

# Workshop on Mathematics & Wild Fires



Department of Mathematics, University of Coimbra  
Coimbra, Portugal  
November 8-9, 2018

**Organising Committee:**

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In 2017, Portugal, mainly the center of Portugal, where the city of Coimbra is localised, was devastated by terrible wild fires. This tragedy has affected all the Portuguese society that is now extremely engaged in avoiding future similar and dramatic events.

Mathematics can play a significant role in predicting fire behaviour and spread, as well as, in defining suitable strategies to prevent and combat fires. Since the main activities of CIM include sponsoring events, and organising lectures for the Mathematical community, this Workshop on Mathematics & Wild Fires is an event of the utmost importance in Portugal.

This Workshop on Mathematics & Wild Fires is also a satellite event of the VIII International Conference on FOREST FIRE RESEARCH (Coimbra, Portugal, 10-16 November 2018).

We invite all researchers interested in Mathematics and Wild Fires to participate in this workshop organised by CIM in cooperation with CMUC (Centre for Mathematics of the University of Coimbra), CMAFcIO (Center for Mathematics, Fundamental Applications and Operations Research) and APMTAC (Portuguese Association of Theoretical, Applied and Computational Mechanics).

<b>Workshop Program</b>	
November 8	November 9
	9:30-10:15 Wenceslao Manteiga
	10:30-11:15 Maria Antónia Turkman
	11:15-11:45 Coffe Break
	11:45-12:30 Dominique Morvan
	Lunch
15:00-15:45 Alberto Bressan	14:30-15:15 José Bioucas-Dias
16:00-16:45 Steen Markvorsen	15:30-16:15 Mélanie Rochoux
16:45-17:15 Coffe Break	16:15-17:15 Coffe Break
17:15-18:00 Gianni Pagnini	16:45-17:30 Final Session: Challenges, Open Problems, Scientific Interaction

Alberto Bressan (Penn State University, USA)	1
Steen Markvorsen (Technical University of Denmark, Denmark)	2
Gianni Pagnini (Basque Center for Applied Mathematics, Spain)	4
Wenceslao Manteiga (Universidad de Santiago de Compostela, Spain)	6
Maria Antónia Turkman (Universidade de Lisboa, Portugal)	7
Dominique Morvan (Aix-Marseille Université, France)	9
José Bioucas-Dias (Universidade de Lisboa, Portugal)	11
Mélanie Rochoux (CECI, CNRS-CERFACS, France)	12

# Dynamic blocking problems for a model of fire confinement

**Alberto Bressan**

Department of Mathematics  
Penn State University  
University Park, PA 16802, USA  
e-mail: [axb62@psu.edu](mailto:axb62@psu.edu)

## ABSTRACT

This talk will be concerned with a new class of variational problems, seeking optimal strategies for the confinement of forest fires. The area burned by the fire at time  $t > 0$  is modeled as the reachable set for a differential inclusion  $\dot{x} \in F(x)$ , starting from an initial set  $R_0$ .

We assume that the spreading of the wild fire can be controlled by constructing barriers. In the mathematical model, these are rectifiable sets  $\gamma(t)$  whose length grows linearly in time. In practice, a barrier can be a thin strip of land which is either soaked with water poured from above (by an airplane or a helicopter), or cleared from all vegetation using a bulldozer. In any case, this will prevent the fire from crossing that particular strip of land.

A first problem that we shall address is the existence (or non-existence) of a strategy that completely blocks the spreading of the fire, within a bounded domain.

Next, we consider functions  $\alpha(x)$  describing the unit value of the land at the location  $x$ , and  $\beta(x)$  accounting for the cost of building a unit length of barrier near  $x$ . This leads to an optimization problem, where one seeks to minimize the total value of the burned region, plus the cost of building the barrier.

For these problems, one can prove the existence of optimal strategies, and derive necessary conditions for optimality. Nearly optimal strategies can be constructed by a numerical algorithm.

At the end of the talk, various open problems will be discussed. An overview of the main results, techniques, and open problems can be found in the survey paper [1].

## REFERENCES

- [1] A. Bressan, Dynamic blocking problems for a model of fire propagation. In *Advances in Applied Mathematics, Modeling, and Computational Science*, pp. 11-40, R. Melnik and I. Kotsireas editors. Fields Institute Communications, Springer, New York, 2013.

# A Finsler geometric paradigm for wildfire spread modelling

Steen Markvorsen

DTU Compute, Mathematics, Technical University of Denmark  
Kgs. Lyngby, Denmark  
e-mail: [stema@dtu.dk](mailto:stema@dtu.dk)

## ABSTRACT

In this talk we first review Gwynfor D. Richards' equations for the parametric spread of ellipsoid-borne wildfires, [1]. We generalize these equations to cover any type of (possibly time-dependent) ovaloid-borne wildfires in any dimension - dimensions 2 and 3 being the most relevant and interesting, of course. In this setting a given ovaloid at a given point in space at a given time is to be thought of as the local indicatrix, i.e. the local firelet, that is obtained by short (unit)time spread of a model fire from that point, ignited at that given time, and under the assumption of homogeneous (linearized) measures of fuel, wind, and topography. To be precise, the linearization is performed at the given point and time so that the ensuing firelet indicatrix is, geometrically speaking, molded in the tangent space of the wildfire domain. In this way we obtain an ovaloid in each tangent space at each time, i.e. a time-dependent ovaloid field on the domain of the wildfire. En passant we will briefly indicate how these ovaloids can be explicitly constructed from observations and experiments that only involve well-controlled and relatively simple line-ignited fires in the respective tangent spaces. Naturally, each ovaloid will contain the tangent space origin (the point of local ignition) in its interior, and it will usually be a strongly convex set in each tangent space. Under this assumption of strong convexity, the ovaloid field is then precisely a time-dependent indicatrix field of a time-dependent Finsler metric  $F$  on the domain  $\mathcal{U}$  under consideration. In this Finsler geometric setting the generalized Richards' equations can now be formulated as a time-dependent eikonal type Finsler-Hamilton-Jacobi equation. We show how to solve these equations – and thence the corresponding wildfire spread problem – using results from the differential geometry of geodesic sprays and/or the control geometry of differential inclusions, see e.g. [2]. Both methods are readily available and well defined directly via the Finsler metric  $F$  on the wildfire domain  $\mathcal{U}$ . In particular we will emphasise the corresponding inherited motion of what we could call fire particles (the Finsler geodesics issuing from a given ignition set) as a natural way to understand both the spread of the frontals of the fire as well as the formation of their ensuing singularities. In differential geometry the latter singularities are known as cut loci. They correspond to places where fire fighters may experience so-called bear hugs, i.e. where the fire particles, and hence the frontal, approaches from more than one direction. Obviously it is of paramount importance to know how, where, and when such cut loci are formed in each given case. Finally we will address the important problem of including the curvature of the frontal into a modification of Richards' equations, and more generally into the Finsler eikonal equation. As summarized by Sullivan in [3, p. 162, 166] a point-ignited fire will naturally increase its width but at the same time it will also increase its rate of forward spread. This latter behaviour, and not least the more significant and observed 'straightening out' of concave portions of the fire front (just before a bear hug would otherwise tend to take place), are curvature dependent phenomena, that cannot be explained by a first order eikonal equation. We show how to modify the Finsler eikonal equation mentioned above into a second order equation, which (by construction) will produce the observed initial increase in fire particle speed from point ignitions. Moreover, the modified equation has speedy fire

particle solutions that will also typically 'evaporate' the first encountered segments of the cut loci mentioned above and thence contribute to the straightening of the wildfire frontal.

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- [2] S.Markvorsen, A Finsler geodesic spray paradigm for wildfire spread modelling, *Non-linear Analysis: Real World Applications*, 28, pp. 208–228 (2016)
- [3] A. Sullivan, Fundamental processes of wildland fire behaviour. Part 2: Heat transfer and interactions, *Current Forestry Reports*, 3, pp. 150–171 (2017)

# Fire-spotting modelling for regional-scale wildfire simulators: a case study with WRF-Sfire

Gianni Pagnini

BCAM - Basque Center for Applied Mathematics  
Ikerbasque - Basque Foundation for Science  
Bilbao, Basque Country - Spain  
e-mail: [gpagnini@bcamath.org](mailto:gpagnini@bcamath.org)

## ABSTRACT

The main aim of the present research is to provide a versatile probabilistic approach for modelling fire-spotting that is also ready to be implemented as a step-by-step post-processing routine in any of existing operational regional-scale wildfire simulators. The proposed approach is based on the weighted superposition of random fronts whose fluctuations of the position are distributed according to proper densities functions including the random effects of turbulent heat transfer and fire-spotting. These fluctuations can be applied to the outputs of a generic operational simulator of wildfire propagation in a way that allows for preserving the existing structure of the simulator and independent of the definition of Rate of Spread [1]. In particular, a Gaussian density is considered for turbulence and a lognormal for the landing distance of the firebrands [2,3]. The final model results to be dependent on a number of macro- and meso-scale parameters. In this respect a global sensitivity analysis (SA) for the proposed fire-spotting model has been performed.

Such SA allows to rank input parameters by the means of the Sobol' indices for what concerns their effect on topology and size of the burned area. An extensive work of Surrogate Modeling is performed to compare the performance of the surrogates for varying size and type of training sets as well as for varying parameterization and choice of algorithms. The different databases built with different sampling techniques allow for cross-validation in order to rank the different surrogate models. The best performance is achieved using a generalized Polynomial Chaos (gPC) strategy based on a sparse least-angle regression (LAR) and a low-discrepancy Haltons sequence or a Gaussian Process strategy based on RBK kernel.

The sensitivity analysis highlights that the importance of the mean wind for the propagation of the main fire as known from literature holds true also for the generation of secondary fires through fire-spotting. Then, only a fire-atmosphere coupling system could provide the dynamics of secondary fires in agreement with the characteristics of the primary fires. For this reason, the proposed fire-spotting model has been implemented in WRF-Sfire, a full-fledged coupled fire-atmosphere model (<http://www.openwfm.org/wiki/WRF-SFIRE>). A test case has been performed and will be discussed at the workshop.

Moreover, the proposed formulation can be extended to include further variables such as moisture, spatial distribution of combustible and orography.

Work in collaboration with I. Kaur, A. Trucchia, V.N. Egorova.

**Acknowledgements:** This research is supported by the Basque Government through the BERC 2014-2017 and BERC 2018-2021 programs and by the Spanish Ministry of Economy



and Competitiveness MINECO through BCAM Severo Ochoa accreditation SEV-2013-0323 and through project MTM2016-76016-R "MIP", and by the PhD grant "La Caixa 2014".

The implementation of the fire-spotting model in WRF-Sfire is freely available at the official git repository of BCAM, Bilbao, <https://gitlab.bcamath.org/atrucchia/randomfront-wrfsfire-1sfire>.

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# Using statistics and optimization techniques to fight against wildfires in Galicia

**Wenceslao González Manteiga**

Universidad de Santiago de Compostela  
15782 Santiago de Compostela, Spain  
e-mail: [wenceslao.gonzalez@usc.es](mailto:wenceslao.gonzalez@usc.es)

## ABSTRACT

Wildfire represents an important social and environmental problem all around the world; particularly, in Galicia (NW Spain) arson fires are the main cause of forest destruction. The need of efficient fire prediction and fighting systems have enhanced a large body of research. We have applied nonparametric inference techniques for spatial and spatio-temporal point processes to understand wildfire behavior, and operations research techniques for an optimal allocation of limited resources, such as aircrafts, in fire extinction. The large number of data (more than 100.000 wildfires between 1999 and 2014), and the large number of covariates that may be involved in wildfire risk increase the computational demand and complexity of these techniques leading us to a Big Data scenario. This talk outlines the challenges found in the analysis of wildfires in Galicia and discusses the nonparametric spatio-temporal inference and operational research techniques applied.

This is joint work with members of the MODESTYA research group (Universidade de Santiago de Compostela).

**Acknowledgments:** This research received financial support from Ministerio de Economía y Competitividad of Spain through grants MTM2016-76969-P and MTM2017-87197-C3-3-P, and from ITMATI, Technological Institute of Industrial Mathematics, Santiago de Compostela, Spain, through the Enjambre project. Both are gratefully acknowledged.

# Statistical methods towards the construction of decision tools to assist wildfire management

Maria Antónia Amaral Turkman <sup>(1)</sup>, Kamil Feridun Turkman <sup>(1)</sup>, Paula Pereira <sup>(2)</sup>,  
Patrícia de Zea Bermudez <sup>(1)</sup>, Ana Sá <sup>(3)</sup>, José Miguel Cardoso Pereira <sup>(3)</sup>, Carlos  
C. Da Câmara <sup>(4)</sup>

(1) Centro de Estatística e Aplicações, Universidade de Lisboa  
Faculdade de Ciências  
Campo Grande, 1749-016 Lisboa, Portugal  
e-mail: [maturkman@fc.ul.pt](mailto:maturkman@fc.ul.pt); [kfturkman@fc.ul.pt](mailto:kfturkman@fc.ul.pt); [patzea@fc.ul.pt](mailto:patzea@fc.ul.pt)

(2) Centro de Estatística e Aplicações, Universidade de Lisboa and ESTSetúbal  
Instituto Politécnico de Setúbal  
Campus do IPS, Estefanilha, 2910-761 Setúbal, Portugal  
e-mail: [paula.pereira@estsetubal.ips.pt](mailto:paula.pereira@estsetubal.ips.pt)

(3) Centro de Estudos Florestais, Universidade de Lisboa  
Instituto Superior de Agronomia  
Tapada da Ajuda, 1349-017 Lisboa, Portugal  
e-mail: [anasa@isa.ulisboa.pt](mailto:anasa@isa.ulisboa.pt); [jmcpereira@isa.ulisboa.pt](mailto:jmcpereira@isa.ulisboa.pt)

(4) Instituto Don Luís, Universidade de Lisboa  
Faculdade de Ciências  
Campo Grande, 1749-016 Lisboa, Portugal  
e-mail: [cdcamara@fc.ul.pt](mailto:cdcamara@fc.ul.pt)

## ABSTRACT

Vegetation fires are inherently random in the timing of their location and occurrence, in the detailed behavior of each individual event, and in the particularities of their effects on soils, water, flora, fauna, and air. Therefore, substantial efforts have been directed towards statistical modeling of several fire-related processes [1]. One important fire-related process is fire likelihood or fire danger, which deals with pre-fire events and aims at predicting the probability of fire occurrence and the extent of area burned over a specific spatial area and temporal period, conditional on a fire occurrence. Data sources coming from satellite images and ground sources are point referenced and therefore are more suited in understanding the spatial point patterns of fire incidences as well as fire sizes. Ideally, the data on point patterns, should be treated as a realization of a spatio-temporal marked point process, discrete in time and continuous in space. Typically such point processes are modeled by marked Poisson point processes and the non-homogeneous intensity function together with fire size distribution of this process become the focal point of the study [2]. Typically Log Gaussian Cox processes are used for modeling point patterns whereas a variety of models, among which the generalized Pareto distribution, are often employed for modeling large fire sizes and extreme value theory is the natural inferential tool to quantify the large fire danger. Further simplifications, at the cost of additional loss of information, can be achieved by transforming the point pattern data into fire incidence data. The marks, namely sizes of the individual fires, can be aggregated into burned area fraction of each areal unit during the temporal units. The spatial support for the analysis is a regular grid, and not individual fire events and consequently large fires that span more than one grid cell will be subdivided and have their area distributed by the corresponding cells. Grid cell records for each year represent the binary data indicating the presence of at least one fire together with the corresponding burned area fraction of the grid cell. We call these binary data the fire incidence data. The burned area fraction, expressed in terms of the percentage of grid size burned each

year, is used as covariate information. Information from fire weather severity and maps of state of vegetation (green/dry) and cumulated biomass at the end of spring can be used to produce annual fire risk maps as in [3], by incorporating the strong spatial and temporal dependence that exists in the data. We will consider for the purpose a Markovian structure for the fire incidence data. The objective of this model is to capture, as much as possible, the strong spatio-temporal dependence structures in the fire incidence data, allowing at the same time for the introduction of any type of dynamic explanatory variables in the model. This will be achieved through Bayesian hierarchical modeling techniques and simulation-based inference.

The above mentioned modelling strategies are exemplified with the fire risk maps for 2018, based on satellite data of fire ignition and burned area in Portugal from 1988 to 2017, that were obtained during May 2018. These maps are considered useful tools for decision makers to allocate fire fighting capacities and to support fire/forest management decisions in space, according to the risks involved.

**Acknowledgements** This research is partially financed by (FCT) Portugal, through the projects UID/AGR/00239/2013, UID/MAT/00006/2013 and PCIF/GRF/0204/2017.

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# Wildfires physics and modelling

**Dominique Morvan**

Department of Mechanics  
Aix-Marseille Université  
60 rue Joliot Curie 13453 Marseille cedex 13, France  
e-mail: [dominique.morvan@univ-amu.fr](mailto:dominique.morvan@univ-amu.fr)

## ABSTRACT

The behavior of wildfires is governed by various physical mechanisms, at different scales in space (and time), ranging between less than  $1mm$  (the flame) to larger than  $100km$  (the plume). Many of these physical mechanisms, such as the decomposition of the vegetation into combustible gas and solid, the chemical reaction inside the flame and its interaction with the atmosphere, are nonlinear, which constitutes an additional difficulty for their predictions using numerical models. For all these reasons, the numerical simulation of wildfires is a high challenging multiscale problem. Despite these difficulties, the resolution of some problems in fire safety engineering such as the propagation of a fire front through a wildland urban interface (WUI), needs to describe a fire at a relative local scale (few hundred meters), with a relatively high level of details. It is in this context, that a new class of fire models, referred in the literature as "fully physical models", has been proposed at the end of 90's [1, 2, 3, 4]. Before developing such kind of models, it is capital to identify all scales (in space and time) associated to the physical mechanisms contributing to the ignition and the propagation of a fire through a vegetation stratum.

To avoid the complete description of the vegetation, impossible task if we consider the fractal nature of the interface between all the elements (leaves twigs ...) constituting a plant, it is represented as an equivalent porous media, characterized by a set of local physical properties such as the density, the volume fraction, the composition. Then the problem is formulated from the balance equations (mass, momentum, energy ...) of the coupled system formed by the vegetation and the surrounding atmosphere. This approach is often referred in the literature as a multiphase formulation.

The objective of this lecture, will be to identify all the physical phenomenon and the associated scales, contributing to the dynamics of a forest fire, followed by a short presentation of what is a fully physical wildfires model and a presentation of some results obtained with this kind of approach.

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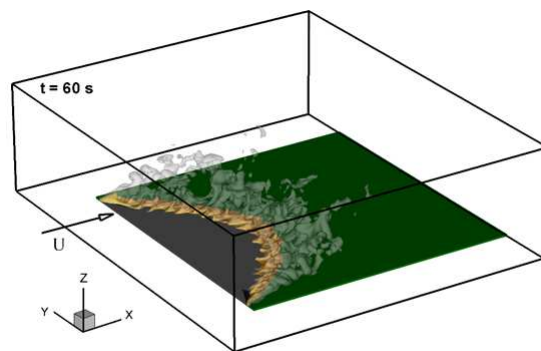


Figure 1: Example of numerical simulation of a surface fire propagating in a grassland (from Frangieh et al Fire Safety Journal 2018, in press)

# Convex and nonconvex formulations for image segmentation with applications to remote sensing

**José Bioucas-Dias**

Instituto de Telecomunicações, Instituto Superior Técnico

Universidade de Lisboa

Av. Rovisco Pais, 1049-001 Lisboa, Portugal

e-mail: [bioucas@lx.it.pt](mailto:bioucas@lx.it.pt)

## ABSTRACT

Image segmentation is fundamentally a discrete problem. It consists of finding a partition of the image domain such that the pixels in each element of the partition exhibit some kind of similarity. Very often, the partitions are obtained via integer optimization, which is NP-hard, apart from few exceptions. We sidestep the discrete nature of image segmentation by formulating the problem in the Bayesian framework and introducing a hidden set of real-valued random fields determining the probability of a given partition. Armed with this model, the original image segmentation is converted into a convex program, in the supervised scenario, and into a non-convex program, in the case of the semi-supervised scenario. To infer the hidden fields, in the convex case, we adopt the Segmentation via the Constrained Split Augmented Lagrangian Shrinkage Algorithm (SegSALSA). The non-convex case is solved via expectation maximization (EM), where the maximization step is similar to SegSALSA. The effectiveness of the proposed methodology is illustrated with simulated and real remote sensing images.

# Overview and challenges of data-driven wildland fire spread modeling

Mélanie C. Rochoux <sup>(1)</sup>, Cong Zhang <sup>(2)</sup>, Ronan Paugam <sup>(1)</sup>, Arnaud Trouvé <sup>(2)</sup>

(1) CECI, CNRS-CERFACS

Toulouse, France

e-mail: [melanie.rochoux@cerfacs.fr](mailto:melanie.rochoux@cerfacs.fr); [paugam@uw.edu](mailto:paugam@uw.edu)

(2) Department of Fire Protection Engineering, University of Maryland  
College Park, MD, USA

e-mail: [cong0129@umd.edu](mailto:cong0129@umd.edu); [atrouve@umd.edu](mailto:atrouve@umd.edu)

## ABSTRACT

Providing accurate predictions of the spread of wildland fires has long been a goal of the fire research community. Whether used as a planning tool prior to prescribed burning or as an operational tool to predict the growth of uncontrolled wildfires, the accuracy of wildland fire spread models and their ability to provide useful information in a timely manner are of paramount importance.

Despite the development of a number of models, the use of wildland fire spread modeling has been relatively limited operationally. Some of this stems from the fact that all models are by nature approximate, simplified versions of reality (the problem of wildfires is particularly complex to model due to the wide range of relevant spatial scales and to the multiple physical processes involved, ranging from biomass pyrolysis, combustion and flow dynamics to atmospheric dynamics and chemistry). Available data to initialize and parametrize these models, such as fuels, topography, and weather, are also subject to large uncertainties and limited resolution, both spatially and temporally. A new approach to this problem is to couple existing models and real-time observations, with the objective of reducing the uncertainties in both model fidelity and input data by using real-time observations of the wildland fire dynamics. This approach is called "data-driven modeling" (or "data assimilation"). Data-driven modeling thus offers to take full advantage of the recent advances in remote sensing technology to improve forecasts of the wildland fire evolution.

Data-driven wildfire spread model is still at an early stage of development. In this paper we will provide an overview of current strategies and we will highlight some of the challenges and opportunities this new data assimilation approach offers.