A Generalized Approach to Portfolio Optimization: Improving Performance By Constraining Portfolio Norms

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Follow-up Workshop on Optimization in Finance
Centro Internacional de Matematica, Coimbra

Portfolio Selection with the Penalty and Conjugate Gradient Methods

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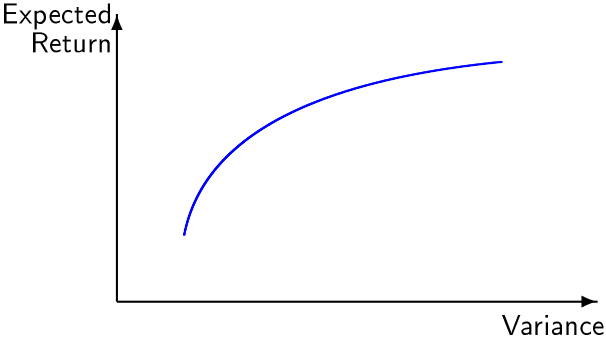
Outline

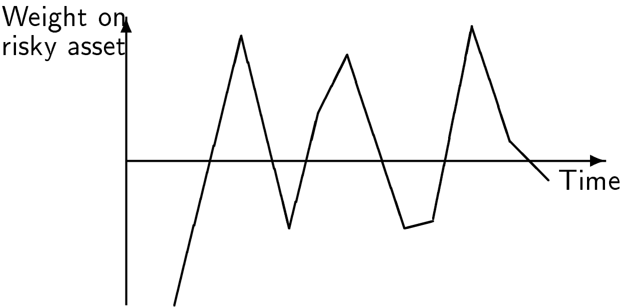
Mean-variance portfolios

- Markowitz: Investor concerned only about mean and variance of returns chooses portfolio on **efficient frontier.**



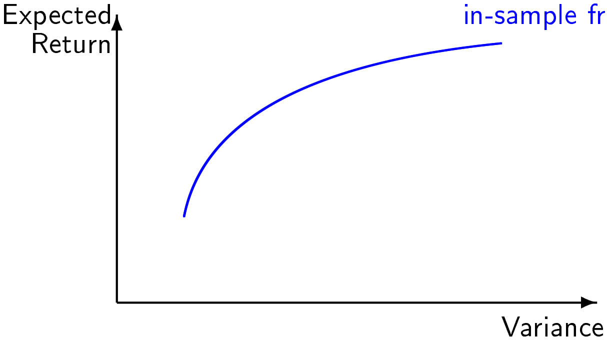
Mean-variance portfolios

- Challenge:
  - Sample estimates of mean and covariance matrix.
  - **Unstable** portfolios: **Extreme weights** that fluctuate a lot over time.
  - *"The Markowitz Optimization Enigma: Is Optimized Optimal?"* Michaud (1989)



Mean-variance portfolios

- **Minimum-variance portfolios** perform at least as good as any efficient portfolio out of sample; Jorion (1985, 1986, 1991).
  - Estimation error in mean larger than in variance; Merton (1980)
  - Jagannathan and Ma (2003): "estimation error in the sample mean is so large that nothing much is lost in ignoring the mean altogether".



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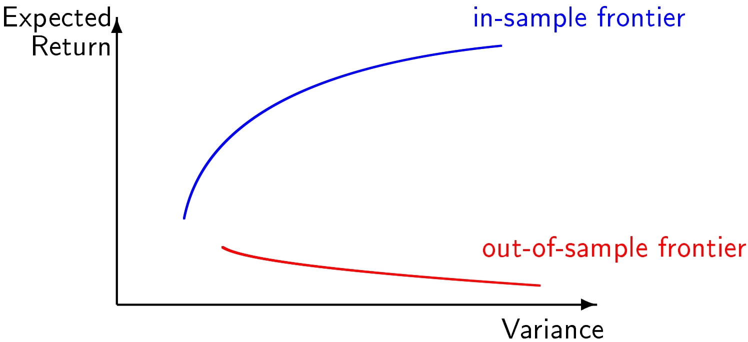

Solutions proposed in the literature

- Ignore estimates of means and focus only on covariances; that is, on minimum-variance portfolios

  Minimum-variance portfolio usually performs better out of sample than mean-variance portfolios—even when performance measure depends on variance and mean

  Jorion (1985, 1986), Jagannathan-Ma (2003), DeMiguel, Garlappi, Uppal (2007)

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- Impose short-sale constraints

  Jagannathan-Ma: "sample covariance matrix [with shortsale constraints] performs almost as well as those constructed using factor models, shrinkage estimators or daily returns."

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- Use weighted average of the sample covariance matrix and the identity matrix—Ledoit and Wolf (2004)

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- Use weighted average of the sample covariance matrix and the identity matrix—Ledoit and Wolf (2004)

- Use the $1/N$ portfolio—DeMiguel, Garlappi and Uppal (2007)

Our contribution

1. Develop a general framework for portfolio selection
    - Based on constraining the portfolio norm
    - Related to ridge regression and lasso for regression analysis
    - Show that this nests Jagannathan and Ma, Ledoit and Wolf, and $1/N$
    - Extend existing and develop new portfolios: 1-norm, 2-norm, and partial (conjugate gradient) minimum-variance portfolios
    - Provide Bayesian and moment-shrinkage interpretations for norm-constrained portfolios

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   - By minimizing portfolio variance
   - By maximizing last period portfolio return

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   - Extend existing and develop new portfolios: 1-norm, 2-norm, and partial (conjugate gradient) minimum-variance portfolios
   - Provide Bayesian and moment-shrinkage interpretations for norm-constrained portfolios
2. Demonstrate how general framework can be used to calibrate model
   - By minimizing portfolio variance
   - By maximizing last period portfolio return
3. Compare empirically out-of-sample performance of norm-constrained portfolios to 9 strategies in the existing literature for 5 datasets.

Outline

Shortsale unconstrained minimum-variance portfolio

No shortsale constraints – solution denoted by w_{MINU}

$$\min_{w} \quad w^\top \hat{\Sigma} w$$
$$\text{s.t.} \quad w^\top e = 1$$

- Sample covariance matrix may be inaccurate:
  - Requires estimating $(N^2 + N)/2$ variances and covariances.
- DeMiguel, Garlappi and Uppal (2007) show that 1/N often outperforms the shortsale-unconstrained minimum-variance portfolio.

Shortsale-constrained minimum-variance portfolio

Jagannathan and Ma (2003) study the effect of imposing shortsale constraints on the minimum-variance portfolio.

Shortsales constrained – solution denoted by w_{MINC}

$$\begin{aligned} \min_{w} \quad & w^\top \hat{\Sigma} w \\ \text{s.t.} \quad & w^\top e = 1 \\ & w \geq 0 \end{aligned}$$

Solution coincides with unconstrained minimum-variance portfolio if the sample covariance matrix $\hat{\Sigma}$ is replaced by

$$\hat{\Sigma}_{JM} = \hat{\Sigma} - \lambda e^\top - e\lambda^\top,$$

where $\lambda \in \mathcal{R}^N$ is vector of Lagrange multipliers for the constraint $w \geq 0$.

"Honey, I have shrunk the sample covariance matrix" Ledoit and Wolf (2004)

Ledoit and Wolf (2004)

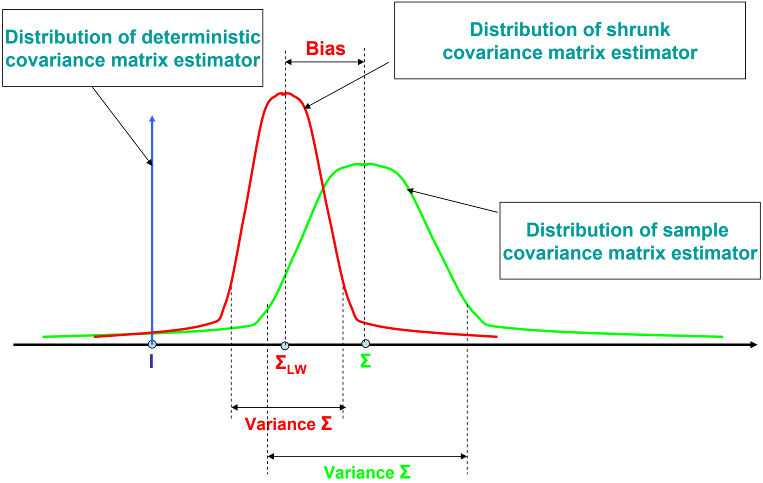
Replace sample covariance matrix $\hat{\Sigma}$ by

$$\hat{\Sigma}_{LW} = \hat{\Sigma} + \nu I,$$

where $\nu \in \mathcal{R}$ is a positive constant and $I \in \mathcal{R}^{N \times N}$ is identity matrix.

They interpret this method as shrinking the sample covariance matrix toward the identity matrix.

Shrinkage estimators



Outline

General norm-constrained minimum-variance portfolio

Solve minimum-variance problem subject to additional constraint that norm of portfolio-weight vector is smaller than a threshold δ:

General norm-constrained portfolio – solution denoted by w_{NC}

$$\begin{aligned} \min_{w} \quad & w^\top \hat{\Sigma} w \\ \text{s.t.} \quad & w^\top e = 1 \\ & \|w\| \leq \delta \end{aligned}$$

- We consider the 1-norm and the 2-norm.
- Shrink the portfolio weight vector.

First Particular Case: The 1-Norm-Constrained Minimum-Variance Portfolios

- Nest shortsale-constrained portfolios

Proposition 1

Solution to 1-norm-constrained problem with $\delta = 1$ coincides with solution to shortsale-constrained problem (Jagannathan-Ma).

- Shortsale-constrained minimum-variance portfolio can be interpreted as shrinking the portfolio weights.
- 1-norm constraint generalizes shortsale constraint—implies a shortsale budget:

$$\|w\|_1 \leq \delta \iff 1 - 2 \sum_{i \in \mathcal{N}(w)} w_i \leq \delta \iff -\sum_{i \in \mathcal{N}(w)} w_i < \frac{\delta - 1}{2}$$

Second Particular Case: The 2-norm-constrained portfolios

The 2-norm constraint can be reformulated equivalently as:

$$\sum_{i=1}^{N} w_i^2 \leq \delta \iff \sum_{i=1}^{N} \left(w_i - \frac{1}{N} \right)^2 \leq \left(\delta - \frac{1}{N} \right).$$

Thus, $1/N$ portfolio is special case of 2-norm-constrained portfolio if $\delta = 1/N$.

Proposition 2

For each $\nu \geq 0$, there exists a δ, such that the Ledoit-Wolf portfolio with shrinkage ν coincides with the 2-norm-constrained portfolio with threshold parameter δ.

Thus, Ledoit-Wolf strategy can be interpreted as shrinking portfolio weights

Third Particular Case: Partial Min-Var Portfolios

- Apply conjugate-gradient method to solve the minimum-variance problem:
  - Obtain $N - 1$ portfolios that join $1/N$ portfolio and shortsale-unconstrained minimum-variance portfolio.
- Relation to 2-norm-constrained portfolios:
  - We show that, like the 2-norm-constrained portfolios, the partial minimum-variance portfolios shrink the 2-norm of the shortsale-unconstrained minimum-variance portfolio-weight vector.
  - We show that the partial minimum-variance portfolios can be viewed as a discrete first-order approximation to the 2-norm-constrained portfolios.

Bayesian interpretation for norm-constrained portfolios

Proposition 3

1-norm-constrained portfolio maximizes the posterior likelihood if prior belief for each shortsale-unconstrained minimum-variance portfolio weights are IID distributed as a double-exponential distribution:

$$\pi(w_i) = \frac{\nu}{2} e^{-\nu |w_i|}.$$

Proposition 4

2-norm-constrained portfolio maximizes the posterior likelihood if prior belief for each shortsale-unconstrained minimum-variance portfolio weights are IID distributed as a normal distribution:

$$\pi(w_i) = \sqrt{\nu/\pi}\, e^{-\nu w_i^2}.$$

Moment-shrinkage interpretation

Proposition 5

The 1-norm-constrained portfolio is shortsale-unconstrained minimum-variance problem if sample covariance matrix, $\hat{\Sigma}$, is replaced by

$$\hat{\Sigma}_{NC1} = \hat{\Sigma} - \nu n e^{\top} - \nu e n^{\top},$$

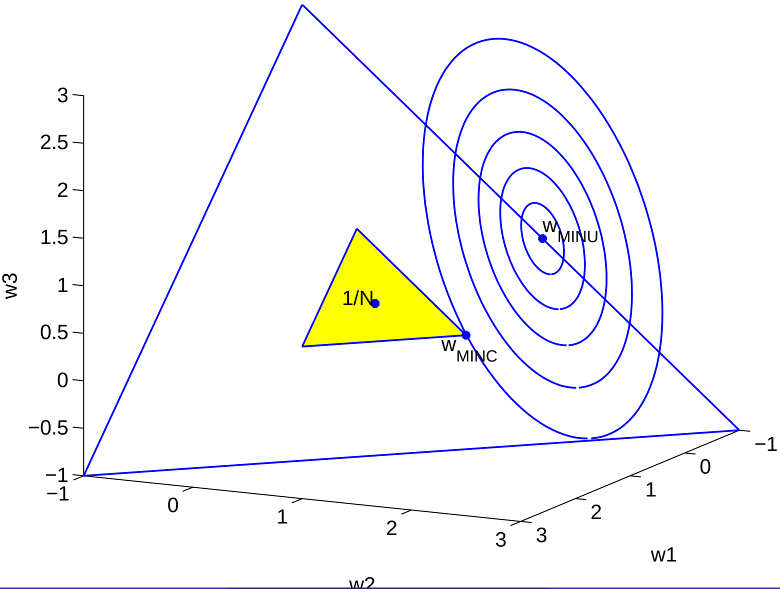
where $\nu \in \mathcal{R}$ is Lagrange multiplier of 1-norm constraint and $n \in \mathcal{R}^N$ is a vector whose i^{th} component is one if the weight on the i^{th} asset is negative and zero otherwise.

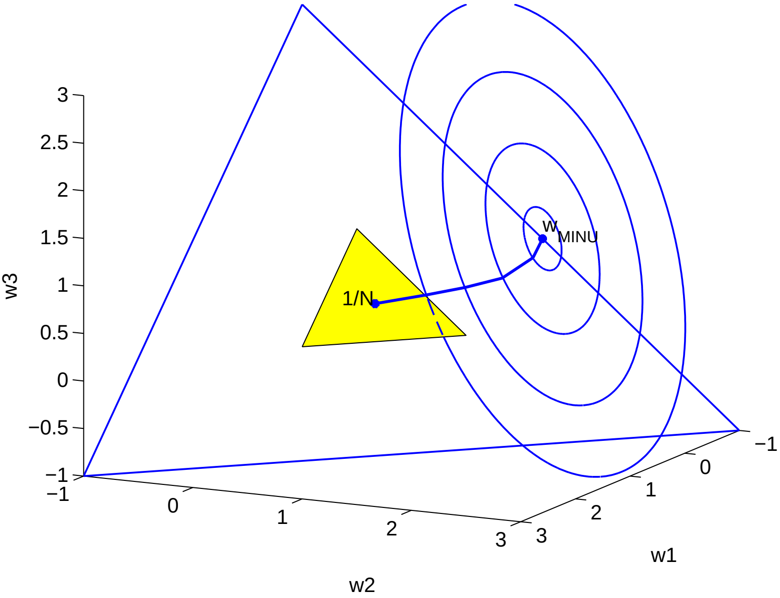
Proposition 6

The 2-norm-constrained portfolios are obtained with

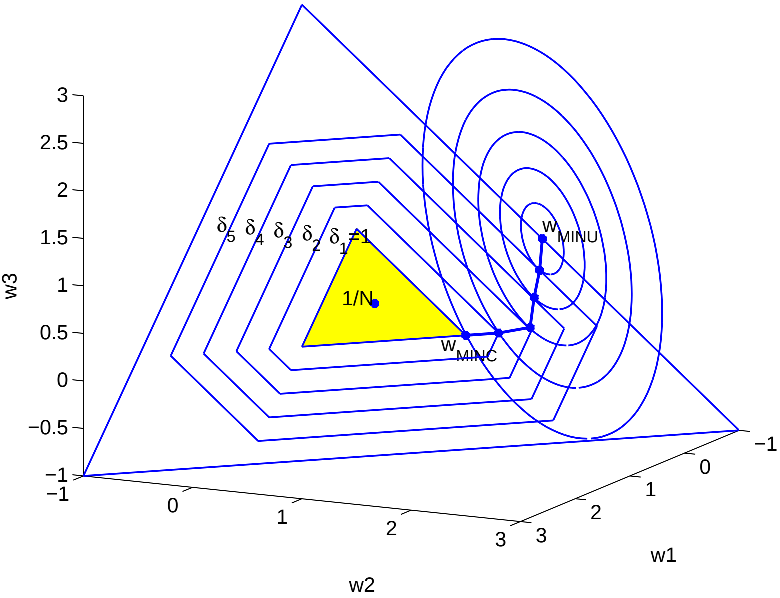
$$\Sigma_{NC2} = \left(\frac{1}{1+\nu}\right)\hat{\Sigma} + \left(\frac{\nu}{1+\nu}\right) I.$$

Geometric interpretation: shortsale-constrained portfolios

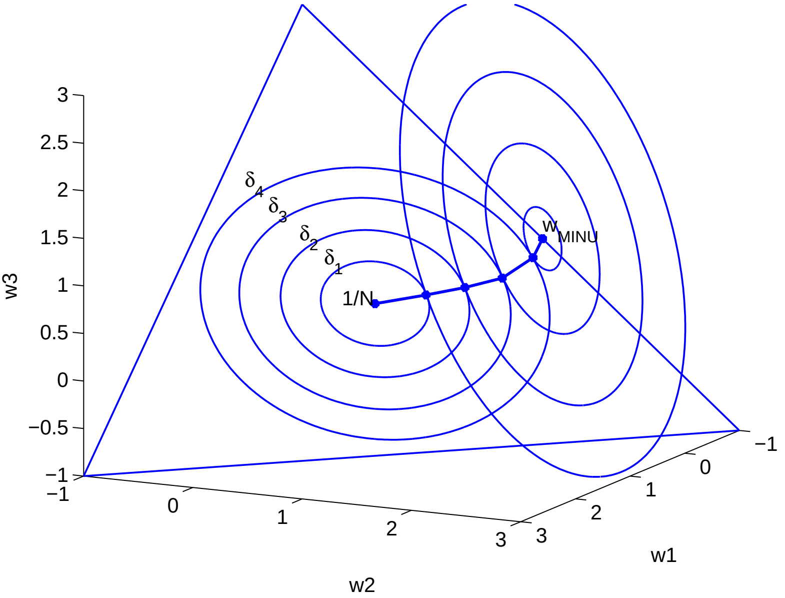



3
2.5
2
1.5
1
0.5
0
−0.5
−1
w3
1/N
w MINU
−1
0
1
2
3
w2
−1
0
1
2
3
w1

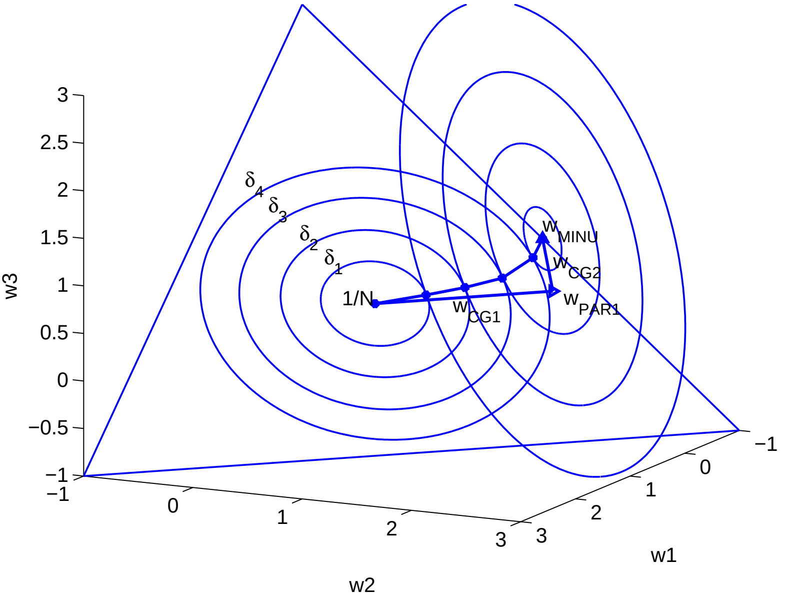
Geometric interpretation: 1-norm-constrained portfolios



Geometric interpretation: 2-norm-constrained portfolios

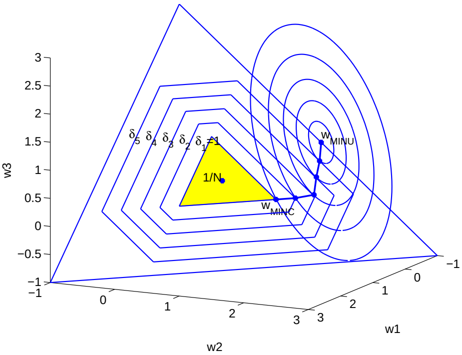


Partial minimum-variance portfolios

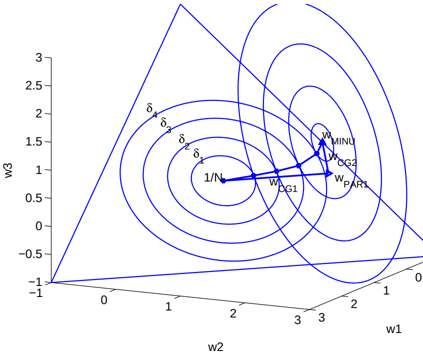


Geometric interpretation: norm-constrained portfolios

1-norm-constrained



2-norm-constrained



Outline

How To Calibrate Norm-Constrained Portfolios

- For 1- and 2-norm constrained portfolios, need to choose δ
- For partial minimum-variance portfolios, need to choose k
- Could set these exogenously
- But, can also choose them to achieve a particular objective or exploit some features of the data
  1. Minimize portfolio variance – we do this using cross validation

How To Calibrate Norm-Constrained Portfolios: Cross Validation

- Given $\tau = 120$ sample returns, for $t = 1 : \tau$:
  1. Remove t^{th} return from sample
  2. Compute sample covariance matrix from other returns
  3. Compute corresponding portfolio
  4. Compute "out-of-sample" return using t^{th} sample return
- Estimate variance as variance of 119 returns generated
- Choose parameter to minimize estimate of "out-of-sample" variance

How To Calibrate Norm-Constrained Portfolios

- For 1- and 2-norm constrained portfolios, need to choose δ
- For partial minimum-variance portfolios, need to choose k
- Could set these exogenously
- But, can also choose them to achieve a particular objective or exploit some features of the data
  1. Minimize portfolio variance – we do this using cross validation
  2. Maximize last period portfolio return to exploit momentum in portfolio returns—Campbell, Lo, and MacKinley (1997)—as opposed to momentum in individual securities returns

Table 1: List of Portfolios Considered

#	Model	Abbreviation
Panel A: Portfolio strategies developed in this paper		
1	**1-norm-constrained minimum-variance portfolio**	
	• With δ calibrated using cross-validation over portfolio variance	NC1V
	• With δ calibrated by maximizing portfolio return in previous period	NC1R
2	**2-norm-constrained minimum-variance portfolio**	
	• With δ calibrated using cross-validation over portfolio variance	NC2V
	• With δ calibrated by maximizing portfolio return in previous period	NC2R
3	**Partial minimum-variance portfolios**	
	• With k calibrated using cross-validation over portfolio variance	PARV
	• With k calibrated by maximizing the portfolio variance in the previous period	PARR
Panel B: Portfolio strategies from the existing literature used for comparison		

These are given on the next slide

Table 1: List of Portfolios Considered

	Panel B: Portfolio strategies from the existing literature used for comparison	
Simple benchmarks		
1	Equally-weighted ($1/N$) portfolio	1/N
2	Value-weighted (market) portfolio	VW
Portfolios that use mean returns		
3	Mean-variance portfolio with shortsales unconstrained	MEAN
4	Bayesian mean-variance portfolio using the approach in Jorion (1985, 1986)	BAYE
Minimum-variance portfolios that ignore mean returns		
5	Minimum-variance portfolio with shortsales unconstrained	MINU
6	Minimum-variance portfolio with shortsales constrained	MINC
7	Minimum-variance portfolio with covariance matrix as in Ledoit and Wolf (2004b)	MINL
Portfolios based on a factor model and parametric portfolios		
8	Minimum-variance portfolio with the market as the single factor	FAC1
9	Brandt, Santa-Clara, and Valkanov (2005) strategy with a risk-aversion parameter of $\gamma = 5$ using the factors Size, Book-to-Market, and Momentum	BSV

Table 2: List of Datasets Considered

#	Dataset	Abbreviation	N	Time Period	Source
1	Ten industry portfolios	10Ind	10	07/1963–12/2004	K. French
2	Forty eight industry portfolios	48Ind	48	07/1963–12/2004	K. French
3	6 Fama and French portfolios	6FF	6	07/1963–12/2004	K. French
4	25 Fama and French portfolios	25FF	25	07/1963–12/2004	K. French
5	500 randomized stocks from CRSP balanced monthly	500CRSP	500	04/1968–04/2005	CRSP

Three criteria to evaluate performance

1. Out-of-sample portfolio variance
2. Out-of-sample portfolio Sharpe ratio
3. Portfolio turnover

$$\begin{aligned}
\text{Mean} &= \hat{\mu}^k = \frac{1}{T-\tau}\sum_{t=\tau}^{T-1} \mathrm{w}_t^{k\top} r_{t+1}, \\
\text{Variance} &= (\hat{\sigma}^k)^2 = \frac{1}{T-\tau-1}\sum_{t=\tau}^{T-1}\left(\mathrm{w}_t^{k\top} r_{t+1} - \hat{\mu}^k\right)^2, \\
\text{Sharpe Ratio} &= \frac{\hat{\mu}_k}{\hat{\sigma}_k}, \\
\text{Turnover} &= \frac{1}{T-\tau-1}\sum_{t=\tau}^{T-1}\sum_{j=1}^{N}\left(\left|\mathrm{w}_{j,t+1}^k - \mathrm{w}_{j,t^+}^k\right|\right).
\end{aligned}$$

"Rolling-horizon" procedure

1. Choose window over which to do the estimation: $\tau = 120$ months
2. Compute the various portfolios using return data over τ
3. Repeat this for the next period, by including data for the new month and dropping data for the earliest month
4. Continue doing this until end of the dataset is reached
5. At the end, we have $T - \tau$ portfolio-weight vectors for each strategy
6. Compute out-of-sample return over the next month
7. Use the time series of $T - \tau$ excess returns, r_t^k, to compute the out-of-sample variance, Sharpe ratio, and turnover.
8. Compute P-values

Table 3: Portfolio Variances

Strategy	10Ind	48Ind	6FF	25FF	500CRSP
Panel A: Portfolio policies developed in this paper					
Norm-constrained portfolio policies					
NC1V	0.00134	0.00126	0.00156	0.00135	0.00074
	(0.18)	(0.00)	(0.98)	(0.28)	(0.00)
NC2V	0.00134	0.00137	0.00156	0.00130	0.00066
	(0.14)	(0.00)	(0.79)	(0.00)	(0.00)
PARV	0.00138	0.00141	0.00159	0.00133	0.00065
	(0.71)	(0.00)	(0.21)	(0.07)	(0.00)
Panel B: Portfolio policies from existing literature					

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	(0.71)	(0.00)	(0.21)	(0.07)	(0.00)
Panel B: Portfolio policies from existing literature					
Simple benchmarks					
1/N	0.00179	0.00221	0.00230	0.00249	0.00169
	(0.00)	(0.15)	(0.00)	(0.00)	(0.00)
VW	0.00158	0.00190	0.00191	0.00186	0.00157
	(0.05)	(0.90)	(0.00)	(0.00)	(0.00)

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Panel B: Portfolio policies from existing literature					
Portfolios that use mean returns					
MEAN	0.01090	0.38107	0.00353	0.00942	0.00626
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
BAYE	0.00264	0.06793	0.00221	0.00400	0.00066
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

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PARV	0.00138	0.00141	0.00159	0.00133	0.00065
	(0.71)	(0.00)	(0.21)	(0.07)	(0.00)
Panel B: Portfolio policies from existing literature					
Minimum-variance portfolio policies					
MINU	0.00138	0.00186	0.00156	0.00143	0.00104
	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)
MINC	0.00134	0.00133	0.00186	0.00176	0.00087
	(0.46)	(0.00)	(0.00)	(0.01)	(0.02)
MINL	0.00138	0.00185	0.00156	0.00143	0.00066
	(0.00)	(0.00)	(0.31)	(0.00)	(0.00)

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PARV	0.00138	0.00141	0.00159	0.00133	0.00065
	(0.71)	(0.00)	(0.21)	(0.07)	(0.00)
Panel B: Portfolio policies from existing literature					
Portfolios based on factor model and parametric portfolios					
FAC1	0.00145	0.00159	0.00201	0.00240	0.00075
	(0.33)	(0.04)	(0.00)	(0.00)	(0.01)
BSV	0.00602	0.00392	0.00306	0.00344	0.00574
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Table 4: Portfolio Sharpe Ratios

Strategy	10Ind	48Ind	6FF	25FF	500CRSP
Panel A: Portfolio policies developed in this paper					
Norm-constrained portfolio policies					
NC1R	0.2890	0.2831	0.3374	0.3553	0.3706
	(0.78)	(0.02)	(0.17)	(0.03)	(0.85)
NC2R	0.3193	0.2891	0.3922	0.4278	0.4672
	(0.21)	(0.02)	(0.31)	(0.73)	(0.01)
PARR	0.3293	0.3166	0.3912	0.4403	0.4768
	(0.10)	(0.00)	(0.29)	(0.48)	(0.01)
Panel B: Portfolio policies from existing literature					
We will see these on the next few slides					

Table 4: Portfolio Sharpe Ratios

Strategy	10Ind	48Ind	6FF	25FF	500CRSP
Panel A: Portfolio policies developed in this paper					
Norm-constrained portfolio policies					
NC1R	0.2890	0.2831	0.3374	0.3553	0.3706
	(0.78)	(0.02)	(0.17)	(0.03)	(0.85)
NC2R	0.3193	0.2891	0.3922	0.4278	0.4672
	(0.21)	(0.02)	(0.31)	(0.73)	(0.01)
PARR	0.3293	0.3166	0.3912	0.4403	0.4768
	(0.10)	(0.00)	(0.29)	(0.48)	(0.01)
Panel B: Portfolio policies from existing literature					
Simple benchmarks					
1/N	0.2541	0.2508	0.2563	0.2565	0.3326
	(0.42)	(0.50)	(0.01)	(0.00)	(0.16)
VW	0.2619	0.2698	0.2437	0.2558	0.2748
	(0.49)	(0.24)	(0.00)	(0.00)	(0.01)

Table 4: Portfolio Sharpe Ratios

Strategy	10Ind	48Ind	6FF	25FF	500CRSP
Panel A: Portfolio policies developed in this paper					
Norm-constrained portfolio policies					
NC1R	0.2890	0.2831	0.3374	0.3553	0.3706
	(0.78)	(0.02)	(0.17)	(0.03)	(0.85)
NC2R	0.3193	0.2891	0.3922	0.4278	0.4672
	(0.21)	(0.02)	(0.31)	(0.73)	(0.01)
PARR	0.3293	0.3166	0.3912	0.4403	0.4768
	(0.10)	(0.00)	(0.29)	(0.48)	(0.01)
Panel B: Portfolio policies from existing literature					
Portfolios that use mean returns					
MEAN	0.0499	-.0334	0.3214	0.2253	0.0723
	(0.00)	(0.01)	(0.37)	(0.01)	(0.00)
BAYES	0.1685	-.0121	0.3666	0.3151	0.4018
	(0.00)	(0.04)	(0.99)	(0.12)	(0.63)

Table 4: Portfolio Sharpe Ratios

Strategy	10Ind	48Ind	6FF	25FF	500CRSP
Panel A: Portfolio policies developed in this paper					
Norm-constrained portfolio policies					
NC1R	0.2890	0.2831	0.3374	0.3553	0.3706
	(0.78)	(0.02)	(0.17)	(0.03)	(0.85)
NC2R	0.3193	0.2891	0.3922	0.4278	0.4672
	(0.21)	(0.02)	(0.31)	(0.73)	(0.01)
PARR	0.3293	0.3166	0.3912	0.4403	0.4768
	(0.10)	(0.00)	(0.29)	(0.48)	(0.01)
Panel B: Portfolio policies from existing literature					
Minimum-variance portfolio policies					
MINU	0.2865	0.2222	0.3640	0.4199	0.3820
	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)
MINC	0.2852	0.2914	0.2629	0.2720	0.3985
	(0.94)	(0.04)	(0.00)	(0.00)	(0.67)
MINL	0.2865	0.2224	0.3640	0.4200	0.4028
	(0.23)	(0.36)	(0.31)	(0.76)	(0.59)

Table 4: Portfolio Sharpe Ratios

Strategy	10Ind	48Ind	6FF	25FF	500CRSP
Panel A: Portfolio policies developed in this paper					
Norm-constrained portfolio policies					
NC1R	0.2890	0.2831	0.3374	0.3553	0.3706
	(0.78)	(0.02)	(0.17)	(0.03)	(0.85)
NC2R	0.3193	0.2891	0.3922	0.4278	0.4672
	(0.21)	(0.02)	(0.31)	(0.73)	(0.01)
PARR	0.3293	0.3166	0.3912	0.4403	0.4768
	(0.10)	(0.00)	(0.29)	(0.48)	(0.01)
Panel B: Portfolio policies from existing literature					
Portfolios based on factor model and parametric portfolios					
FAC1	0.3060	0.2674	0.2485	0.2486	0.4166
	(0.31)	(0.18)	(0.00)	(0.00)	(0.53)
BSV	0.1157	0.3314	0.3908	0.4047	0.2674
	(0.00)	(0.06)	(0.57)	(0.81)	(0.38)

Table 5: Portfolio Turnovers

Strategy	10Ind	48Ind	6FF	25FF	500CRSP
Panel A: Portfolio policies developed in this paper					
Norm-constrained portfolio policies					
NC1V	0.1494	0.2680	0.1729	0.2407	0.6141
NC2V	0.1448	0.3266	0.1946	0.4570	0.5808
PARV	0.1689	0.3838	0.2600	0.4628	0.5743
NC1R	0.6013	0.8232	1.0064	0.9767	0.9753
NC2R	1.0177	2.7556	1.6594	3.6275	1.0443
PARR	1.0414	2.4846	1.6407	3.5657	1.0984
Panel B: Portfolio policies from existing literature					

Table 5: Portfolio Turnovers

Strategy	10Ind	48Ind	6FF	25FF	500CRSP
Panel A: Portfolio policies developed in this paper					
Norm-constrained portfolio policies					
NC1V	0.1494	0.2680	0.1729	0.2407	0.6141
NC2V	0.1448	0.3266	0.1946	0.4570	0.5808
PARV	0.1689	0.3838	0.2600	0.4628	0.5743
NC1R	0.6013	0.8232	1.0064	0.9767	0.9753
NC2R	1.0177	2.7556	1.6594	3.6275	1.0443
PARR	1.0414	2.4846	1.6407	3.5657	1.0984
Panel B: Portfolio policies from existing literature					
Simple benchmarks					
1/N	0.0232	0.0311	0.0155	0.0174	0.0595
Portfolios that use mean returns					
MEAN	1.0135	105.6126	0.7987	4.2495	3.0014
BAYES	0.3565	6.6314	0.5388	2.1264	0.6191
Minimum-variance portfolio policies					
MINU	0.1656	0.8286	0.2223	0.7953	0.7769
MINC	0.0552	0.0741	0.0461	0.0841	0.4222
MINL	0.1656	0.8207	0.2222	0.7935	0.6111
Factor model portfolio					
FAC1	0.0935	0.2047	0.1152	0.2398	0.3650
BSV	0.4685	0.9066	0.5381	0.5564	2.1926

Outline

Conclusion: Main Contributions

1. Provide general framework for portfolio selection
    - Based on constraining the norm of the portfolio weight vector
    - Nests Jagannathan and Ma (2003), Ledoit and Wolf (2004), $1/N$.
    - Interpretation: Bayesian, moment-shrinking, regression analysis
2. Show how to calibrate norm-constrained portfolios
3. Compare out-of-sample performance of the norm-constrained polices to 9 strategies across 5 datasets.
    - Norm-constrained portfolios outperform the ones studied in Jagannathan and Ma (2003), Ledoit and Wolf (2004), and $1/N$.
    - Perform similar to Brandt, Santa-Clara and Valkanov (2005) without relying on firm-specific characteristics.

Thank you

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Victor DeMiguel

Paper available at

http://www.london.edu/avmiguel/