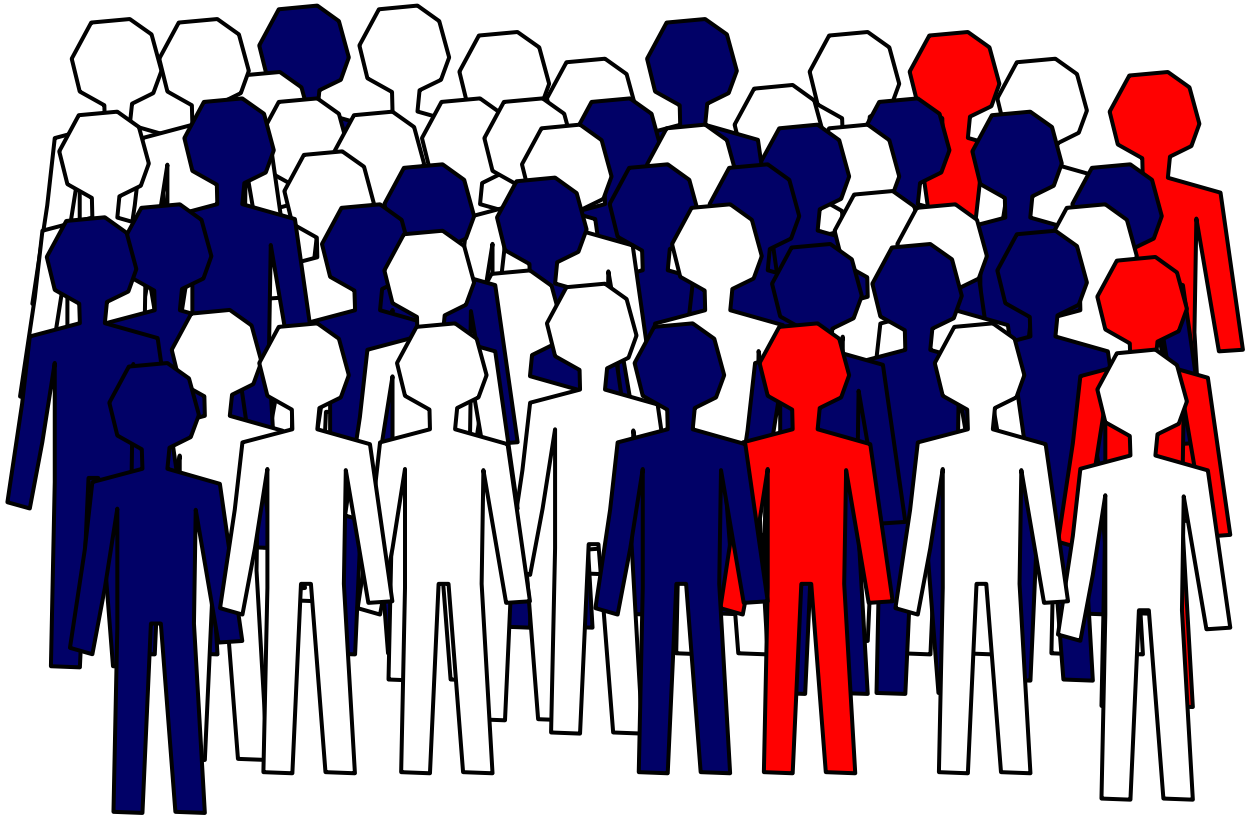
Gabriela Gomes

Models establish bridges between levels of organisation

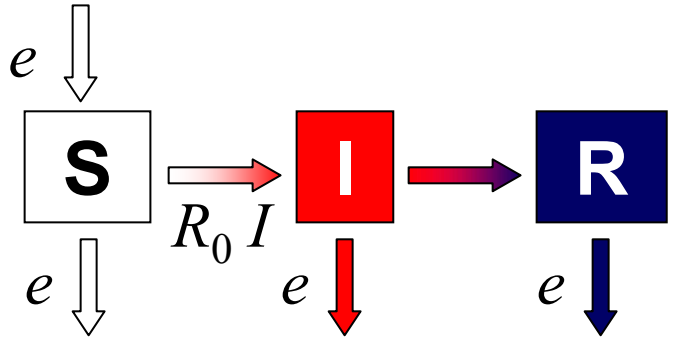


SIR model – totally protective immunity



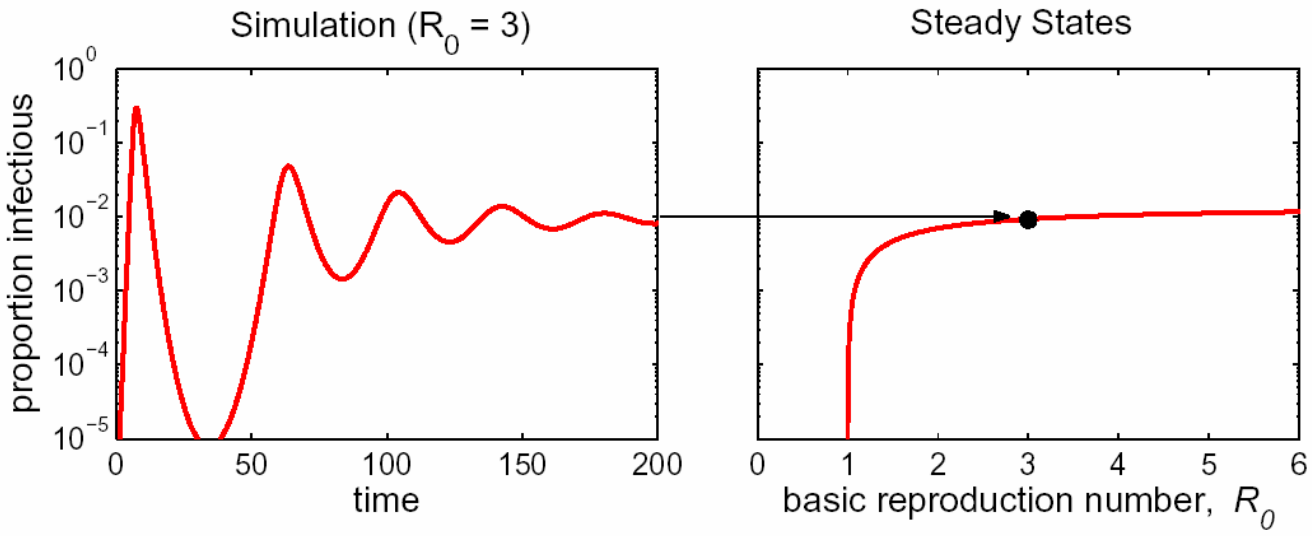


Endemic threshold – $R_0 = 1$



$$\frac{dS}{dt} = e - R_0 IS - eS$$

$$\frac{dI}{dt} = R_0 IS - I$$



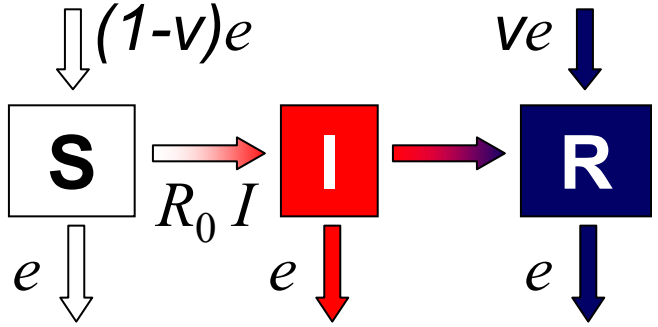
The basic reproduction number, R_0

The expected number of secondary cases arising from a single infectious individual in a totally susceptible population.

Disease	R_0
Smallpox	4
Measles	17
Rubella (England and Wales)	6
Rubella (Gambia)	15

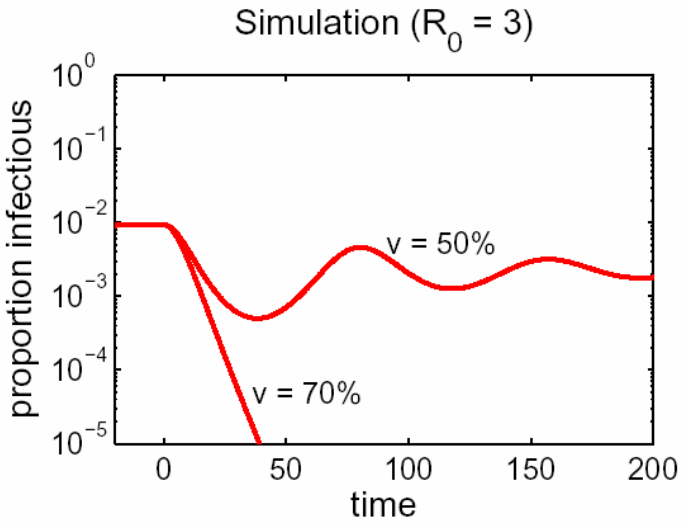
R_0 depends on the disease and the population.

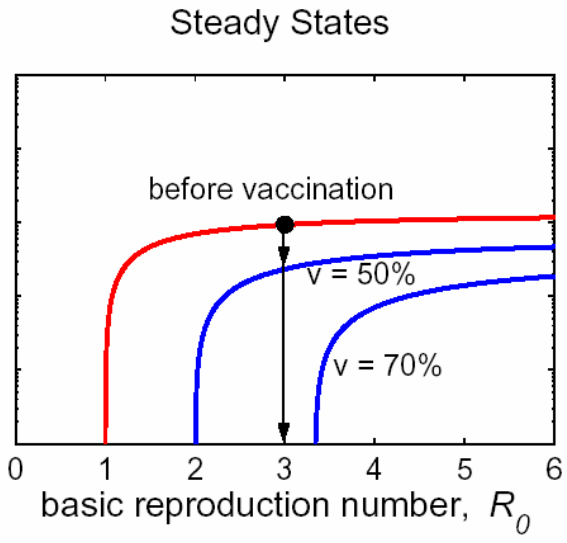
Vaccination reduces the susceptibility pool



$$\frac{dS}{dt} = (1-v)e - R_0 IS - eS$$

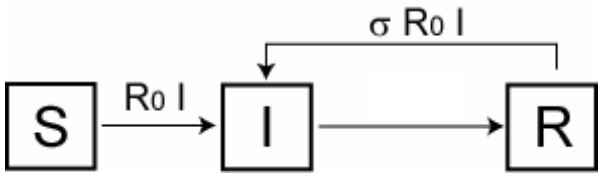
$$\frac{dI}{dt} = R_0 IS - I$$

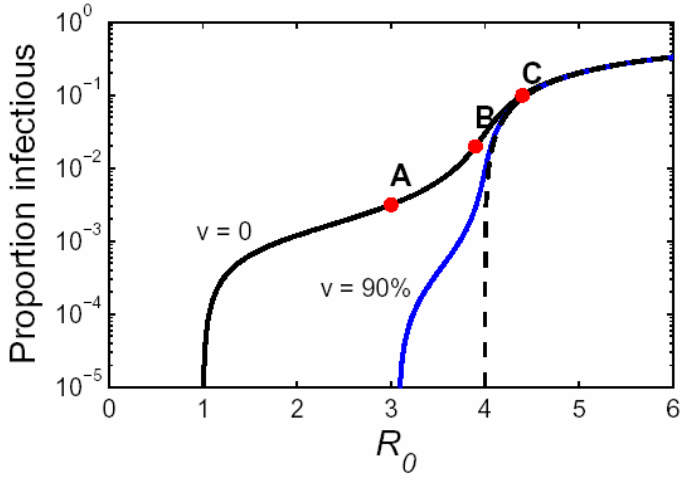


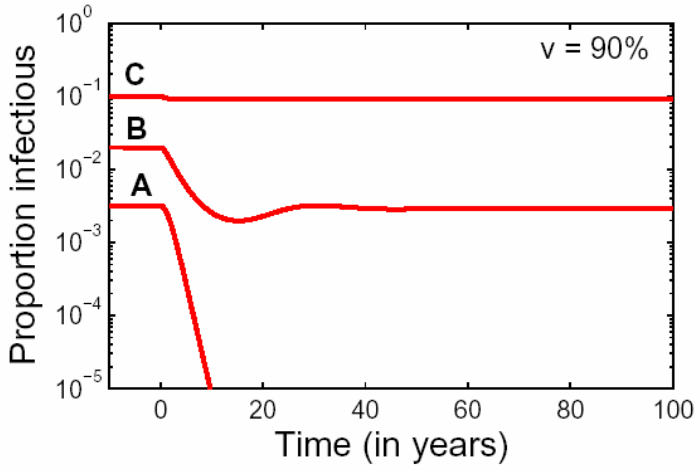


Partial immunity induces a **reinfection threshold**, $R_0 = 1 / \sigma$, above which the prevalence of infection is very high and insensitive to vaccination.

σ: reduction in the risk of infection due to partial immunity.







Tuberculosis – geographical variability

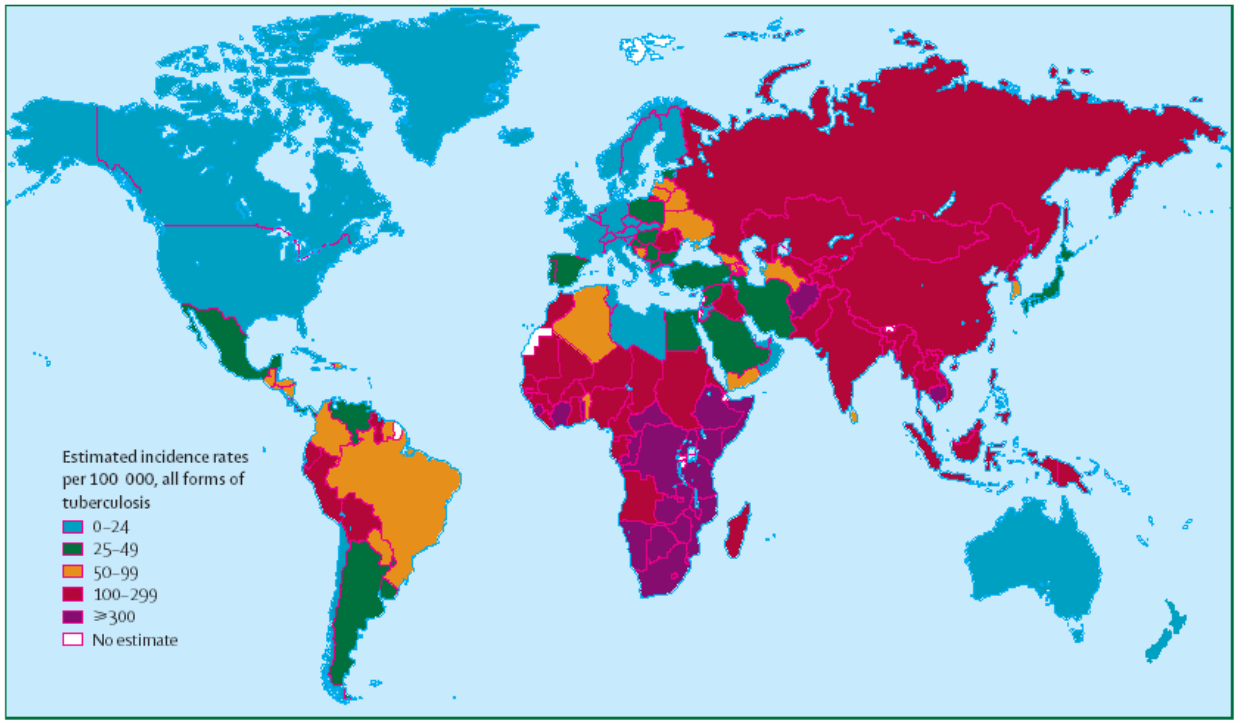


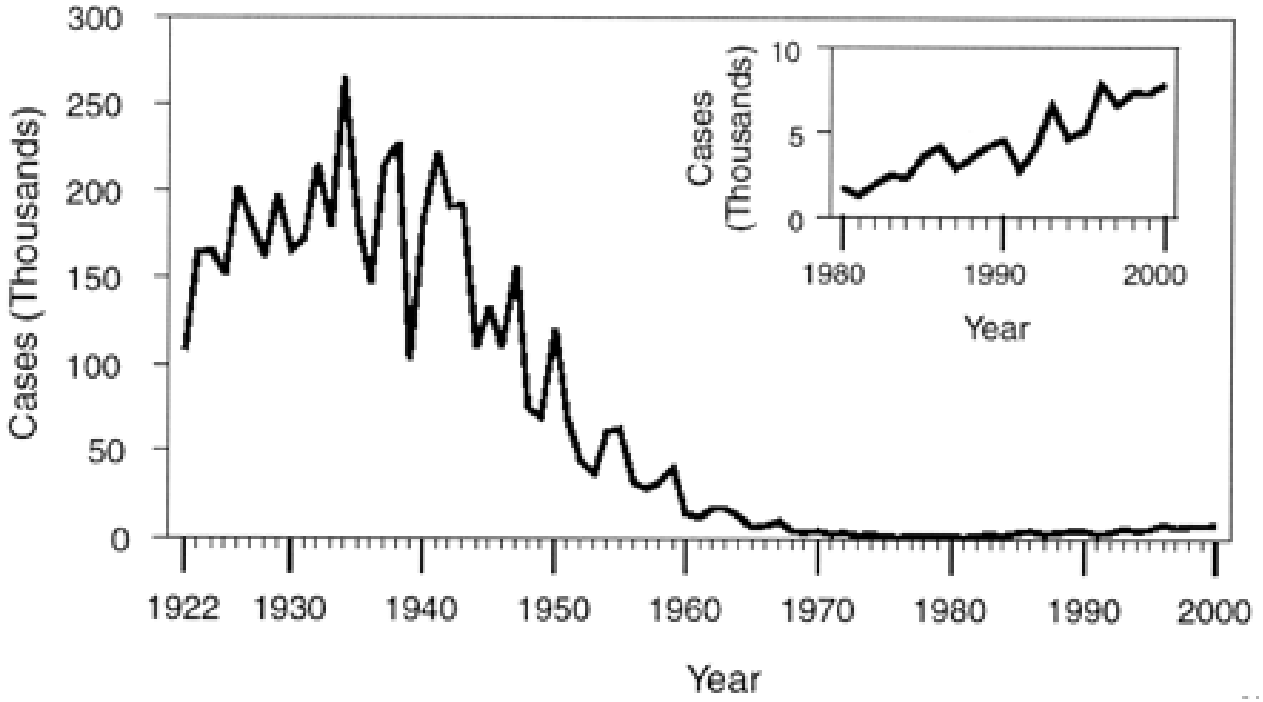
Figure 1: Distribution of tuberculosis in the world in 2003[4]

Gomes MGM, Franco AO, Gomes MC, Medley GF 2004 The reinfection threshold promotes variability in tuberculosis epidemiology and vaccine efficacy. *Proc. R. Soc. Lond.* B 271, 617-623.

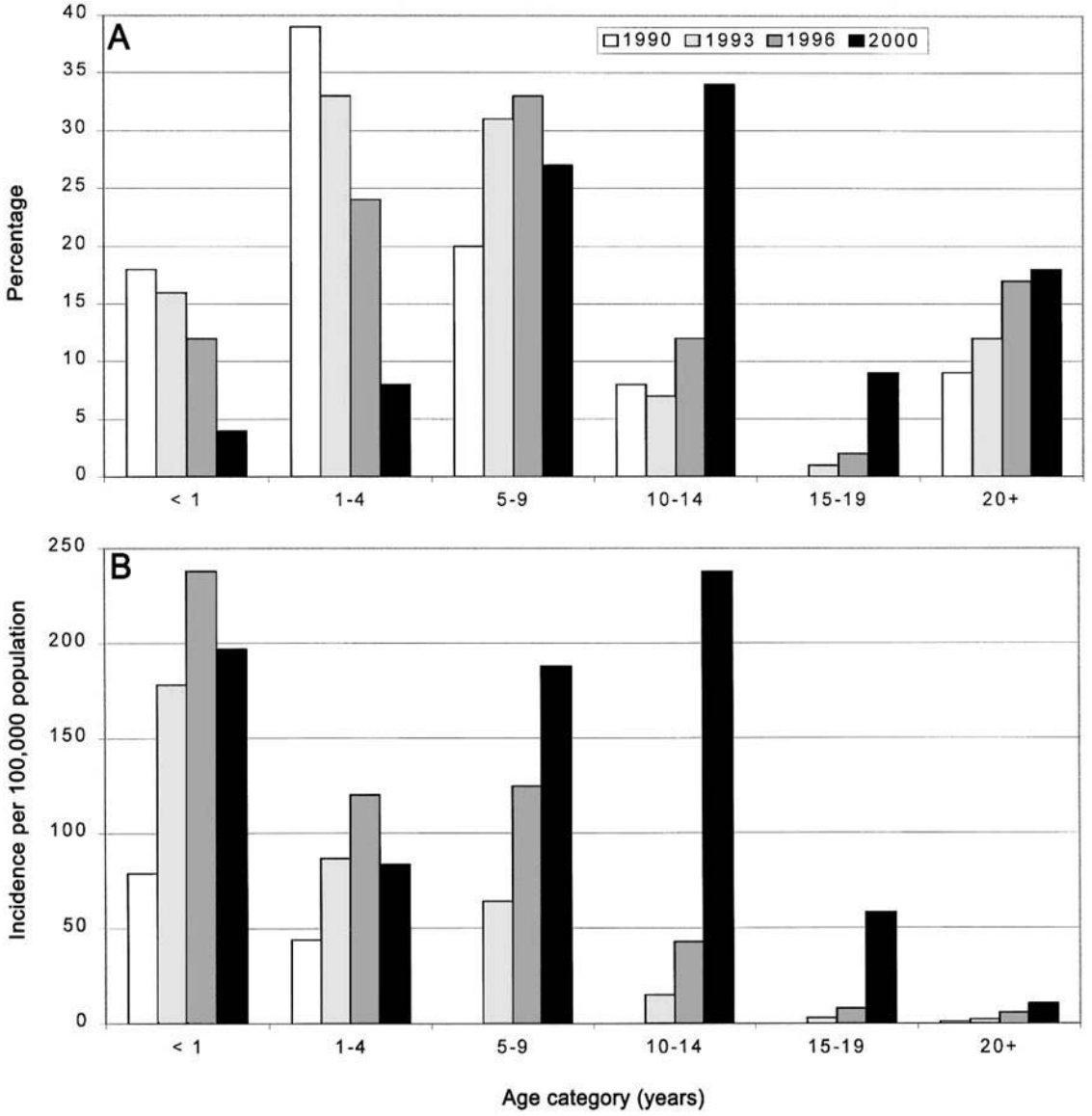
Gomes MGM, Rodrigues P, Hilker FM, Mantilla-Beniers NB, Muehlen M, Paulo ACAS, Medley GF 2006 The potential of post-exposure interventions in global tuberculosis control *(submitted).*

Rodrigues P, Rebelo C, Gomes MGM 2007 Resistant tuberculosis: A reinfection model. *Theoretical Population Biology* 71, 196-212.

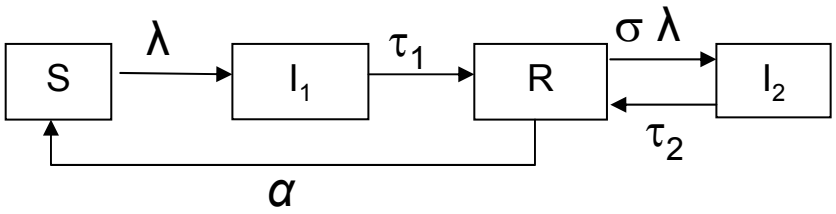
Pertussis - resurgence



Reinfection Threshold
April 2005 – March 2009

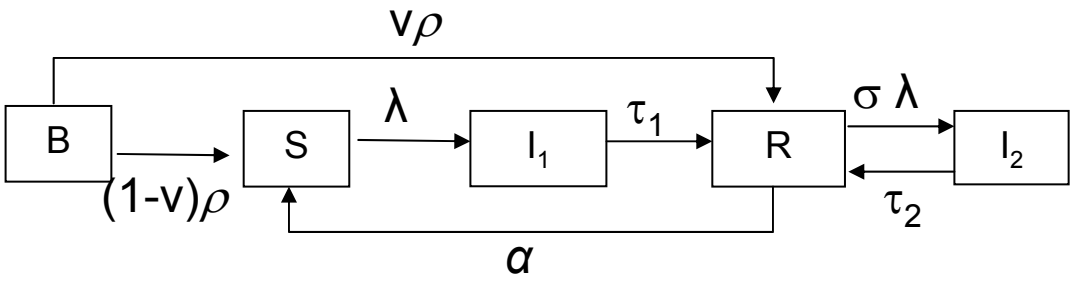


Ricardo Águas - IGC



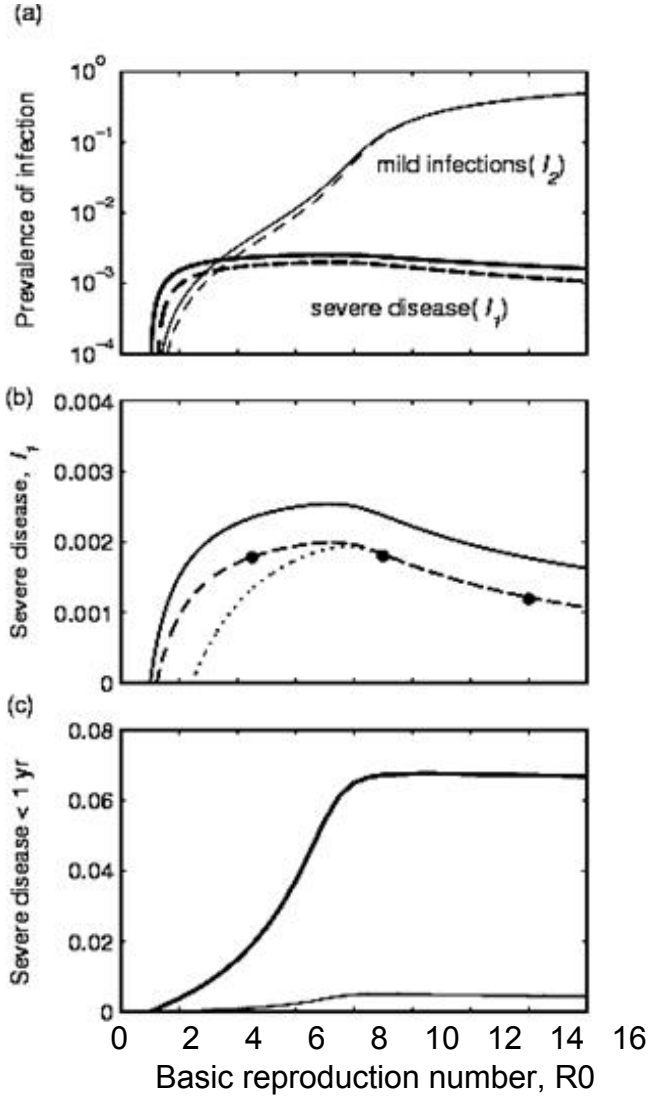
Águas R, Gonçalves G, Gomes MGM (2006) Pertussis: Increasing disease as a consequence of reducing transmission. *Lancet Infectious Diseases* 6, 112-117.

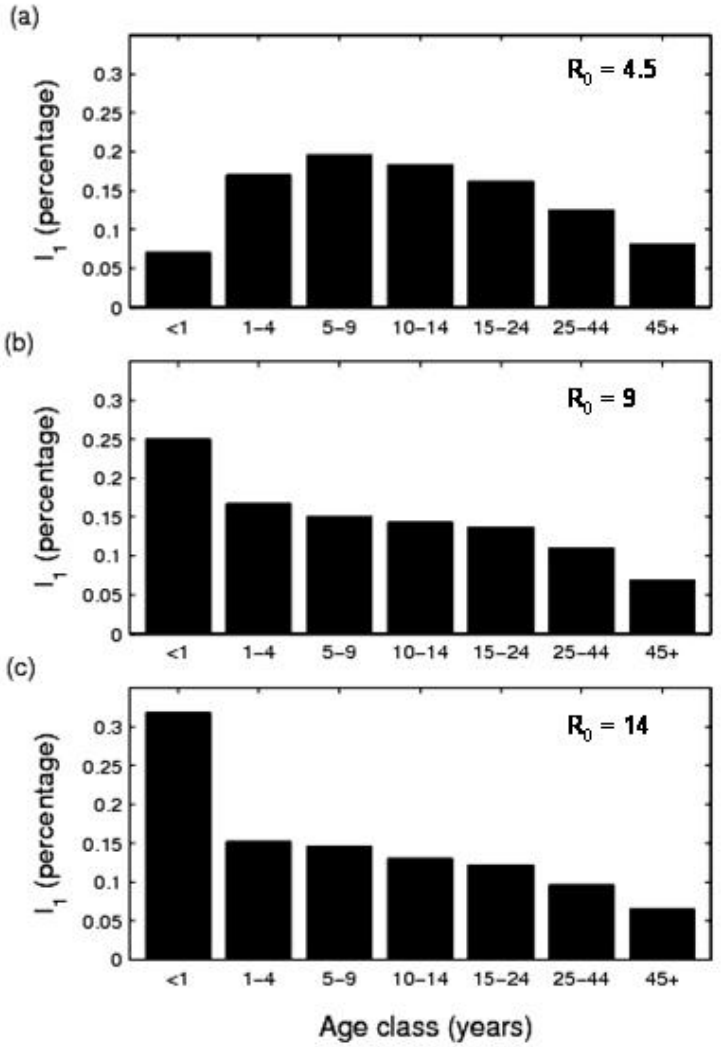
Ricardo Águas - IGC



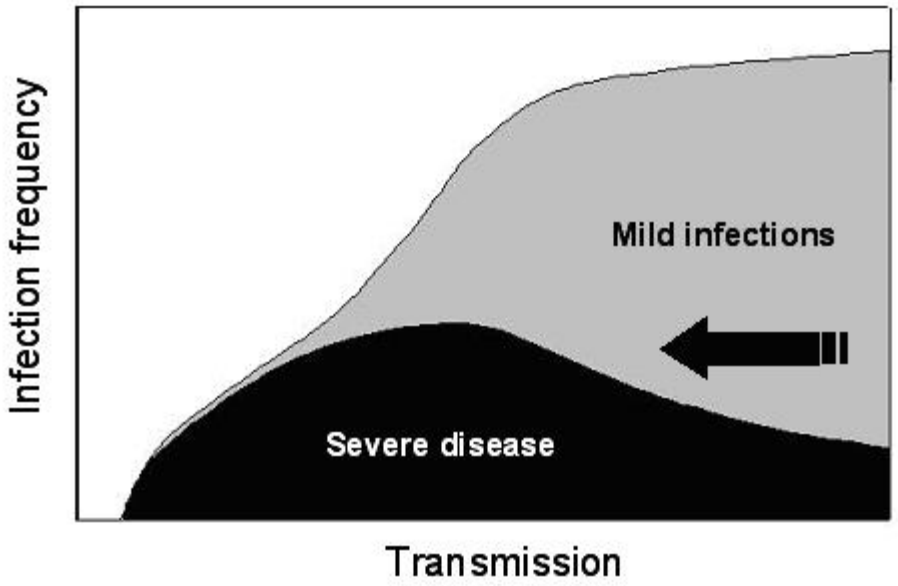
Águas R, Gonçalves G, Gomes MGM (2006) Pertussis: Increasing disease as a consequence of reducing transmission. *Lancet Infectious Diseases* 6, 112-117.

Pertussis - resurgence





Increasing disease with decreasing transmission



Model equations

$$\frac{dS_0}{dt} = (1-v_0)\mu + \alpha R - S_0(\lambda + \mu)$$

$$\frac{dR}{dt} = v_0\mu + \tau_1 I_1 + \tau_2 I_2 - R(\sigma\lambda + \alpha + \mu)$$

$$\frac{dI_1}{dt} = \lambda S_0 - I_1(\tau_1 + \mu)$$

$$\frac{dI_2}{dt} = \sigma\lambda R - I_2(\tau_2 + \mu)$$

Age-dependent model equations

$$\frac{\partial S_0}{\partial t} + \frac{\partial S_0}{\partial a} = \alpha R - S_0(\lambda + v_0\mu + \mu)$$

$$\frac{\partial R}{\partial t} + \frac{\partial R}{\partial a} = v_0\mu + \tau_1 I_1 + \tau_2 I_2 - R(\sigma\lambda + \alpha + \mu)$$

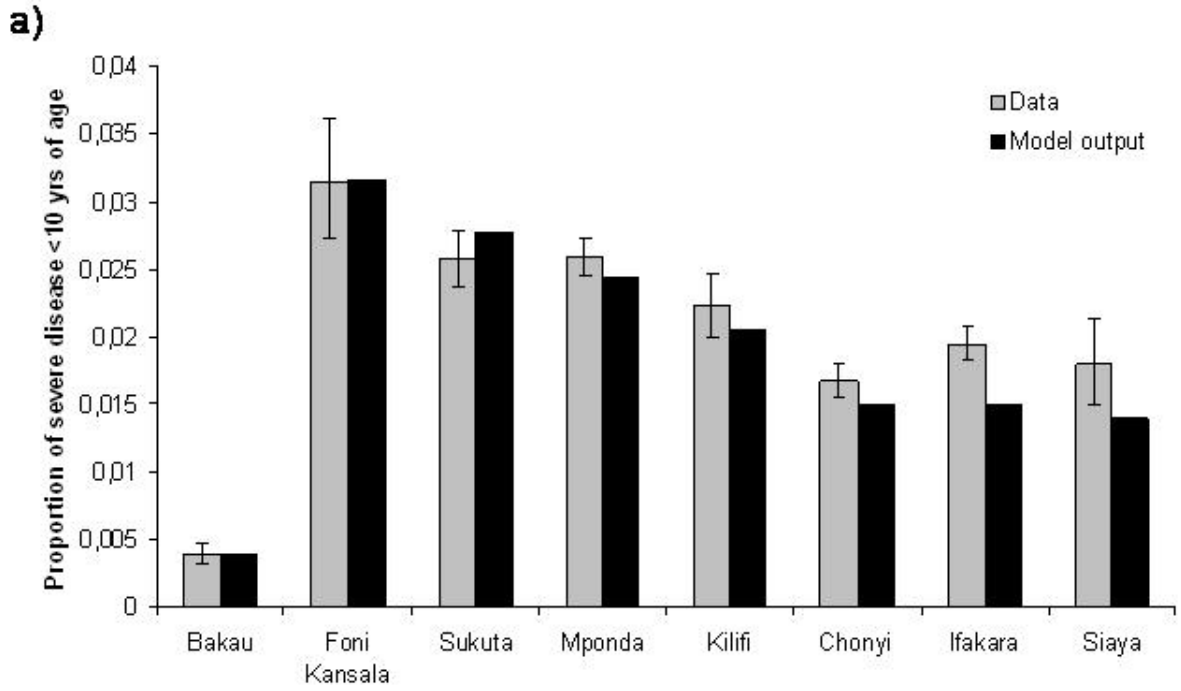
$$\frac{\partial I_1}{\partial t} + \frac{\partial I_1}{\partial a} = \lambda S_0 - I_1(\tau_1 + \mu)$$

$$\frac{\partial I_2}{\partial t} + \frac{\partial I_2}{\partial a} = \sigma\lambda R - I_2(\tau_2 + \mu)$$

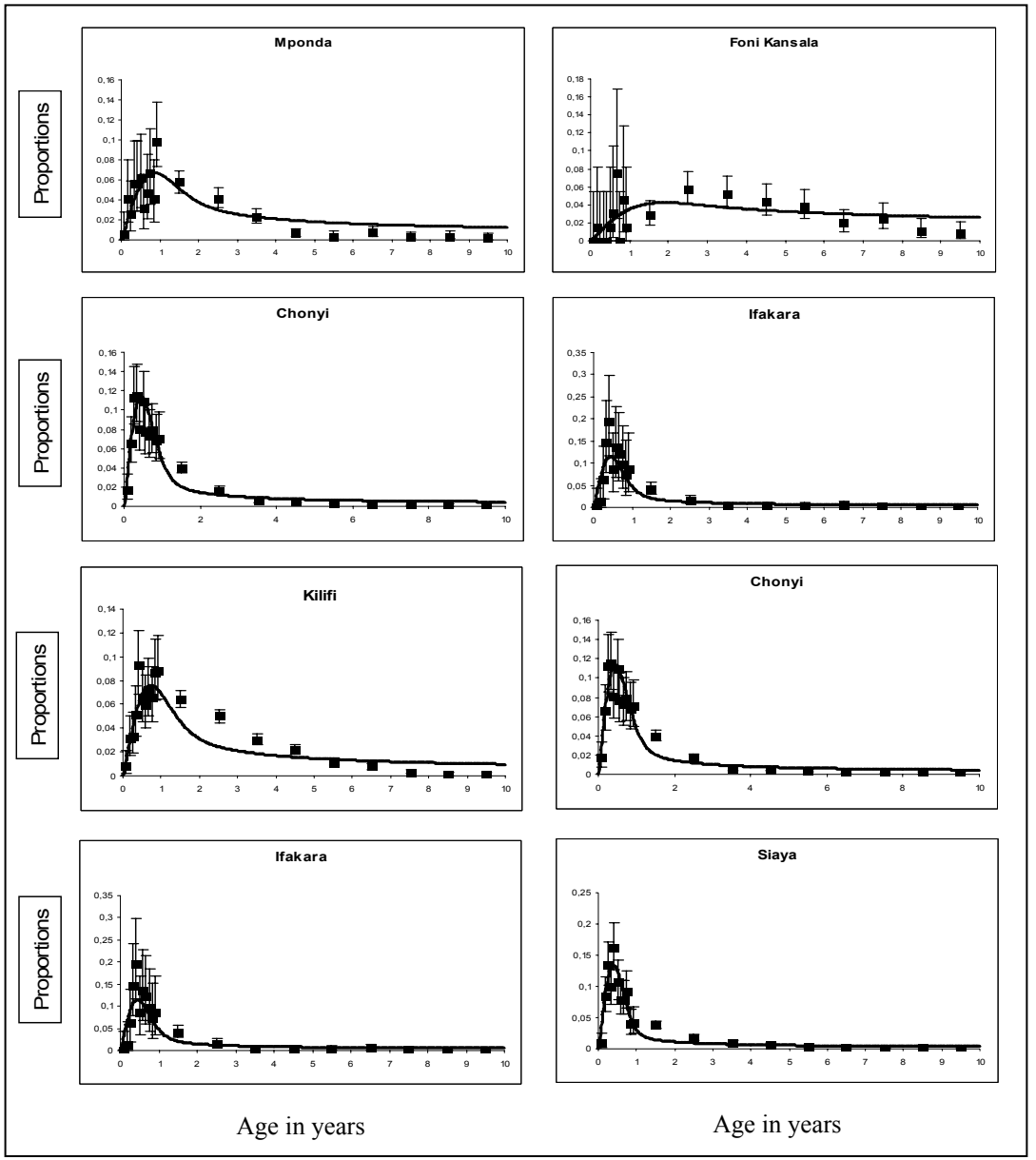
Boundary conditions:

$$S_0(\mathrm{t},0) = \mu \qquad R(\mathrm{t},0) = I_1(\mathrm{t},0) = I_2(\mathrm{t},0) = 0$$

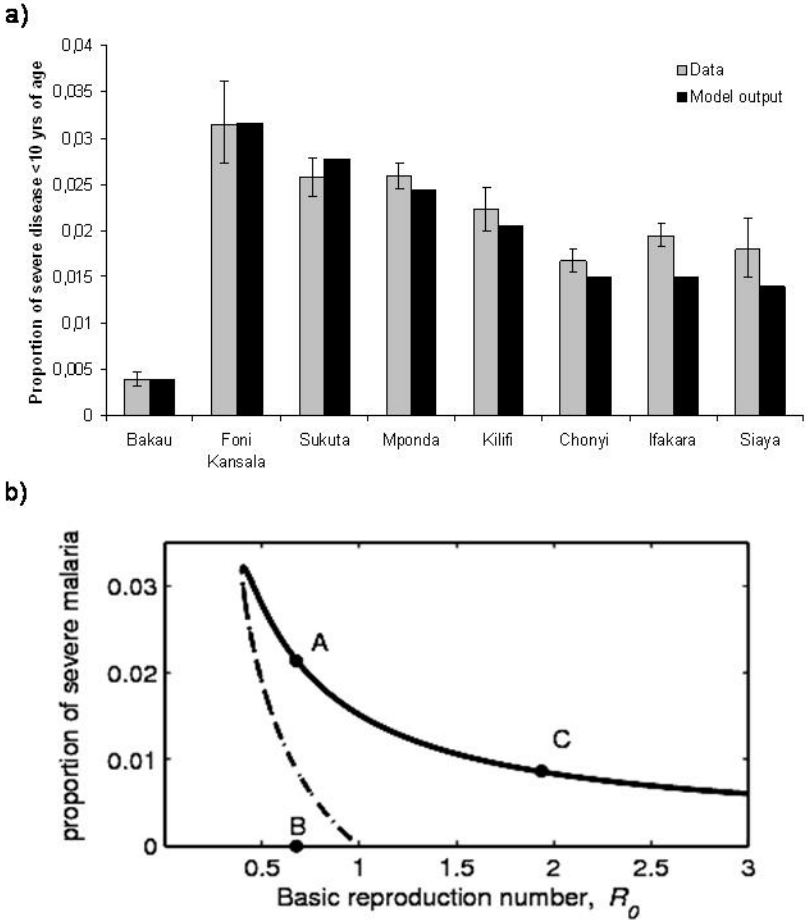
Malaria – data from sub-Saharan Africa



Malaria – data from Sub-Saharan Africa

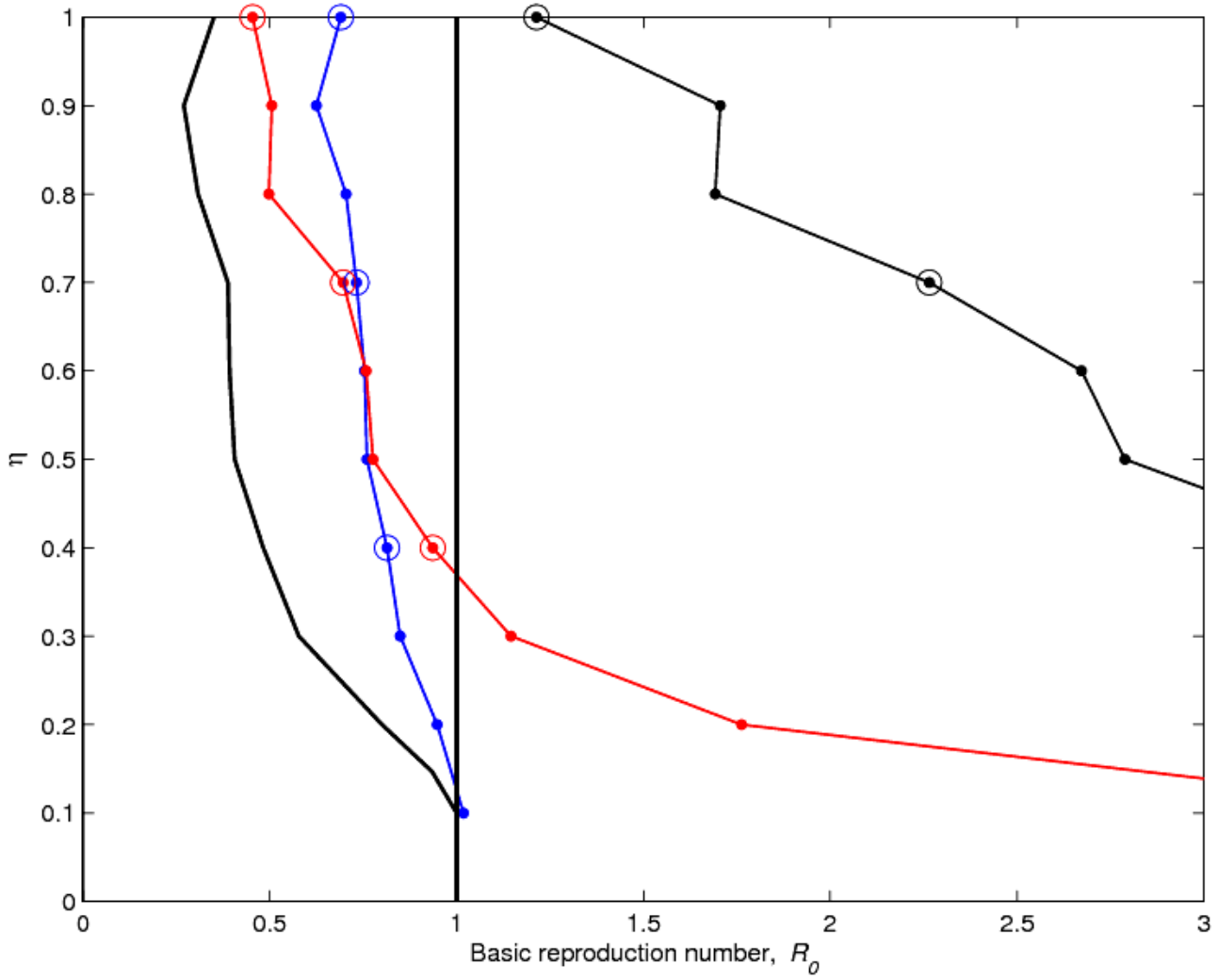


Malaria – sustainability of interventions

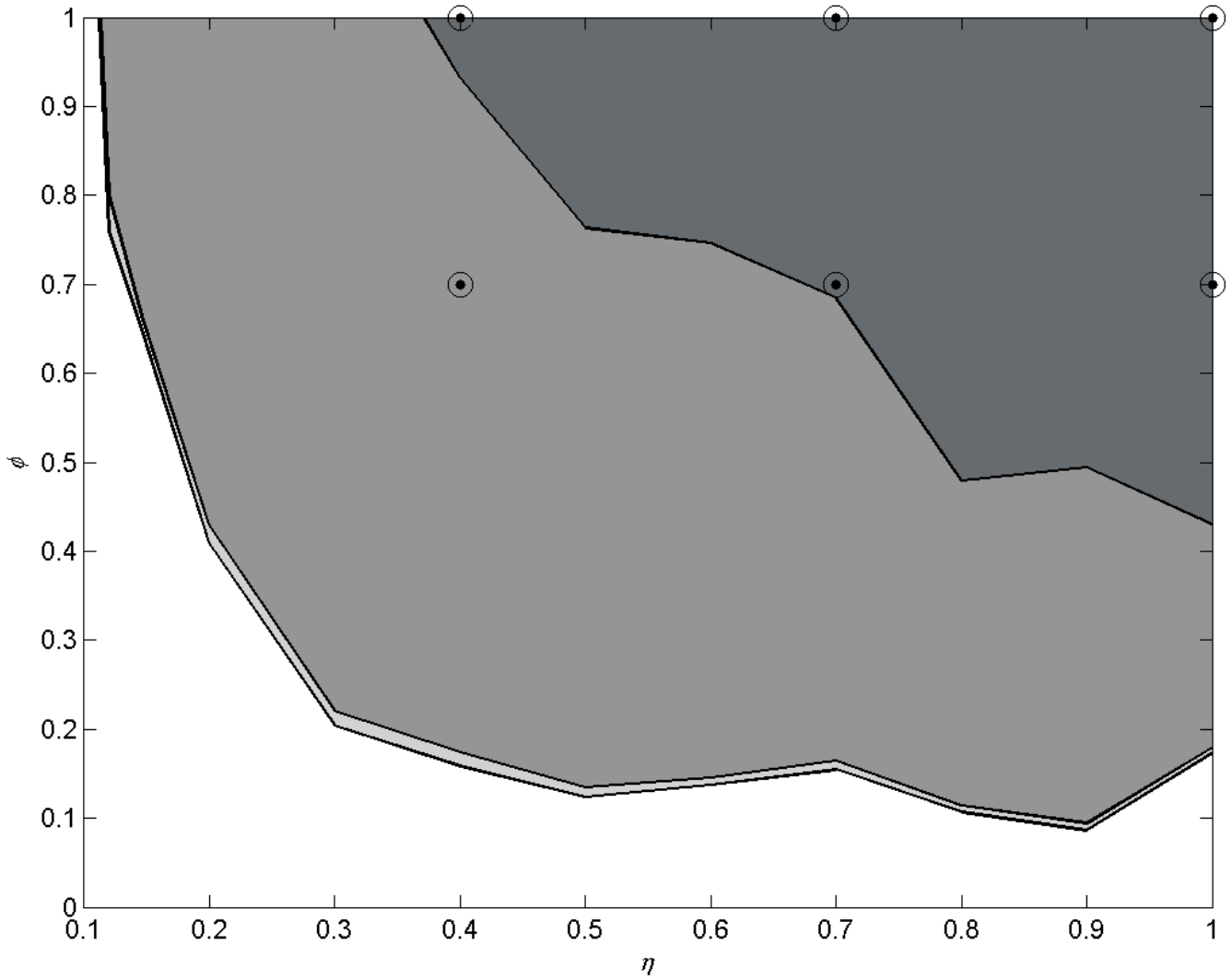


Águas R, White LJ, Snow RW, Gomes MGM 2007 Prospects for sustainable malaria control in sub-Saharan areas of mesoendemic transmission *(submitted)*.

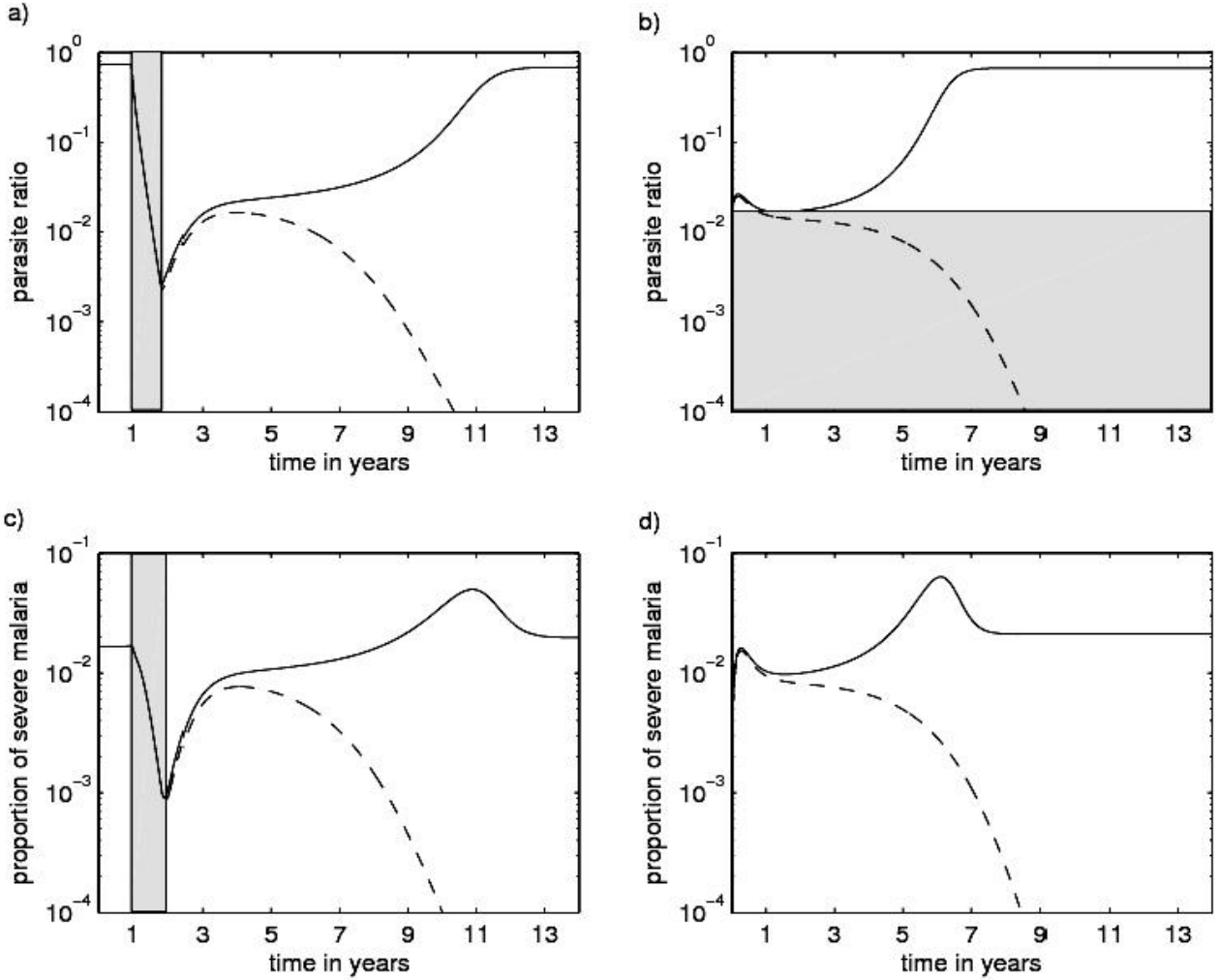
Malaria – sensitivity to reporting rate



Malaria – sensitivity to infectively reduction



Malaria – intervention design

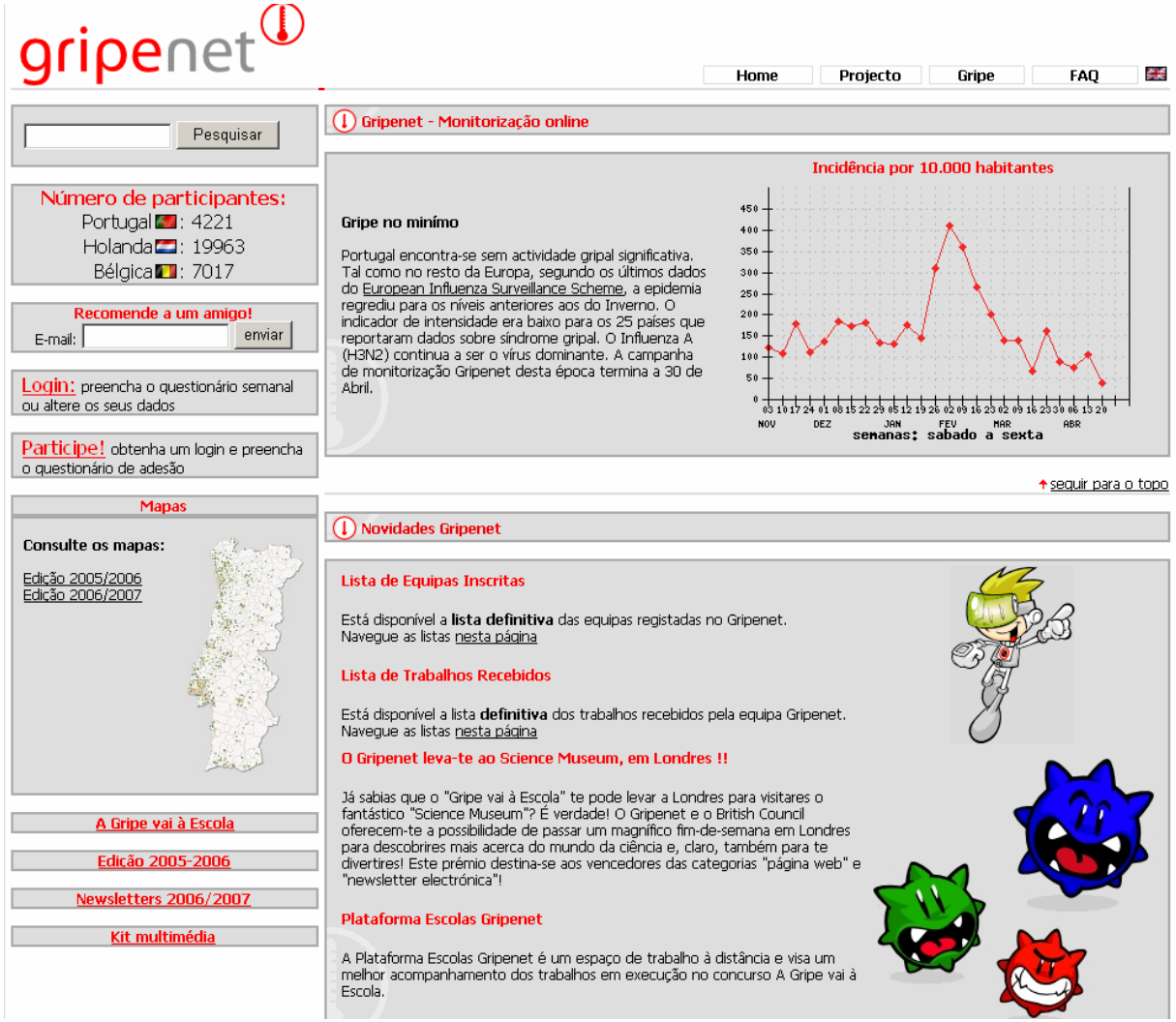


Advanced Courses and Workshops:

http://sites.igc.gulbenkian.pt/ggomes/meetings.php
First Summer School on “Mathematics in Biology and Medicine”
Advanced Course on “Tuberculosis: Scientific Basis for Control”
First Workshop on “Pathogen Diversity and Disease Epidemiology”
Science in Society Workshops - “Gripenet”
Advanced Course on “Infectious Disease Epidemiology”
Second Summer School on “Mathematics in Biology and Medicine”
Second Workshop on “Pathogen Diversity and Disease Epidemiology”

Epidemic monitoring and forecast

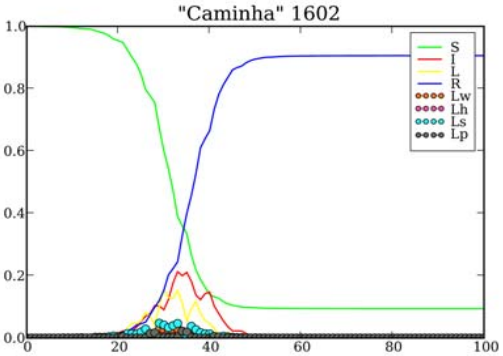
- Fast
- Transparent
- Uniform

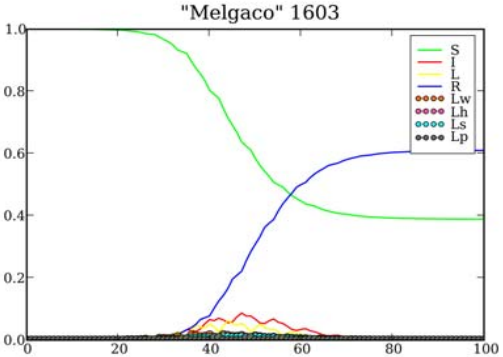


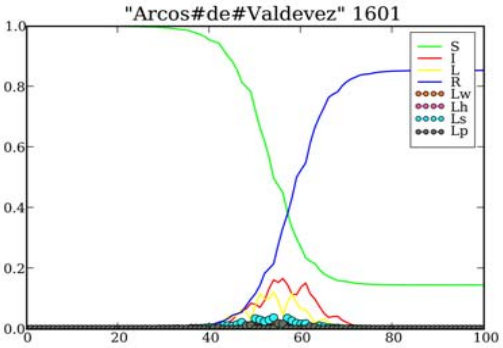
Marquet RL, Bartelds AIM, van Noort SP, Koppeschaar CE, Paget J, Schellevis FG, van der Zee J 2006 An internet.based monitoring of influenza-like illness (ILI) in the general population of the Netherlands during the 2003-2004 influenza season. BMC Public Health 6, 242.

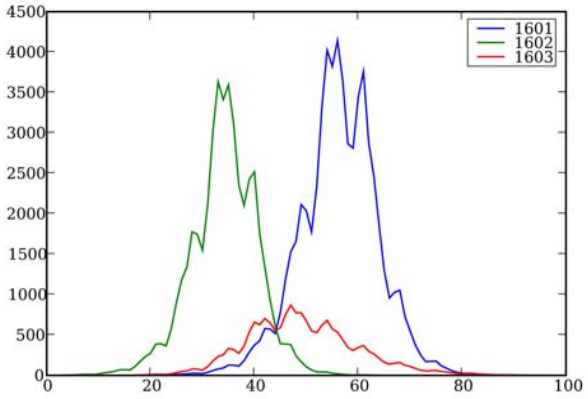
van Noort SP, Lourenço J, Rebelo de Andrade H, Muehlen M, Gomes MGM 2007 Internet-based surveillance of ILI performs iniformly in the Netherlands, Belgium and Portugal (submitted).

José Lourenço - IGC

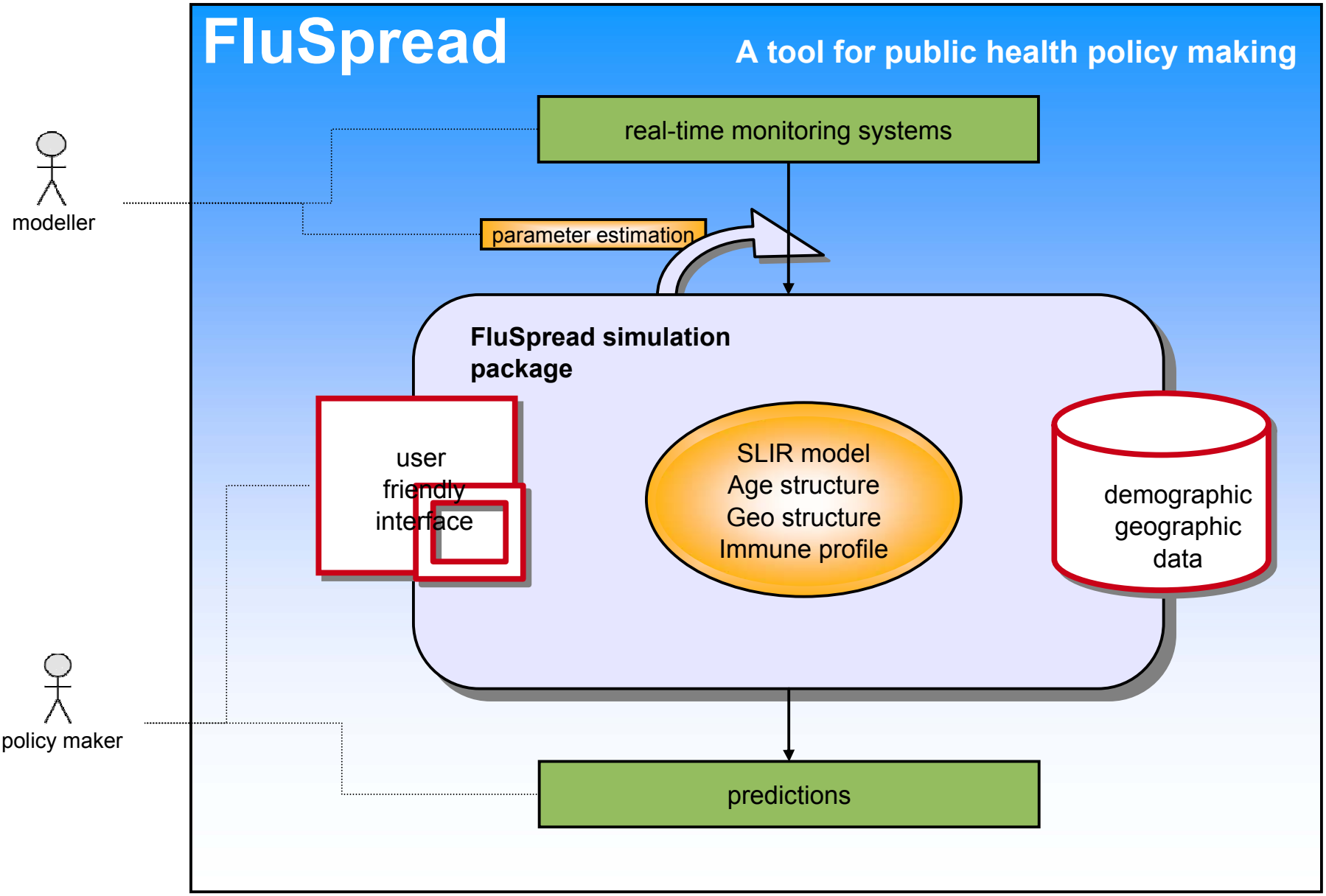








Demographic and Geographical Information


FluSpread
A tool for public health policy making
real-time monitoring systems
modeller
parameter estimation
FluSpread simulation package
user friendly interface
SLIR model
Age structure
Geo structure
Immune profile
demographic geographic data
policy maker
predictions

Reinfection Threshold
April 2005 – March 2009