# Propagation, dispersion, control and numerical approximation of waves

Enrique Zuazua

Departamento de Matemáticas, Universidad Autónoma, 28049 Madrid. Spain

In this lecture we shall discuss several topics related with numerical approximation of waves.

Control Theory is by now and old subject, ubiquitous in many areas of Science and Technology. There is a quite well-established finite-dimensional theory and many progresses have been done also in the context of PDE (Partial Differential Equations). But gluing these two pieces together is often a hard task from a mathematical point of view.

This is not a merely mathematical problem since it affects modelling and computational issues. In particular, the following two questions arise: Are finite-dimensional and infinite-dimensional models equally efficient from a control theoretical point of view? Are controls built for finite-dimensional numerical schemes efficient at the continuous level?

In this talk we shall briefly analyze these issues for the wave equation as a model example of propagation without damping. We shall show that high frequency spurious oscillations may produce the divergence of the most natural numerical schemes. This confirms the fact that finite and infinite-dimensional modelling may give completely different results from the point of view of control. We shall then discuss some remedies like filtering of high frequencies, multi-grid techniques and numerical viscosity.

Similar questions arise when building numerical approximation schemes for nonlinear Schrödinger equations.

We first consider finite-difference space semi-discretizations and show that the standard conservative scheme does not reproduce at the discrete level the properties of the continuous Schrödinger equation. This is due to high frequency numerical spurious solutions. In order to damp out these high-frequencies and to reflect the properties of the continuous problem we add a suitable extra numerical viscosity term at a convenient scale. We prove that the dispersive properties of this viscous scheme are uniform when the mesh-size tends to zero. Finally we prove the convergence of this viscous numerical scheme for a class of nonlinear Schrödinger equations with nonlinearities that may not be handeled by standard energy methods and that require the so-called Strichartz inequalities. Finally, we show that similar convergence results may be obtained by a two-grid algorithm based on the idea of resolving on a fine grid slow oscillations of the initial data and nonlinearity. This is a joint work with Liviu Ignat.