BULLETIN

INTERNATIONAL CENTER FOR MATHEMATICS

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CONTENTS

COMING EVENTS

January 27-28: Follow-up Workshop Mathematics and the Environment

ORGANIZERS

J. Videman (Technical University of Lisbon), J. M. Urbano (University of Coimbra).

AIMS

The objective of the Follow-up Workshop is to assess the impact of the Thematic Term "Mathematics and the Environment", two years after the event. Two mini symposia are scheduled: "Oceanography, Lakes and Rivers", and "Atmospheric Sciences and Climate Dynamics".

We aim at a rather informal atmosphere to incite the discussion about the different perspectives on the development of the subject in Portugal.

The event will be held at the Observatório Astronómico of the University of Coimbra.

INVITED SPEAKERS

for the mini symposia *Oceanography, Lakes and Rivers*:

A. dos Santos (Technical University of Lisbon)

for the mini symposia *Atmospheric Sciences and Climate Dynamics*: J. Teixeira (NATO Undersea Research Centre, La Spezia, Italy)

For more information about the event, see

www.cim.pt/wme2006

April 10-12: Aveiro Workshop on Graph Spectra

Organizers

D. M. Cardoso (University of Aveiro) (Chairman), R. Cordovil (Technical University of Lisbon), A. L. Duarte (University of Coimbra), C. J. Luz (Politechnical Institute of Setúbal), A. G. de Oliveira (University of Porto).

Aims

The theory of graph spectra is now a well established field of research in mathematics and in several applied sciences (e.g. chemistry), and many results have been published over the last few decades. In recognition of the strong developments in the subject, this workshop has been organized as a forum for the many researchers around the world.

The main goals of the workshop are to bring together the leading researchers on graph spectra and related topics, to establish the state of the art, and to discuss recent achievements and challenges. The topics include applications of graph spectra to chemistry and other branches of science.

The members of the Scientific Committee are among the renowned specialists on spectral graph theory who will deliver 10 plenary presentations. Additionally, we have planned a problem session and a few parallel contributions where international experts will be able to present their most recent results.

The event will be held at the Department of Mathematics of the University of Aveiro.

Plenary Presentations

Old and new results on algebraic connectivity of graphs
N. Abreu (Federal Univ. of Rio de Janeiro, Brazil)

Spectral radius of tournaments and bipartite graphs
R. A. Brualdi (Univ. of Wisconsin, Madison, USA)

Signless Laplacians of finite graphs
D. Cvetkovic (Univ. of Belgrade, Serbia & Montenegro)

Graph Spectra and Graph Isomorphism
C. Godsil (Univ. of Waterloo, Canada)

The Laplacian and Cheeger inequalities for directed graphs
F. C. Graham (Univ. of California, San Diego, USA)

Generalized adjacency matrices
W. H. Haemers (Tilburg Univ., The Netherlands)

Constructing graphs with integral Laplacian spectra
S. Kirkland (Univ. of Regina, Canada)

Star complements in finite graphs
P. Rowlinson (Univ. of Stirling, Scotland)

Some extremal problems for the eigenvalues of simple graphs
S. Simic (Univ. of Belgrade, Serbia & Montenegro)

Spectral characterizations of distance-regular graphs
E. van Dam (Tilburg Univ., The Netherlands)

For more information about the event, see

ceoc.mat.ua.pt/conf/graph2006

June 28-30: Workshop From Lie Algebras to Quantum Groups

Organizers

H. Albuquerque (University of Coimbra), S. Lopes (University of Porto), J. Teles (University of Coimbra).

Aims

This workshop will bring together leading specialists in the topics of Lie algebras, quantum groups and related areas. It aims to present the latest developments in these areas as well as to stimulate the interaction between young researchers and established specialists in these fields.

The school will be held at the Department of Mathematics of the University of Coimbra.

Keynote Speakers

H. Albuquerque (University of Coimbra)

G. Benkart (University of Wisconsin-Madison, USA)

A. Elduque (University of Zaragoza, Spain)

G. Lusztig (Massachusetts Institute of Tech., USA)

S. Majid (University of London, UK)

C. Moreno (University Complutense of Madrid, Spain)

M. Semenov-Tian-Shansky (Univ. Bourgogne, France)

For more information about the event, see

www.cim.pt/wlaqg

July 19-21: Mathematics in Chemistry

Organizers

J.-C. Zambrini (Univ. of Lisbon), J. M. P. Paixão (Univ. of Lisbon), F. B. Pereira (Univ. of Lisbon).

Aims

To identify and discuss research problems in the area of the chemical sciences whose development is strongly dependent on mathematical techniques. To foster the collaboration between leading researchers in chemistry and mathematics.

Chemistry is an exact science since it relies on quantitative models that can be described and applied by using the mathematical language. For instance, the theory of chemical bonding and molecular structure, rates and equilibria of chemical reactions, molecular thermodynamics, relationships involving energy, structure and reactivity, modeling of solvation, are swarming with problems whose solutions require sophisticated mathematical techniques. Mathematics also plays a central role in many areas of "applied" chemistry and chemical engineering. Important examples include atmospheric chemistry, biochemistry, and the broad field of computer simulations. The development of faster and more accurate spectroscopic techniques, the design of molecular devices, biomolecular computers, and of new empirical methods to predict reliable chemical data, and the conception of more efficient chemical reactors are just a few of a vast number of other topics that have strong links to applied mathematics. A closer interaction between chemists and mathematicians may therefore lead to significant progress in many key problems in chemistry. The proposed workshop will foster that interaction since it will identify a number of important research issues which will benefit from a joint effort.

Intended Audience are Researchers and post-graduate students on mathematics or chemical sciences.

The event will take place at Complexo Interdisciplinar of the University of Lisbon

Invited Speakers

S. Canuto (University of São Paulo, Brazil)

D. C. Clary (University of Oxford, UK)

I. Fonseca (Carnegie Mellon Univ., Pittsburgh, USA)

J. T. Hynes (Universities of Paris, France, and of Colorado, Boulder, USA)

C. Leforestier (University of Montpellier, France)

J. A. Perdew (University of New Orleans, USA)

P. Piecuch (University of Michigan, USA)

J.-L. Rivail (University of Nancy, France)

M. N. Berberan e Santos (University of Lisbon)

J. A. de Sousa (University of Lisbon)

A. Varandas (University of Coimbra)

M. Viana (IMPA, Rio de Janeiro, Brazil)

H.-J. Werner (University of Stuttgart, Germany)

J. H. Zhang (University of New York, USA)

For more information about the event, see

www.math-chem.org/home.do

September 4-8: 3rd International Workshop on Mathematical Techniques and Problems in Telecommunications

Organizers

A. Navarro (Univ. of Aveiro), C. Rocha (Tech. Univ. of Lisbon), C. Salema (Tech. Univ. of Lisbon).

Aims

The goals are three fold. Firstly, to identify and possibly find solutions for a number of mathematical problems in the field of Telecommunications. Secondly to disseminate among telecommunications engineers some mathematical techniques which are not widely known in this community even if they are being applied in modern communication techniques. Thirdly, to improve mutual understanding and recognition between mathematicians and telecommunication engineers, heavy users of mathematical techniques in the field of engineering.

The intended audience includes telecommunications engineers, mostly those providing the problems and being introduced to new mathematical tools, and mathematicians, mainly providing solutions and being introduced to real life problems that may influence the direction of their research. A strong participation of young scientists, mainly those attending undergraduate degrees is also expected.

The event will take place at the Polytechnic Institute of Leiria.

Invited speakers

Cross-Layer Issues in Wireless Networks
V. Poor (Princeton University, USA)

Encryption
J. Rosenthal (Notre Dame University, USA)

Mathematical Needs for Behavioural Modelling of Telecommunication Circuits and Systems
J. C. Pedro (University of Aveiro)

A multiobjective routing optimisation framework for multiservice networks - a heuristic approach
J. Craveirinha (University of Coimbra)

Signal Processing And Compression
C. Guillemot (INRIA, France)

For more information about the event, see

www.mtpt.it.pt

September 20-24: Summer School on Mathematics in Biology and Medicine

Organizers

J. Carneiro, F. Dionísio, G. Gomes, I. Gordo (Gulbenkian Institute of Science, Oeiras).

Aims

The aim of this event is to promote the use of mathematical modelling in biology and medicine. This will be accomplished by bringing some of international experts to give a short course on their area of expertise. The lecturing team combines researchers with a diversity of backgrounds in mathematics, biology and medicine, who will share their experience with the participants.

The school is aimed at postgraduate students from mathematics,physics, biology or medicine, who are motivated to develop biomathematical research approaches.

The role of mathematical formalisms in providing insight into biological and medical processes became apparent at the beginning of the 20th century. The approach has since increased in popularity, especially during the past 10-20 years. This new phase of expansion is, to a large degree, stimulated by new developments in molecular biology and computation. Appropriate mathematical models are in great demand in many areas of biology.

Given the high stands of mathematical and biomedical research in Portugal, it is disappointing that only a few research groups integrate the two disciplines. This can be promoted by organizing interdisciplinary activities as proposed here.

The school will include a broad range of areas in biology and medicine where mathematical modelling is established. We plan to include six courses covering several research areas such as evolution, populations genetics, epidemiology, population biology, developmental biology and immunology. Each course will consist of three lectures.

The event will take place at the Gulbenkian Institute of Science, Oeiras.

Short Courses

Epidemiology and Population Biology
S. Levin (Princeton University, USA)

Immunology
D. Coombs (University of British Columbia, Vancouver, Canada)

Developmental Biology
R. Azevedo (University of Houston, USA)

Evolutionary Biology
T. Day (Queens University, Ontario, Canada)

Population Genetics and Disease Mapping
G. McVean (University of Oxford, UK)

Neurobiology
C. Brody (Cold Spring Harbor, USA)

Other CIM events in 2006:

Working Afternoons SPM/CIM

CIM, Coimbra

A joint initiative of the Portuguese Mathematical Society (SPM) and the International Center for Mathematics (CIM). Programme for 2006:

7 January 2006 - Dynamical Systems
Organizer: José Ferreira Alves (Univ. Porto)

4 March 2006 - Statistics
Organizer: Paulo Teles (Univ. Porto)

6 May 2006 - Optimization
Organizer: Domingos Cardoso (Univ. Aveiro)

For more information, see

www.spm.pt/investigacao/spmcim/spmcim.phtml

CIM News

Annual Scientific Council Meeting 2006

Hotel Quinta das Lágrimas, Coimbra

The CIM Scientific Council will meet in Coimbra on February 11, to discuss the CIM scientific programme for 2007.

Timetable:

10:30-16:30 Scientific council working session (with lunch)

17:00 J. M. Martínez (State Univ. of Campinas, Brazil), *Lower-sum order-value optimization*

18:30 E. Zuazua (Univ. Autónoma de Madrid, Spain), *Propagation, dispersion, control, numerical approximation of waves*

20:00 Dinner

For the detailed programme and registration, see

www.cim.pt/?q=cscam06

Meeting of the General Assembly of CIM

The General Assembly of CIM will meet on May 20, 2006 in the CIM premises at the Astronomical Observatory of the University of Coimbra. The members of the General Assembly will also have the opportunity to attend two talks in that same day. We shall report on this in the next number of the Bulletin.

Research in Pairs at CIM

CIM has facilities for research work in pairs and welcomes applications for their use for limited periods.

These facilities are located at Complexo do Observatório Astronómico in Coimbra and include:

- office space, computing facilities, and some secretarial support;
- access to the library of the Department of Mathematics of the Univ. of Coimbra (30 minutes away by bus);
- lodging: a two room flat.

At least one of the researchers should be affiliated with an associate of CIM, or a participant in a CIM event.

Applicants should fill in the electronic application form in www.cim.pt/?q=research

CIM Thematic Term on Optimization

Scientific Report

Workshop on Optimization in Finance (Coimbra, July 5-8, 2005)

The Workshop on Optimization in Finance was the first of four optimization events of the CIM 2005 Thematic Term on Optimization. It took place in the Faculty of Economics of the University of Coimbra, during July 5–8, 2005. The total number of participants was 84 (and among those there were 22 Portuguese).

Optimization is one of the mathematical tools most frequently used by practitioners in Finance. Also, Optimization in Finance is an area of intense academic research. This workshop has covered most of the relevant topics in Optimization in Finance and brought to Coimbra a significant number of the best researchers in the field.

The workshop started with a short course in the first day, consisting of two lectures:

- *Optimization Problems in Pricing and Hedging Options*, by S. Herzel (Univ. Perugia, Italy) — 2h30m,
- *Robust Optimization in Finance*, by R. H. Tütüncü (Carnegie Mellon Univ., USA) — 2h30m.

There were six plenary lectures of 45m given by the following invited speakers:

- J. R. Birge (Northwestern Univ., USA),
- J. M. Mulvey (Princeton Univ., USA),
- R. T. Rockafellar (Univ. of Washington, USA),
- N. Touzi (CREST, France),
- S. Uryasev (Univ. of Florida, USA),
- S. A. Zenios (Univ. of Cyprus, Cyprus).

For a meeting of this type, there was a large number (42) of 30m contributed talks, divided in two streams of parallel sessions. The number of submissions was so high and unexpected that a considerable number of talks had unfortunately to be rejected. Among the overall 14 sessions, 6 were especially organized by A. Consiglio (Univ. Palermo, Italy), R. H. Tütüncü (Carnegie Mellon Univ., USA), H. Pham (Univ. Paris 7, France), M. Pinar (Univ. Bilkent, Turkey), and L. Zuluaga (Univ. New Brunswick, Canada).

The organizing committee was formed by A. M. Monteiro (Fac. de Economia, Univ. Coimbra, Portugal), R. H. Tütüncü (Carnegie Mellon Univ., USA), and L. N. Vicente (Dep. Matemática, Univ. Coimbra, Portugal).

Summer School on Algebraic and Geometric Approaches to Integer Programming (Lisbon, July 11-15, 2005)

The second event of the CIM 2005 Thematic Term on Optimization was the Summer School on Algebraic and Geometric Approaches to Integer Programming, which took place in the campus of the Faculty of Sciences of the University of Lisbon, during July 11–15, 2005.

The school was composed by 5 short courses (reaching a total of 33h), given by the following well-known invited lecturers:

- Alexander Barvinok (Univ. Michigan, USA): *Generating Functions for Lattice Points* (6h).
- Gerard Cornuéjols (Carnegie Mellon Univ., USA): *Geometric Approaches to Cutting Plane Theory* (8h).
- Friedrich Eisenbrand (Max-Planck-Institut, Germany): *Fast Algorithms for Integer Programming in Fixed Dimension* (6h).
- Jesus De Loera (Univ. California, Davis, USA): *Experimenting and Applying the Rational Function Method: A LattE Tutorial* (1,5h) and *Transportation Polytopes: Structure, Algorithms, and Applications to Optimization and Statistics* (4,5h).

- Robert Weismantel (Otto-von-Guericke Univ., Magdeburg, Germany): *The Integral Basis Method and Extensions* (7h).

There were 66 attendees, 17 of them Portuguese and 42 of them PhD students. The school was organized by Miguel Constantino, Luís Gouveia, João Telhada (Fac. de Ciências, Univ. Lisboa, Portugal) and Robert Weismantel (Otto-von-Guericke Univ., Magdeburg, Germany).

Workshop on Optimization in Medicine (Coimbra, July 20-22, 2005)

It took place in the Institute of Biomedical Research in Light and Image of the University of Coimbra, during July 20–22, 2005, the Workshop on Optimization in Medicine, which was the third event of the CIM 2005 Thematic Term on Optimization.

The workshop was jointly organized by C. J. S. Alves (Dep. Matemática, IST), A. L. Custódio (Dep. Matemática, FCT-UNL), P. M. Pardalos (Univ. Florida, USA), and L. N. Vicente (Dep. Matemática, Univ. Coimbra).

The workshop was structured around 7 plenary talks, given by the following invited speakers:

- M. C. Ferris (Univ. of Wisconsin, USA),
- H. W. Hamacher (Univ. of Kaiserslautern, Germany),
- L. D. Iasemidis (Arizona State Univ., USA),
- A. K. Louis (Univ. of Saarbrücken, Germany),
- J. P. Kaipio (Univ. of Kuopio, Finland),
- E. K. Lee (Georgia Institute of Technology, USA),
  A. Rangarajan (Univ. of Florida, USA).

The program was complemented by two sessions especially organized by A. Sofer (George Mason Univ., USA) and T. Ha-Duong (Univ. Tech. Compiègne, France) and by three sessions of contributed talks, representing a total of 19 (30m long) talks. There were 13 Portuguese among the 44 participants.

The workshop covered applications of Optimization in Medicine. It was the first international event of its kind and counted with most of the best international researchers in the field. It was possible to treat the role of Optimization in Medicine in a comprehensive way, for a clear benefit of the participants.

Workshop on PDE Constrained Optimization (Tomar, July 26-29, 2005)

The CIM 2005 Thematic Term on Optimization came to an end with the Workshop on PDE Constrained Optimization, which took place in the Hotel dos Templários, Tomar, during July 26–29 2005, with the local support of the Polytechnical Institute of Tomar.

The scientific program started with a short course, with the duration of 5h, titled *Introduction to the Theory and Numerical Solution of PDE Constrained Optimization Problems*. The invited lecturers of this short course were M. Heinkenschloss (Rice Univ., USA) and F. Tröltzsch (Tech. Univ. Berlin, Germany). The next three days of the workshop included seven plenary talks of 60m, given by the following invited speakers:

- M. D. Gunzburger (Florida State Univ., USA),
- R. H. W. Hoppe (Univ. of Augsburg, Germany),
- K. Kunisch (Univ. of Graz, Austria),
- G. Leugering (Univ. of Erlangen-Nuernb., Germany),
- A. T. Patera (MIT, USA),
- R. Rannacher (Univ. of Heidelberg, Germany),
- E. W. Sachs (Univ. of Trier, Germany).

Three sessions were especially organized by B. Mohammadi (Univ. Montpellier, France), E. Casas (Univ. Cantabria, Spain), and M. Hintermueller (Univ. Graz, Austria), yielding a total of 11 talks of 30m. The scientific program included 14 other (20m long) contributed talks scheduled in 4 different sessions.

It was relatively large the spectrum of topics treated in the area of Optimization governed by Partial Differential Equations. Some of the best world experts in finite element methods, optimal control, and nonlinear optimization were present in the workshop.

The members of the organizing committee were L. M. Fernandes (Escola Superior de Tecnologia de Tomar, Portugal), M. Heinkenschloss (Rice Univ., USA), and L. N. Vicente (Dep. Matemática, Univ. Coimbra, Portugal). There was a total of 50 participants (including 11 Portuguese).

Luís Nunes Vicente (University of Coimbra)

International Conference On Semigroups And Languages

Scientific Report

The International Conference on Semigroups and Languages, in honour of the 65th birthday of Donald B. McAlister, was organized within the activities of the International Center for Mathematics (CIM) and of the Center for Algebra of the University of Lisbon (CAUL). The Conference was held at CAUL, Av. Prof. Gama Pinto 2, Lisbon, from July 12 to 15, 2005.

The Conference was a great success and the attendance exceeded the expectations: there were 82 participants from whom 33 were Portuguese and the others were from 15 foreign countries (Canada, Czech Republic, Estonia, Finland, France, Germany, Hungary, Israel, Japan, Latvia, Mozambique, Russia, Spain, United Kingdom, USA).

The scientific programme included 13 invited lectures of 50 minutes presented by J. Almeida (Univ. of Porto), R. Gilman (Stevens Institute of Technology), M. Lawson (Heriot-Watt Univ.), D. McAlister (Northern Illinois Univ.), D. Munn (Univ. of Glasgow), F. Otto (Univ. of Kassel), J.-E. Pin (Univ. Paris 7), P. Silva (Univ. of Porto), B. Steinberg (Carleton Univ.), M. Szendrei (Univ. of Szeged), D. Thérien (McGill Univ.), M. Volkov (Ural State Univ.) and P. Weil (Univ. Bordeaux I) as well as 31 talks of 25 minutes.

The talks were dedicated to a variety of subjects considered of great importance to the study of Semigroups and Languages, two areas intrinsically connected, namely in the finite case through Eilenberg's Theory. Some of the talks were specially dedicated to showing how McAlister's work continues to influence Semigroup Theory.

The proceedings, subjected to a refereeing system, will be published by World Scientific.

The social programme included a visit to Palmela's Castle followed by a dinner, on July 15, on the occasion of D. McAlister's birthday.

The organizers express their thanks to the City Hall of Lisbon, the Center for Algebra of the University of Lisbon, the International Center for Mathematics, the Interdisciplinary Complex of the University of Lisbon, the Portuguese Council of University Rectors, the Faculty of Science of the University of Lisbon, the Foundation of the University of Lisbon, the Luso-American Foundation, the Portuguese Foundation for Science and Technology, the International Relations Unit of the Portuguese Ministry for Science and Higher Education, the Blue Coast Tourism Board.

The Organizing Committee: Jorge M. André (FCTUNL/CAUL), Mário Branco (FCUL/CAUL), Vitor Hugo Fernandes (FCTUNL/CAUL), John Fountain (University of York), Gracinda M.S. Gomes (FCUL/CAUL), John Meakin (University of Nebraska)

Workshop on Statistics in Genomics and Proteomics

Scientific Report

The *Workshop on Statistics in Genomics and Proteomics* took place in Monte Estoril, Portugal, from 5-8 October 2005.

Genomics and proteomics aim to identify biomarkers that can answer specific clinical questions. The most obvious are markers that can be used for diagnosis and prognosis. Another important issue is to predict a patient's response to a specific drug. Diagnostic markers can themselves be candidates for drug targets. Therefore Pharmaceutical companies pursue genomics and proteomics to identify markers that predict toxicity of candidate drugs. They also investigate biological interactions between all small organic molecules and construct metabolomic networks using multivariate approaches. Researchers from several areas are involved in this process, from the identification of the problems, realization of adequate experiments, collection of data, interpretation of results, etc. The analysis of such amount of data offer real challenges to statisticians. By joining their efforts with geneticists, biologists, and computer scientists, statisticians can be of great help in all this process.

There has been in Portugal a growing interest among statisticians to cooperate with researchers in these areas. The organization of this event brought fruitful discussions among statisticians and non-statisticians and we hope that will have a great impact for future collaboration and joint research.

The workshop, organized under the auspices of CIM and the *Center of Statistics and its Applications* (CEAUL, www.ceaul.fc.ul.pt), brought together leading researchers in the areas of statistics in genomics and proteomics, who described the state of the art and presented several challenging problems for researchers in Biostatistics and Bioinformatics.

This workshop, organized by Wolfgang Urfer, Antónia Amaral Turkman, Lisete Sousa , Luzia Gonçalves and Feridun Turkman, had the participation of 7 keynote speakers and 5 invited speakers, covering the following topics

- Ruedi Aebersold - *Challenges in Data Analysis and Statistical Validation*
- Chris Cannings - *Random Networks in Genetics*
- Dirk Husmeier - *Detecting Mosaic Structures in DNA Sequence Alignments*
- Sophie Schbath - *Statistical Problems Arising in Physical Mapping*
- Terry Speed - *Probabilistic Modelling of Tandem Mass Spectrometry Data*
- Korbinian Strimmer - *Small Sample Statistical Modeling and Inference of Genetic Networks*
- Simon Tavaré - *Statistical Issues for Expression Analysis of Illumina Bead-Based Microarrays*
- Margarida Amaral - *Genomic and Proteomics Approaches to Study the Genetic Disease Cystic Fibrosis*
- Pedro Fernandes - *Systems Biology Approaches Based on Biological Information*
- Mário Silva - *Information Integration Of Biological Data Sources*
- Rogério Tenreiro - *Phylogenies, Genome Organization and Taxonomy in Prokaryotes: Dream or Reality?*
- Libia Zé-Zé - *Physical and Genetic Mapping in Whole Genome Sequencing Era: an Overview*

The idea of calling two speakers to approach the same problem, one from the statistical point of view and the other from the biological point of view, was quite appreciated and was very successful, particularly in themes such as “Microarray Data Analysis” (Simon Tavaré versus Margarida Amaral), “Physical Mapping” (Sophie Schbath versus Libia Zé-Zé) and “Modelling Mass Spectrometry Data” (Terry Speed versus Ruedi Aebersold).

The first Keynote Speaker, Simon Tavaré.

Apart from the invited talks there were also 12 contributed papers and 17 posters.

A selection of papers (5) will appear in a special issue of REVSTAT - *Statistical Review* and another selection of refereed papers (8) will appear in an edition of CIM in the form of Proceedings.

There were altogether 70 participants falling in the categories such as

- Young researchers, at the M.Sc. and Ph.D. levels, in the areas of Mathematics, Probability and Statistics, Genetics, Biology, Health Sciences, Bioinformatics. The organization offered several grants for research students to participate.
- Applied Statisticians who were interested in facing new challenges in the field of applications.
- Researchers inside and outside the academic life, related to the research fields of the workshop, who were interested in evaluating how Statisticians can help them providing answers to problems in their field.

The organizing committee would like to express their deep gratitude to all the invited speakers for their contribution to the high scientific standards of the *Workshop on Statistics in Genomics and Proteomics.*

Antónia Turkman (on behalf of the Organizing Committee)

Feature Article

Convolution operators on intervals and their use in diffraction theory

L.P. Castro and F.-O. Speck

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and

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Instituto Superior Técnico, UTL, Portugal

1 Introduction

Classically speaking, convolution operators $\mathscr{W}$ on intervals Ω are one-dimensional linear integral operators where the integration kernels depend on the difference of the arguments and the domain of integration as well as the range of the independent variable are given by the same interval:

$$\mathscr{W}\varphi(x) = c\varphi(x) + \int_\Omega k(x-y)\varphi(y)\,dy = f(x), \quad x \in \Omega$$

Ω may be bounded or semi-infinite, or even consist of a union of intervals. Here and in various other papers $\mathscr{W}$ is briefly called "convolution type operator", although this name stands sometimes for the wider class of convolution type operators with variable coefficients, considered in the Lebesgue spaces $L^p(\Omega)$, for instance.

In applications, the composition with differential operators is very important, then naturally considered in Sobolev spaces, Bessel potential spaces, etc. This leads us rapidly to distribution theory and the world of pseudo-differential operators, to admit an adequate generality of settings. However we shall consider in this article only convolution operators on intervals in spaces of Bessel potentials in order to demonstrate some new important aspects of their theory and use for applications in mathematical physics, explained in the context of certain diffraction problems.

The crucial key of our recent approach is the study of operator relations (presented in the form of operator matrix identities) between the operators associated with linear boundary value problems and their boundary integral (or pseudo-differential) equations. Particular interest is devoted to the construction of relations of "very good quality" that allow explicit analytical solution and, e.g., an exact description of their singularities.

2 Convolution type operators

2.1 The general setting

We start giving the formal definition of the class of operators that we shall consider. These are the so-called *convolution type operators*

$$\mathscr{W}_{\Phi_{\mathscr{A}},\Omega} = r_\Omega \mathscr{A}_{|\widetilde{H}^{r,p}(\Omega)} : \widetilde{H}^{r,p}(\Omega) \to H^{s,p}(\Omega)\,, \qquad (2.1)$$

acting between Bessel potential spaces, where $r, s \in \mathbb{R}$, $p \in]1,\infty[$, Ω is a finite interval or a half-line, $\mathscr{A}$ denotes a bounded, translation invariant operator from $H^{r,p}(\mathbb{R})$ into $H^{s,p}(\mathbb{R})$ (which can therefore be represented as a distributional convolution due to *Hörmander's theorem*) and r_Ω stands for the restriction of distributions from $\mathscr{S}'(\mathbb{R})$ to Ω. More precisely we have

$$\Omega =]0,a[\quad \text{or} \quad \Omega =]0,+\infty[\qquad (0 < a < +\infty) \qquad (2.2)$$

$$\mathscr{A}\varphi = \mathscr{K} * \varphi = \mathscr{F}^{-1}\Phi_{\mathscr{A}} \cdot \mathscr{F}\varphi, \qquad \varphi \in \mathscr{S} \qquad (2.3)$$

$$H^{r,p} = H^{r,p}(\mathbb{R}) = \Lambda^{-r}L^p = \mathscr{F}^{-1}\lambda^{-r}\cdot\mathscr{F}L^p(\mathbb{R}) \qquad (2.4)$$

$$\widetilde{H}^{r,p}(\Omega) = \left\{\varphi \in H^{r,p} : \operatorname{supp}\varphi \subset \overline{\Omega}\right\}, \; H^{s,p}(\Omega) = r_\Omega H^{s,p}\,.$$

As usual, for $1 \le p < +\infty$, $L^p(\mathbb{R})$ denotes the Lebesgue space of all measurable functions ϕ on $\mathbb{R}$ with finite norm

$$\|\phi\|_{L^p(\mathbb{R})} = \left(\int_\mathbb{R} |\phi(y)|^p\,dy\right)^{1/p}.$$

$\mathscr{S} = \mathscr{S}(\mathbb{R})$ denotes the Schwartz space of rapidly decreasing smooth functions, $\mathscr{S}'$ the dual space of generalized functions of slow growth, and $\mathscr{K}*$ indicates a convolution (operator) with $\mathscr{K} \in \mathscr{S}'$. $\mathscr{F}$ is the Fourier transformation first of all defined by

$$\mathscr{F}\phi(\xi) = \int_{\mathbb{R}} \phi(x) \exp(ix\xi)\, dx, \qquad \xi \in \mathbb{R},$$

as a bijection from $\mathscr{S}$ onto $\mathscr{S}$, and secondly extended to the much larger set of distributions in $\mathscr{S}'$ due to the rule

$$\langle \mathscr{F}u, \phi\rangle = \langle u, \mathscr{F}\phi\rangle, \qquad u \in \mathscr{S}', \quad \phi \in \mathscr{S}$$

(where $\langle v, \varphi\rangle$ is the value of the functional $v \in \mathscr{S}'$ on $\varphi \in \mathscr{S}$).

We would like to point out that this last distributional definition of $\mathscr{F}$ is very useful for making a great range of calculations legal and, additionally, for providing a uniform definition of the Bessel potential spaces $H^{r,p}$ in (2.4), for all real values of smoothness orders r and $1 < p < \infty$, due to the use of the Bessel potential operator $\Lambda^{-r} = \mathscr{F}^{-1}\lambda^{-r}\cdot\mathscr{F}$, where $\lambda(\xi) = (\xi^2+1)^{1/2}$ for $\xi \in \mathbb{R}$.

$H^{r,p}$ is equipped with the norm of the corresponding functions in L^p, according to (2.4), $\widetilde{H}^{r,p}(\Omega)$ with the subspace topology and $H^{s,p}(\Omega)$ with the quotient space topology, respectively. Multi-indexed spaces ($r, s \in \mathbb{R}^n$) can be considered by analogy, as well as scales of Sobolev-Slobodečkiĭ spaces. For the above indicated range of indices, all those spaces are Banach spaces.

In formula (2.3) (sometimes called *convolution theorem*), $\Phi_{\mathscr{A}} \in L^\infty_{loc}(\mathbb{R})$ is known as the *Fourier symbol* of the convolution operator $\mathscr{A} = \mathscr{K}*$.

Starting with a distribution $\phi \in \mathscr{S}'$, we are able to write a convolution operator characterized by ϕ as $\mathscr{F}^{-1}\phi\cdot\mathscr{F} : \mathscr{S} \to \mathscr{S}'$. The set of Fourier symbols ϕ for which $\mathscr{F}^{-1}\phi\cdot\mathscr{F}$ has a bounded extension $\mathscr{F}^{-1}\phi\cdot\mathscr{F} : L^p(\mathbb{R}) \to L^p(\mathbb{R})$ is usually denoted by $\mathscr{M}^p$, and their elements are called L^p*–Fourier multipliers*. The set $\mathscr{M}^p$ endowed with the norm $\|\phi\|_{\mathscr{M}^p} = \|\mathscr{F}^{-1}\phi\cdot\mathscr{F}\|_{\mathscr{L}(L^p(\mathbb{R}))}$, and point-wise multiplication, forms a Banach algebra. Knowing these facts, and considering the influence of the smoothness orders in the spaces in (2.1), it is possible to conclude that if $\lambda^{s-r}\phi \in \mathscr{M}^p$ then $\mathscr{W}_{\Phi_{\mathscr{A}},\Omega}$ is a well defined and bounded (linear) operator.

2.2 About the case $\Omega = \mathbb{R}_+ =]0, +\infty[$

The probably best known convolution type operator $\mathscr{W}_{\Phi_{\mathscr{A}},\Omega}$ is that one for which $r = s = 0$ and $\Omega = \mathbb{R}_+$. In this case we are working with the famous *Wiener-Hopf operators* acting between Lebesgue spaces. These operators received their name due to the pioneering work of Norbert Wiener and Eberhard Hopf [36] about the study of integral equations of the form

$$\mathscr{W}_{c+\mathscr{F}k,\mathbb{R}_+}\varphi(x) := \\ c\varphi(x) + \int_0^\infty k(x-y)\varphi(y)\, dy = f(x)\,, \quad x \in \mathbb{R}_+\,, \tag{2.5}$$

for an unknown φ from $L^p(\mathbb{R}_+)$ where $f \in L^p(\mathbb{R}_+)$ is arbitrarily given and $c \in \mathbb{C}$ and $k \in L^1(\mathbb{R})$ are fixed and known. The *Wiener algebra* is defined by

$$\mathbb{W} = \left\{\phi \,:\, \phi = c + \mathscr{F}k\,,\ c \in \mathbb{C}\,,\ k \in L^1(\mathbb{R})\right\}$$

which is a Banach algebra when endowed with the natural norm $\|c + \mathscr{F}k\|_{\mathbb{W}} = |c| + \int_{\mathbb{R}} |k(y)|\, dy$ and the usual multiplication operation. The Wiener algebra is a subalgebra of $\mathscr{M}^p$.

For those who had the opportunity to read last year's *Feature Article* "Mathematical Finance - a glimpse from the past challenging the future" in CIM Bulletin 17, we would like to note that the Norbert Wiener mentioned there is the same person to whom we are referring here. In fact, Norbert Wiener appeared already in 1921 with a work about Brownian motion. Later on, he moved by invitation of his engineering colleagues to the *MIT* where he generalized his work on Browian motion to more general stochastic processes. This in turn led him to study harmonic analysis around 1930. In this way, his work on generalized harmonic analysis led him to study *Tauberian theorems* in 1932, and his contributions on this topic won the Böcher Memorial Prize in 1933 (a prize awarded in memory of Professor Maxime Böcher) [15].

Wiener-Hopf operators have a similar structure as the so-called *Toeplitz operators*. Therefore they are often studied together, see the famous work of Mark Kreĭn [19]. The basic result for L^p spaces reads as follows.

Theorem 2.1 (Kreĭn [19]). *The Wiener-Hopf operator $\mathscr{W}_{c+\mathscr{F}k,\mathbb{R}_+}$ in (2.5), acting between L^p spaces, is one-sided invertible if and only if*

$$c + (\mathscr{F}k)(\xi) \neq 0\,, \quad \text{for all } \xi \in \mathbb{R}\cup\{\infty\}\,. \tag{2.6}$$

Moreover, if (2.6) holds true, then $\mathscr{W}_{c+\mathscr{F}k,\mathbb{R}_+}$ is invertible, only left-sided invertible or only right-sided invertible in case of the integer $\varkappa = \operatorname{wind}(c + \mathscr{F}k)$ being zero, positive or negative, respectively.

Additionally, under the assumption (2.6), it follows that

$$\dim\ker \mathscr{W}_{c+\mathscr{F}k,\mathbb{R}_+} = \max\{-\varkappa, 0\}$$
$$\dim \operatorname{coker} \mathscr{W}_{c+\mathscr{F}k,\mathbb{R}_+} = \max\{\varkappa, 0\}\,.$$

In the last theorem the notation $\operatorname{wind}\phi$ refers to the winding number of the graph of ϕ. As the name suggests, that is the number of windings around the origin carried out by the point $\phi(\xi)$ when ξ runs from $-\infty$ to $+\infty$.

Knowledge about the kernel and co-kernel of an operator belongs to the so-called *Fredholm theory* of this operator. By definition, an operator with closed image and with a finite dimensional kernel and co-kernel is called a *Fredholm operator*. For Fredholm operators, the notion of Fredholm index is important. The *Fredholm index* of such an operator is the difference between the dimension of the kernel and the dimension of the co-kernel.

A matrix version of the last theorem was obtained by Gohberg and Kreĭn [14]. Instead of (2.6) a corresponding condition for the determinant of the matrix Fourier symbol characterizes the Fredholm property of the Wiener-Hopf operator $\mathscr{W}$, which is then not automatically one-sided invertible, but an index formula holds still true:

$$\dim\ker\mathscr{W} - \dim\,\mathrm{coker}\,\mathscr{W} \;=\; \mathrm{wind}\det(cI+\mathscr{F}k)\,.$$

In the last decades great advances were made in the Fredholm study of Wiener-Hopf operators, for much more complicated classes of Fourier symbols than the Wiener algebra. An example is the class of piecewise continuous functions for which, in the Fredholm case, an auxiliary new function can be constructed by filling up the gaps in the graph of the symbol that correspond with its discontinuities. Such extensions of the initially non-closed graphs are found with the help of some well defined arcs so that the winding number of the resulting continuous curve provides also the value for the Fredholm index of the corresponding initial Wiener-Hopf operator (following therefore the spirit of the above theorem of Kreĭn). As expected, the shape of the arcs depend on the integrability index p. For details see the work of Roland Duduchava [11].

2.3 About the case $\Omega =]0, a[$

In view of the above background, the idea to treat the more difficult case of $\Omega =]0, a[$ consists of reducing it somehow to the situation where $\Omega = \mathbb{R}_+$. That is why one of the objectives in the theory of convolution type operators is to construct operator relations between convolution type operators with $\Omega \neq \mathbb{R}_+$ and others with $\Omega = \mathbb{R}_+$, although both structures are of great difference in general.

Due to the interest originating from mathematical physics applications, several papers were directly devoted to the study of the Fredholm property, index formulas, and invertibility of $\mathscr{W}_{\Phi_{\mathscr{A}},\Omega}$, for the case where Ω is a finite interval; see for instance the work of M. P. Ganin [13], B. V. Pal'cev [22, 23], V. Yu. Novokshenov [29], Yu. I. Karlovich, I. M. Spitkovsky [17, 18], M. A. Bastos, A. F. dos Santos [3], M. A. Bastos, A. F. dos Santos, R. Duduchava [4] and L. P. Castro, F.-O. Speck [8]. While the early work of M. P. Ganin is written in a classical form focusing explicit solution in certain special cases and reduction to Riemann-Hilbert boundary value problems on $\mathbb{R}$, most of the recent work is based on the construction of an *algebraic equivalence after extension relation*, see [20, 21] and [4] as well,

$$\begin{bmatrix} \mathscr{W}_{\Phi_{\mathscr{A}},\Omega} & 0 \\ 0 & I_Y \end{bmatrix} = E \begin{bmatrix} \mathscr{W}_{\Phi,\mathbb{R}_+} & 0 \\ 0 & I_Z \end{bmatrix} F \tag{2.7}$$

with a matrix Wiener-Hopf operator $\mathscr{W}_{\Phi,\mathbb{R}_+}$ according to the previous case where $\Omega = \mathbb{R}_+$, i.e. to find, beside of $\mathscr{W}_{\Phi,\mathbb{R}_+}$, additional Banach spaces Y, Z and invertible linear operators E, F acting between dense subspaces of the corresponding direct topological sums such that (2.7) holds. If E and F are homeomorphisms, (2.7) is said to be a *(topological) equivalence after extension relation* and we write in this case

$$\mathscr{W}_{\Phi_{\mathscr{A}},\Omega} \quad \overset{*}{\sim} \quad \mathscr{W}_{\Phi,\mathbb{R}_+}. \tag{2.8}$$

This is equivalent to the fact that $\mathscr{W}_{\Phi_{\mathscr{A}},\Omega}$ and $\mathscr{W}_{\Phi,\mathbb{R}_+}$ are *matricially coupled* and $\mathscr{W}_{\Phi,\mathbb{R}_+}$ is an *indicator* for $\mathscr{W}_{\Phi_{\mathscr{A}},\Omega}$, see [1, 2]. In fact, it is known from [2, Theorem 1] that two (general) bounded linear operators acting between Banach spaces, T and S, are (topologically) equivalent after extension if and only if they are matricially coupled, that is, if and only if there are additional operators T_j and S_j (with $j = 0, 1, 2$) so that

$$\begin{bmatrix} T & T_2 \\ T_1 & T_0 \end{bmatrix} \;:\; X_1 \oplus Y_2 \to X_2 \oplus Y_1$$

$$\begin{bmatrix} S_0 & S_1 \\ S_2 & S \end{bmatrix} \;:\; X_2 \oplus Y_1 \to X_1 \oplus Y_2$$

are bounded invertible linear operators satisfying

$$\begin{bmatrix} T & T_2 \\ T_1 & T_0 \end{bmatrix}^{-1} = \begin{bmatrix} S_0 & S_1 \\ S_2 & S \end{bmatrix}. \tag{2.9}$$

The notion of matricial coupling was introduced and used in [1], already within the spirit of finding solutions for integral equations, and we can also find some of the roots of this notion in the early work of Allen Devinatz and Marvin Shinbrot [10].

Since 1984 it is known that matricial coupling implies (topological) equivalence after extension, but only in 1992 it was proved by Bart and Tsekanovskiĭ [2] that the converse holds also true.

Thinking of the related work of several Portuguese researchers on *matrix completion problems* [5, 9, 30, 32, 33], we would like to point out that matrices T and S of complex numbers (of size $m_T \times n_T$ and $m_S \times n_S$, respectively), are matricially coupled if and only if

$$\mathrm{rank}\,T - \mathrm{rank}\,S = m_T - m_S = n_T - n_S. \tag{2.10}$$

This means that once given matrices T and S of arbitrary size such that (2.10) holds true, we can solve the *completion problem* of constructing additional matrices

T_0, T_1, T_2, S_0, S_1 and S_2 such that (2.9) occurs (and vice versa).

Both relations, the algebraic equivalence after extension relation (2.7) and the topological one (2.8), are reflexive, symmetric and transitive. But evidently (2.8) has much stronger *transfer properties*:

- $\mathscr{W}_{\Phi_{\mathscr{A}},\Omega}$ belongs to the same regularity class (invertibility, Fredholm property, generalized invertibility, normal solvability etc.) as $\mathscr{W}_{\Phi,\mathbb{R}_+}$ does, since the operators have isomorphic kernels and co-kernels; indices and defect numbers are the same;
- explicit formulas for generalized inverses or regularizers of $\mathscr{W}_{\Phi,\mathbb{R}_+}$ imply corresponding formulas for those of $\mathscr{W}_{\Phi_{\mathscr{A}},\Omega}$, and vice versa;
- qualitative properties of solutions can be concluded, dependent on the particular form of E and F: singular behavior, asymptotic expansion etc. [7, 31];
- operator theoretical conclusions are possible: description of the spectrum, numerical range, reduction of order, perturbation, positivity, application of the fixed point principle, normalization [27] etc.

For the finite interval case, Kuijper [20] proposed an extension method to construct algebraic equivalence after extension relations (2.7) based on certain injective and surjective operators which are determined by the geometry of Ω. Kuijper's method guarantees the existence of invertible linear operators E and F that are constructed just by an algebraic decomposition of the domain and image spaces into the corresponding defect spaces of the two operators and their algebraic complements.

Theorem 2.2 (Kuijper). *For $\Omega=]0,a[$, the convolution type operator $\mathscr{W}_{\Phi_{\mathscr{A}},\Omega}$ introduced in (2.1) is algebraically equivalent after extension to a new Wiener-Hopf matrix operator*

$$\mathscr{W}_{\Phi_{\mathscr{C}},\mathbb{R}_+} = r_{\mathbb{R}_+}\mathscr{C} : [L^p_+(\mathbb{R})]^2 \to [L^p(\mathbb{R}_+)]^2 \tag{2.11}$$

where $\mathscr{C}=\mathscr{F}^{-1}\Phi_{\mathscr{C}}\cdot\mathscr{F}$, $\Phi_{\mathscr{C}}\in[L^\infty(\mathbb{R})]^{2\times 2}$, and

$$\Phi_{\mathscr{C}} = \begin{bmatrix} \tau_{-a}\zeta^r & 0 \\ \lambda_-^s\Phi_{\mathscr{A}}\lambda_+^{-r} & \tau_a\zeta^s \end{bmatrix},$$

with $\tau_a(\xi)=\exp(ia\xi)$, $\lambda_\pm(\xi)=\xi\pm i$, for $\xi\in\mathbb{R}$, and $\zeta=\lambda_-/\lambda_+$.

Here we used the abbreviations $L^p_+(\mathbb{R}) := \widetilde{H}^{0,p}(\mathbb{R}_+)$ and $L^p(\mathbb{R}_+) := H^{0,p}(\mathbb{R}_+)$.

For most cases of the smoothness orders r and s, the *Kuijper theorem* was already improved, in the sense that it is possible to present a *topological* equivalence after extension relation in explicit form; namely for non-critical orders, i.e., if $s-1/p\in\mathbb{R}\backslash\mathbb{Z}$. In the critical cases only existence of a stronger relation could be proved. We will not go into details here but the interested reader can proceed into this direction by consulting [1, 8]. However, until now there is no general unifying method to obtain a topological equivalence after extension relation for all orders $r,s\in\mathbb{R}$ (and $1<p<+\infty$) in the finite interval variant.

It is clear that explicit relations (in the form of operator matrix identities) have their direct profits. One of the consequences of such explicit formulas can be seen, e.g., in the fact that if we know a (generalized) inverse of $W_{\Phi_{\mathscr{C}},\mathbb{R}_+}$, say $W^-_{\Phi_{\mathscr{C}},\mathbb{R}_+}$, then the explicit equivalence after extension relation

$$\begin{bmatrix} \mathscr{W}_{\Phi_{\mathscr{A}},\Omega} & 0 \\ 0 & I_Y \end{bmatrix} = E \begin{bmatrix} W_{\Phi_{\mathscr{C}},\mathbb{R}_+} & 0 \\ 0 & I_Z \end{bmatrix} F \tag{2.12}$$

allows a quick way to find an explicit (generalized) inverse of $\mathscr{W}_{\Phi_{\mathscr{A}},\Omega}$ since

$$\begin{bmatrix} \mathscr{W}^-_{\Phi_{\mathscr{A}},\Omega} & 0 \\ 0 & I_Y \end{bmatrix} = F^{-1} \begin{bmatrix} W^-_{\Phi_{\mathscr{C}},\mathbb{R}_+} & 0 \\ 0 & I_Z \end{bmatrix} E^{-1}$$

is a (generalized) inverse of the matricial operator in the right-hand side of (2.12), and therefore $\mathscr{W}^-_{\Phi_{\mathscr{A}},\Omega}$ is a generalized inverse of $\mathscr{W}_{\Phi_{\mathscr{A}},\Omega}$. Naturally, a similar reasoning is obtained for one-sided inverses if they exist.

One can say that Theorem 2.2 provides a reduction of complexity: We start with a convolution type operator acting between Bessel potential spaces on the interval Ω and arrive at a Wiener-Hopf operator acting between Lebesgue spaces on the interval $\mathbb{R}_+$. The price consists of a larger size of the operator and a more complicated Fourier symbol, which contains terms oscillating at infinity.

2.4 On Fourier symbols $\Phi_{\mathscr{A}}$ from the Wiener algebra

Let us now consider operator (2.1) when $\Phi_{\mathscr{A}}$ is an invertible element in the Wiener algebra on the real line

$$\Phi_{\mathscr{A}} \in \mathscr{G}\,\mathbb{W} \tag{2.13}$$

and

$$\Omega=]0,a[,\quad p=2,\quad r=s=k\in\mathbb{Z}. \tag{2.14}$$

In this case, the Fourier symbol $\Phi_{\mathscr{C}}$ of the last theorem takes the particular form

$$\Phi_{\mathscr{P}} = \begin{bmatrix} \tau_{-a}\zeta^k & 0 \\ \Phi_{\mathscr{A}}\zeta^k & \tau_a\zeta^k \end{bmatrix}.$$

A so-called *Wiener-Hopf factorization* [34] of $\Phi_{\mathscr{P}}$ is well-known in the case of $\Phi_{\mathscr{A}}\equiv 1$ and $k=0$, because

$$\begin{bmatrix} \tau_{-a} & 0 \\ 1 & \tau_a \end{bmatrix} = \begin{bmatrix} \tau_{-a} & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & \tau_a \\ 0 & -1 \end{bmatrix}, \tag{2.15}$$

which implies a formula for the inverse of $\mathscr{W}_{\Phi_\mathscr{A},\Omega}$ in the case of a *sectorial symbol*

$$\Phi_\mathscr{A} = c\,(1+\epsilon), \qquad \|\epsilon\|_{L^\infty(\mathbb{R})} < 1 \tag{2.16}$$

in terms of a Neumann series (provided $c \in \mathbb{C}\setminus\{0\}$, $k=0$ and $p=2$ are satisfied).

We will now describe a procedure to obtain a generalized inverse of (2.1) under the assumptions (2.13)–(2.14) for general $k \in \mathbb{Z}$ and some restrictions; more precisely the formulas are:

(a) explicit in closed analytical form, if $\Phi_\mathscr{A}$ is rational;

(b) explicit in analytical form plus Neumann series, if $\Phi_\mathscr{A}$ is not rational.

The strategy is as follows

(i) Letting $w = $ wind $\Phi_\mathscr{A}$, we consider, instead of

$$\mathscr{W}_{\Phi_\mathscr{A},\Omega} = r_\Omega \mathscr{F}^{-1}\Phi_\mathscr{A}\cdot\mathscr{F} : \widetilde{H}^{k,p}(\Omega) \to H^{k,p}(\Omega),$$

the restricted or continuously extended operator

$$\mathscr{W}^{(s)}_{\Phi_\mathscr{A},\Omega} : \widetilde{H}^{s,p}(\Omega) \to H^{s,p}(\Omega),$$

$$\mathscr{W}^{(s)}_{\Phi_\mathscr{A},\Omega} = \begin{cases} \text{Rst } r_\Omega \mathscr{F}^{-1}\Phi_\mathscr{A}\cdot\mathscr{F} &, \quad s>k \\ \text{Ext } r_\Omega \mathscr{F}^{-1}\Phi_\mathscr{A}\cdot\mathscr{F} &, \quad s<k \\ \mathscr{W}_{\Phi_\mathscr{A},\Omega} &, \quad s=k \end{cases} \tag{2.17}$$

respectively, where $s=-w$.

(ii) We relate the operator (2.17) (in the sense of Theorem 2.2) with a Wiener-Hopf operator

$$\begin{aligned} \mathscr{W}^{(s)}_{\Phi_\mathscr{A},\Omega} &\overset{*}{\sim} \mathscr{W}_{\Phi_\mathscr{P}{}^{(s)},\mathbb{R}_+} = r_{\mathbb{R}_+}\mathscr{F}^{-1}\Phi_\mathscr{P}{}^{(s)}\cdot\mathscr{F} \\ &\qquad : \left[L^2_+(\mathbb{R})\right]^2 \to \left[L^2(\mathbb{R}_+)\right]^2 \\ \Phi_\mathscr{P}{}^{(s)} &= \begin{bmatrix} \tau_{-a}\zeta^s & 0 \\ \Phi_\mathscr{A}\zeta^s & \tau_a\zeta^s \end{bmatrix} \end{aligned}$$

where wind $(\Phi_\mathscr{A}\zeta^s) = 0$.

(iii) Now we consider the particular cases $\Phi_\mathscr{A}\zeta^s = \Phi_0\, r_+$ if $s \geq 0$ or $\Phi_\mathscr{A}\zeta^s = \Phi_0\, r_-$ if $s \leq 0$ where Φ_0 satisfies (2.16) and $r_\pm \in \mathscr{G}\mathscr{R}_\pm(\dot{\mathbb{R}})$, i.e., $r_\pm$ are invertible rational functions which are restrictions to $\dot{\mathbb{R}} = \mathbb{R}\cup\{\infty\}$ of holomorphic functions in the upper/lower half-plane and continuous in the union of the upper/lower half-plane with the real line. It is known [13] that the previous factorization exists (due to the sign of s) and that it can be constructed by means of Weierstrass approximation.

(iv) So we have to factor (after elementary factorization)

$$\begin{aligned} G &= \begin{bmatrix} \tau_{-a}\zeta^s r_+^{-1} & 0 \\ \Phi_0 & \tau_a\zeta^s r_-^{-1} \end{bmatrix} \\ &= \begin{bmatrix} \tau_{-a}\rho_1 & 0 \\ \Phi_0 & \tau_a\rho_2 \end{bmatrix} \end{aligned}$$

where either $r_- = 1$ (for $s \geq 0$) or $r_+ = 1$ (for $s \leq 0$), and all non-oscillating symbols are 1 at ∞

$$\Phi_0(\infty) = r_+(\infty) = r_-(\infty) = \rho_1(\infty) = \rho_2(\infty) = 1.$$

(v) We reduce G to a non-oscillating symbol G_0 by using (2.15)

$$G = \begin{bmatrix} \tau_{-a} & 1 \\ 1 & 0 \end{bmatrix} G_0 \begin{bmatrix} 1 & \tau_a \\ 0 & -1 \end{bmatrix}$$

with

$$G_0 = \begin{bmatrix} \Phi_0 & \tau_a(\Phi_0-\rho_2) \\ \tau_{-a}(\rho_1-\Phi_0) & \rho_1-\Phi_0+\rho_2 \end{bmatrix}.$$

(vi) Consider the principal part of G_0

$$G_1 = \begin{bmatrix} 1 & \tau_a(1-\rho_2) \\ \tau_{-a}(\rho_1-1) & \rho_1-1+\rho_2 \end{bmatrix}$$

separately in the two cases $s \geq 0$ or $s \leq 0$, respectively:

(vi$_+$) $s \geq 0$, $\rho_1 = \zeta^s r_+^{-1}$, $\rho_2 = \zeta^s$

$$\begin{aligned} G_1 &= \begin{bmatrix} 1 & \tau_a(1-\zeta^s) \\ \tau_{-a}(\zeta^s r_+^{-1}-1) & \zeta^s r_+^{-1} - 1 + \zeta^s \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ \tau_{-a}(\zeta^s r_+^{-1}-1) & \zeta^{2s} r_+^{-1} \end{bmatrix} \\ &\qquad\qquad \times \begin{bmatrix} 1 & \tau_a(1-\zeta^s) \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ b_- & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \zeta^{2s} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ b_+ & r_+^{-1} \end{bmatrix} \\ &\qquad\qquad \times \begin{bmatrix} 1 & \tau_a(1-\zeta^s) \\ 0 & 1 \end{bmatrix} \end{aligned}$$

where $b_- + b_+\zeta^{2s} = \tau_{-a}(\zeta^s r_+^{-1} - 1)$, i.e., $b_- = P_-\,\tau_{-a}(\zeta^s r_+^{-1} - 1)$ and $b_+ = P_+\,\tau_{-a}(\zeta^{-s} r_+^{-1} - \zeta^{-2s})$, with $P_\pm = \mathscr{F}\chi_\pm\mathscr{F}^{-1}$ being the Cauchy projection operators due to the characteristic functions $\chi_\pm$ of $\mathbb{R}_\pm$;

(vi$_-$) $s \le 0$, $\rho_1 = \zeta^s, \rho_2 = \zeta^s r_-^{-1}$

$$
\begin{aligned}
G_1 &= \begin{bmatrix} 1 & \tau_a(1-\zeta^s r_-^{-1}) \\ \tau_{-a}(\zeta^s - 1) & \zeta^s - 1 + \zeta^s r_-^{-1} \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 \\ \tau_{-a}(\zeta^s - 1) & 1 \end{bmatrix} \begin{bmatrix} 1 & \tau_a(1-\zeta^s r_-^{-1}) \\ 0 & \zeta^{2s} r_-^{-1} \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 \\ \tau_{-a}(\zeta^s - 1) & 1 \end{bmatrix} \begin{bmatrix} 1 & c_- \\ 0 & r_-^{-1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \zeta^{2s} \end{bmatrix} \\
&\qquad \times \begin{bmatrix} 1 & c_+ \\ 0 & 1 \end{bmatrix}
\end{aligned}
$$

where $c_+ + c_- \zeta^{2s} = \tau_a(1-\zeta^s r_-^{-1})$, i.e., $c_+ = P_+ \tau_a(1-\zeta^s r_-^{-1})$ and $c_- = P_- \tau_a(\zeta^{-2s} - \zeta^{-s} r_-^{-1})$.

(vii) Thus, since one-sided inverses are stable against small perturbations, we obtain a one-sided inverse of $\mathscr{W}_{\Phi_{\mathscr{P}}(s),\mathbb{R}_+}$ in both cases and $\operatorname{ind} \mathscr{W}_{\Phi_{\mathscr{P}}(s),\mathbb{R}_+} = -2s = 2w$. From (ii) we have a one-sided inverse of $\mathscr{W}^{(s)}_{\Phi_{\mathscr{A}},\Omega}$. Consequently, due to the so-called *Shift Theorem* [6, 12] and (i), we are able to obtain a generalized inverse of $\mathscr{W}_{\Phi_{\mathscr{A}},\Omega}$, for the present case.

3 Applications in diffraction theory

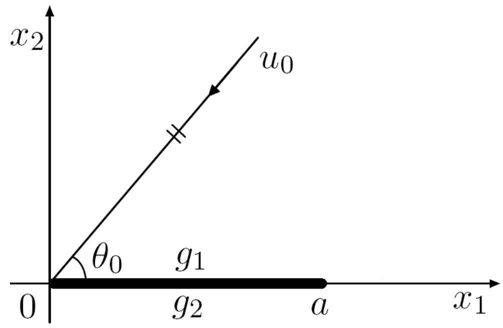
In this section we would like to exemplify the use of convolution type operators in some boundary value and/or transmission problems in weak formulation which originate from diffraction of time-harmonic waves by an infinite strip, see [16, 25, 26, 28, 35] for a detailed background. The proofs of the results presented below can be found in [8].

A. Sommerfeld was the first to formulate and solve a canonical boundary value problem for the Helmholtz equation which governs time-harmonic scalar waves. In his famous Habilitation Thesis of 1896 he was dealing with geometries formed by half-planes and wedges. He used series expansions and Riemann surface concepts to arrive at the solutions of corresponding Dirichlet boundary value problems. The so-called Sommerfeld integrals were afterwards systematically used by authors from Soviet Union and culminated in what is now known as the *Maliuzhinets method* [24]. Western authors preferred using the so-called *Wiener-Hopf method*, based on the Fourier transformation and factorization of the Fourier symbol of the corresponding convolution type operators – in the spirit of the last section.

Here we will consider the diffraction by a strip of an incoming plane wave u_0 of the form

$$u_0 = \exp\left[-ik(x_1 \cos\theta_0 + x_2 \sin\theta_0)\right],$$

where θ_0 is the angle of incidence (see the Figure), and we have omitted the time harmonic factor $\exp(-i\omega_0 t)$.



A plane wave incident upon a strip located on the x_1 axis, between 0 and a, and having boundary data g_1 and g_2 on its banks.

The wave number $k = \omega_0\sqrt{\varepsilon\mu}$ is assumed to be complex satisfying $\Im m\,(k) > 0$, i.e., ε and μ are parameters of a lossy medium. The electromagnetic theory yields, for a large spectrum of materials, a quasi-homogeneous refracted wave, which propagates perpendicularly to the boundary regardless of the incident angle. The x_3–dependence is therefore cancelled due to the perpendicular wave propagation, leading us to the consideration of a $\mathbb{R}^2$ situation with the strip Σ and its x_1–complement Σ' here represented by

$$
\begin{aligned}
\Sigma &= \left\{x = (x_1, x_2) \in \mathbb{R}^2 : x_1 \in [0,a], x_2 = 0\right\} \\
&= [0,a] \times \{0\} &(3.18)\\
\Sigma' &= (\mathbb{R}\backslash]0,a[) \times \{0\}\,. &(3.19)
\end{aligned}
$$

The diffracted or scattered field then satisfies the Helmholtz equation as well as the total field does, and several possible boundary conditions can be valid on the banks of Σ corresponding to different material behavior.

These considerations lead us to the problem of finding an element $u \in L^p(\mathbb{R}^2)$ such that

$$
\begin{aligned}
&u^\pm = u_{|\mathbb{R}\times\mathbb{R}_\pm} \in H^{l,p}(\mathbb{R}\times\mathbb{R}_\pm) &(3.20)\\
&Lu^\pm = (\Delta + k^2)u^\pm = 0 \qquad \text{in} \quad \mathbb{R}\times\mathbb{R}_\pm &(3.21)\\
&[u]_{\Sigma'} = (u^+(x) - u^-(x))_{|x\in\Sigma'} = 0 &(3.22)\\
&\left[\frac{\partial u}{\partial x_2}\right]_{\Sigma'} = \left(\frac{\partial u^+}{\partial x_2}(x) - \frac{\partial u^-}{\partial x_2}(x)\right)_{|x\in\Sigma'} = 0 &(3.23)
\end{aligned}
$$

where $p \in]1,\infty[$, $l > 1/p$, $l - 1/p \notin \mathbb{N}$, $k \in \mathbb{C}$ with $\Im m\,(k) > 0$ are given and

$$
\begin{aligned}
B_j u(x_1) &= \Sigma_{\sigma_1+\sigma_2\le m_j} b^+_{\sigma,j} D^\sigma u^+(x_1,0) \\
&\qquad + b^-_{\sigma,j} D^\sigma u^-(x_1,0) \\
&= g_j(x_1), \qquad x_1 \in [0,a], \quad j = 1,2 &(3.24)
\end{aligned}
$$

where $\sigma = (\sigma_1,\sigma_2), m = (m_1,m_2) \in \mathbb{N}_0^2$, $b^\pm_{\sigma,j} \in \mathbb{C}$ and $g_j \in H^{l-\frac{1}{p}-m_j,p}(]0,a[)$ are assumed to be known.

From the physical point of view, one is mainly interested in solutions in the energy space, $u \in H^{1,2}(\mathbb{R}^2 \setminus \Sigma)$ [26]. But we know already from the study of half-plane problems that many of these are ill-posed in that space setting [27]. They need a normalization which is often implemented by change of the space parameters l, p of $H^{l,p}$. Another reason to consider the problem in a scale of spaces is to look for regularity results and asymptotic expansion [31].

The boundary values of u are taken in the sense of the *trace theorem*. The choice of the other data spaces results from the *representation formula* [26] (cf. Prop. 3.1 later on) as a consequence of (3.24). The orders of the boundary operators B_j are arbitrary (from the mathematical point of view).

We associate an operator with the problem, say

$$\mathscr{P}^{(l,p)} : u \mapsto g = (g_1, g_2) \tag{3.25}$$

where the domain $\mathscr{D}(\mathscr{P}^{(l,p)})$ of $\mathscr{P}^{(l,p)}$ is characterized by (3.20)–(3.23), the action and the image space of $\mathscr{P}^{(l,p)}$ are described by (3.24) with the corresponding norms. Evidently the problem (3.18)–(3.24) is well-posed in this space setting if and only if the operator

$$\begin{aligned} &\mathscr{P}^{(l,p)} : \mathscr{X} \to \mathscr{Y}, \\ &\mathscr{X} = H^{l,p}(\mathbb{R} \times \mathbb{R}_+) \times H^{l,p}(\mathbb{R} \times \mathbb{R}_-), \\ &\mathscr{Y} = \times_{j=1}^{2} H^{l-\frac{1}{p}-m_j,p}(]0,a[) \end{aligned} \tag{3.26}$$

is boundedly invertible.

The main objectives are:

(i) to find the spaces in which the operator $\mathscr{P}^{(l,p)}$ is boundedly invertible and those where it is normally solvable (which implies the Fredholm property in the elliptic case and the existence of a generalized inverse in terms of factorization);

(ii) to determine the defect numbers (not only the index) of $\mathscr{P}^{(l,p)}$ by computing the partial indices of a matrix symbol, which appears in a topological (not only algebraic) equivalence after extension relation in the above-mentioned sense;

(iii) to get a generalized inverse of $\mathscr{P}^{(l,p)}$, if possible

  (a) in closed analytical form, or

  (b) in terms of a uniformly convergent series under physically reasonable assumptions on the parameters. As a matter of fact it is not possible to deduce these results only from an algebraic equivalence after extension relation between $\mathscr{P}^{(l,p)}$ and a Wiener-Hopf operator, cf. [4].

The convolution type operators enter here in the scene because we are able to recognize a relation between $\mathscr{P}^{(l,p)}$ and such an operator in form of an operator matrix identity.

Proposition 3.1. *The operator $\mathscr{P}^{(l,p)}$ is (algebraically and topologically) equivalent to a convolution type operator on the interval $[0,a]$ acting in the corresponding boundary data spaces of Bessel potentials. More precisely $\mathscr{P}^{(l,p)} = \mathscr{W}_{\Phi_{\mathscr{A}},\Omega}\, B_-\, \mathscr{T}_0$ where the trace operator*

$$\begin{aligned} &\mathscr{T}_0 : \mathscr{D}(\mathscr{P}^{(l,p)}) \to \left[H^{l-\frac{1}{p},p}(\mathbb{R})\right]^2, \\ &\mathscr{T}_0 u = u_0 = \left(u_0^+, u_0^-\right)^T = \left(u^+_{|x_2=0}, u^-_{|x_2=0}\right)^T \end{aligned}$$

is bounded invertible by the representation formula

$$\begin{aligned} u &= \mathscr{K}\, u_0 \\ u(x_1,x_2) &= \mathscr{F}^{-1}_{\xi \mapsto x_1} \left\{ \exp[-t(\xi)x_2]\, \widehat{u}_0^+(\xi)\, \chi_+(x_2) \right. \\ &\qquad \left. + \exp[t(\xi)x_2]\, \widehat{u}_0^-(\xi)\, \chi_-(x_2) \right\}. \end{aligned}$$

Here $\mathscr{K}$ is called a Poisson operator, $\widehat{\varphi}$ denotes the Fourier transform of φ, and $t(\xi) = (\xi^2 - k^2)^{1/2}$. Further

$$\begin{aligned} B_- &= \mathscr{F}^{-1} \begin{bmatrix} 1 & -1 \\ -t & -t \end{bmatrix} \cdot \mathscr{F} \\ &: \left[H^{l-\frac{1}{p},p}(\mathbb{R})\right]^2 \to \mathscr{X}_0 = H^{l-\frac{1}{p},p}(\mathbb{R}) \times H^{l-\frac{1}{p}-1,p}(\mathbb{R}) \end{aligned}$$

maps the Dirichlet trace vector $u_0 = \left(u_0^+, u_0^-\right)^T = \mathscr{T}_0 u$ (for $u \in \mathscr{D}(\mathscr{P}^{(l,p)})$) into the jump vector of Dirichlet and Neumann data

$$f = \left(u_0^+ - u_0^-, u_1^+ - u_1^-\right)^T = \left([u]_{\Sigma \cup \Sigma'}, \left[\frac{\partial u}{\partial x_2}\right]_{\Sigma \cup \Sigma'} \right)^T.$$

The operator $\mathscr{W}_{\Phi_{\mathscr{A}},\Omega} : \mathscr{X}_0 \to \mathscr{Y}$ is an operator of the form (2.1) where $r = (l - 1/p, l - 1 - 1/p)$, $s = (l - m_1 - 1/p, l - m_2 - 1/p)$ and

$$\Phi_{\mathscr{A}}(\xi) = \begin{bmatrix} \sum_{|\sigma| \le m_1} \frac{1}{2} \left(b^+_{\sigma,1} - b^-_{\sigma,1}\right) (-i\xi)^{\sigma_1} (-t(\xi))^{\sigma_2} & \sum_{|\sigma| \le m_1} \frac{-1}{2t(\xi)} \left(b^+_{\sigma,1} + b^-_{\sigma,1}\right) (-i\xi)^{\sigma_1} (t(\xi))^{\sigma_2} \\ \sum_{|\sigma| \le m_2} \frac{1}{2} \left(b^+_{\sigma,2} - b^-_{\sigma,2}\right) (-i\xi)^{\sigma_1} (-t(\xi))^{\sigma_2} & \sum_{|\sigma| \le m_2} \frac{-1}{2t(\xi)} \left(b^+_{\sigma,2} + b^-_{\sigma,2}\right) (-i\xi)^{\sigma_1} (t(\xi))^{\sigma_2} \end{bmatrix} \tag{3.27}$$

Corollary 3.2. *The system of equations $\mathscr{W}_{\Phi_{\mathscr{A}},\Omega} f = g$ decouples, i.e. $\Phi_{\mathscr{A}}$ is triangular after multiplication with a constant matrix, if and only if some linear combination of the two boundary conditions (3.24) contains only a linear combination of either "difference" or "sum data", i.e. it can be written as*

$$Bu(x_1) = \sum_{|\sigma| \le m_0} b_\sigma D^\sigma (u^+ \pm u^-)(x_1, 0), \quad x_1 \in [0,a].$$

Using the theory of the last section we can now proceed with finding the concrete form of the related Wiener-Hopf operator.

Theorem 3.3. *Let $\mathscr{P}^{(l,p)}$ be the operator defined by (3.18)–(3.26). Then we have the equivalence after extension relation*

$$\mathscr{P}^{(l,p)} \overset{*}{\sim} \mathscr{W}_{\Phi_{\mathscr{U}},\mathbb{R}_+} \in \mathscr{L}\left(\left[L^p_+(\mathbb{R})\right]^{2n}, [L^p(\mathbb{R}_+)]^{2n}\right) \quad (3.28)$$

where $n = 2$ or 1 (in certain decomposing cases described below) and

$$\Phi_{\mathscr{U}} = \begin{bmatrix} \tau_{-a}\zeta^r I_n & 0 \\ \lambda^s_- \Phi_{\mathscr{A}} \lambda^{-r}_+ & \tau_a \zeta^s I_n \end{bmatrix} \quad (3.29)$$

where $\Phi_{\mathscr{A}}$ is given by (3.27) or can be replaced by a scalar symbol according to Corollary 3.2 in the case $n = 1$, respectively. The orders are $r = (l-1/p, l-1-1/p)$, $s = (l - m_1 - 1/p, l - m_2 - 1/p)$ or, for $n = 1$, are components of these two vectors.

In the last result the equivalence after extension is an algebraic one but for certain cases we can perform a topological relation. This is the case of $p = 2$, by using Theorem 3 of [2].

We notice that in the decomposing case, $\Phi_{\mathscr{A}}$ is triangular (say upper). Therefore, one can identify and use (in the following way) an operator $\mathscr{W}_{\Phi_{\mathscr{U}_2},\mathbb{R}_+}$ that has the form (3.28)–(3.29) with $n = 1$. If it is invertible we have

$$\mathscr{P}^{(l,p)} \overset{*}{\sim} \begin{bmatrix} \mathscr{W}_{\Phi_{\mathscr{U}_1},\mathbb{R}_+} & * \\ 0 & \mathscr{W}_{\Phi_{\mathscr{U}_2},\mathbb{R}_+} \end{bmatrix} =$$
$$= \begin{bmatrix} I & * \\ 0 & \mathscr{W}_{\Phi_{\mathscr{U}_2},\mathbb{R}_+} \end{bmatrix} \begin{bmatrix} \mathscr{W}_{\Phi_{\mathscr{U}_1},\mathbb{R}_+} & 0 \\ 0 & I \end{bmatrix},$$

i.e., equivalence after extension to $\mathscr{W}_{\Phi_{\mathscr{U}_1},\mathbb{R}_+}$ and the remainder operator has the same form (3.28)–(3.29) with $n = 1$.

For technical reasons we extend in this final part the definition of $\lambda_\pm(\xi) = \xi \pm i$ and work now with

$$\lambda_\pm(\xi) = \xi \pm k\,, \qquad \Im m\,(k) > 0\,.$$

This makes the Fourier symbols simpler since we can combine factors $\lambda^s_\pm$ with t, but does not change the principal nature of factorizations or the topology of Bessel potential spaces $H^{s,p} = \lambda^{-s}_+ L^p = \lambda^{-s}_- L^p$.

If $\mathscr{W}_{\Phi_{\mathscr{U}_2},\mathbb{R}_+}$ is not invertible, but a shifted one $W^{(w)}_{\Phi_{\mathscr{U}_2},\mathbb{R}_+} : \widetilde{H}^{w,p}(\mathbb{R}_+) \to H^{w,p}(\mathbb{R}_+)$ (defined by restriction or continuous extension) is invertible for some $w = (w_1, w_1) \in \mathbb{R}^2$, one can try to consider $W^{(w,w)}_{\Phi_{\mathscr{U}},\mathbb{R}_+}$ first and then "shift back", i.e. express results for $\mathscr{W}_{\Phi_{\mathscr{U}},\mathbb{R}_+}$ in terms of results for $W^{(w,w)}_{\Phi_{\mathscr{U}},\mathbb{R}_+}$.

Corollary 3.4. *The symbol $\Phi_{\mathscr{U}}$ can be written in the form*

$$\Phi_{\mathscr{U}} = \zeta^r \begin{bmatrix} \tau_{-a} I_n & 0 \\ \lambda^{s-r}_- \Phi_{\mathscr{A}} & \tau_a \zeta^{s-r} I_n \end{bmatrix}$$

where $s - r \in \mathbb{Z}^n$, i.e. λ^{s-r}_- and ζ^{s-r} are rational, precisely $s - r = (-m_1, 1 - m_2)$ if $n = 2$, which admits integer components up to 1. Moreover, the elements of $\lambda^{s-r}_- \Phi_{\mathscr{A}}$ are:

- *Hölder continuous functions with a possible jump at infinity, and*
- *algebraic compositions of $\zeta^{1/2}$ and rational functions.*

Arriving at this point, a Wiener-Hopf factorization of $\Phi_{\mathscr{U}}$ and the operator relations of the former section would lead us to (generalized) inverses of $\mathscr{P}$, and therefore to solutions of the initial boundary value problem (3.24). However the factorization problem *in general* is not solved and remains an open problem for challenging future research.

4 Conclusion

The operator theoretic approach enables us to identify clearly the relations between the (operator associated with the) given problem and the (operator associated with the) boundary pseudodifferential equations. One can analyze simultaneously classes of problems with different boundary conditions and space settings with respect to the questions mentioned before: Solvability, explicit analytical presentation (in particular cases) and qualitative results like regularity, singular behavior and asymptotic expansion. The method was demonstrated for a prototype class of problems from diffraction theory.

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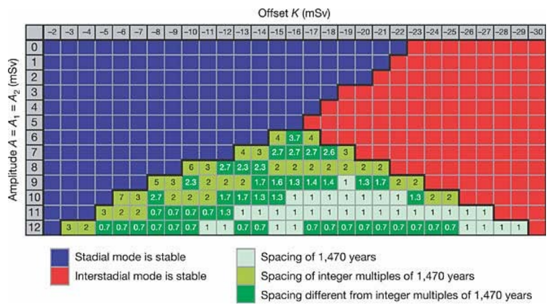
Glacial climate cycles and the least common multiple. In part of the late glacial period severe climate oscillations occurred with a period of almost exactly 1470 years; these are documented by ice-core samples from Greenland, and are called Dansgaard-Oescher (DO) events. The period of these oscillations has been mysterious, because there are no excitations of that frequency either in the solar record or in the variation of the Earth's inclination and orbit. Holger Braun and his colleagues in Heidelberg, Potsdam and Bremerhaven report in the November 10 2005 *Nature* on a possible explanation. There are two "pronounced and stable centennial-scale solar cycles," the DeVries-Suess (period 210 years) and the Gleissberg (86.5 years); the German group designed a model to test the hypothesis that the *sum* of these two excitations could be driving the DO oscillations. In general two different periodic astronomical phenomena will have irrationally related frequencies unless there is "phase locking," but there are *approximate* common periods: waiting long enough one can get the two back as close as one wants to their initial relative position. It turns out that for the DeVries-Suess and Gleissberg cycles, 1470 years is a very good approximation to a common period (it equals 7×210 and almost exactly 17×86.5). The team used CLIMBER-2 (www-lsce.cea.fr/pmip/docs/climberdoc.html), a global climate and biosphere simulation model that has been around since 1998, forcing it with

$$F(t) = -A_1 \cos(\omega_1 t + \varphi_1) - A_2 \cos(\omega_2 t + \varphi_2) + K$$

where ω_1 and ω_2 are the DeVries-Suess and Gleissberg frequencies, and K represents changes in the background climate compared with a baseline.



The response of the model for different combinations of periodic excitation amplitude $A = A_1 = A_2$ (vertical) and K — a baseline measurement of the general warmth of the climate (horizontal). The pale green squares represent the parameter ranges for which the model manifests a 1470-year periodicity. Image from *Nature* **438** 208, used with permission.

The results of the simulation show a region where the 1470-year period would be stable under perturbation. As the authors remark, the simulation also shows that similar oscillations could not happen today.

Math on the Millennium Bridge. The Millennium Bridge, a 325-meter footbridge spanning the Thames in London, opened on June 10, 2000. The November 3 2005 *Nature* ran a Brief Communication entitled "Crowd synchrony on the Millennium Bridge," describing what happened and giving a mathematical analysis. "Soon after the crowd streamed on ... , the bridge started to sway from side to side; many pedestrians fell spontaneously into step with the bridge's vibrations, inadvertently amplifying them." This is not the classical example of marchers across a bridge exciting a resonance of the structure. Rather there was a positive feedback loop in which the bridge invited the initially unorganized pedestrians into synchrony. The authors of the *Nature* communication — a five-man team led by Steven Strogatz of Cornell — modeled the phenomenon by "adapting ideas originally developed to describe the collective synchronization of biological oscillators such as neurons and fireflies." Their model starts with the differential equation for a forced, damped harmonic oscillator:

$$Md^2X/dt^2 + BdX/dt + KX = G(\sin\Theta_1 + ... + \sin\Theta_N)$$

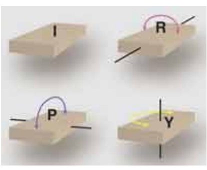
where $X(t)$ is the lateral displacement, and each pedestrian "imparts an alternating sideways force G $\sin\Theta_i$ to the bridge; ... $\Theta_i(t)$ increases by 2π during a full left/right walking cycle." What you wouldn't have seen in Introductory Differential Equations is *feedback*. Since feedback works through the phase difference between the natural oscillation of the bridge and the gait of the pedestrian, the authors make the pair $(X, dX/dt)$ into an angular variable by setting $X = A\sin\Psi$, $dX/dt = (K/M)A\cos\Psi$. Then the feedback is expressed in the set of equations

$$d\Theta_i/dt = \Omega_i + CA\sin(\Psi - \Theta_i + a);$$

here Ω_i is the natural walking rhythm of the i-th pedestrian and a is a phase lag. The model, once tuned by the adjustment of the parameter C, gives a close simulation of the actual event: as the number of pedestrians increases, nothing untoward happens until a critical number is reached, "when the bridge starts to sway and the crowd starts to synchronize, with each process pumping the other in a positive feedback loop."

A new topology for the internet. *Science News* for October 8, 2005 ran a short report by Katie Greene with the title "Untangling a Web. The Internet gets a new look." Greene is describing work to be published in the *PNAS*, in which John Doyle (Caltech) and his colleagues "offer a new mathematical model of the Internet." The conventional ("scale-free") model "indicates that a few well-connected master routers direct Internet traffic to numerous, less essential routers in the network's periphery." Doyle et al. prefer HOT models (the letters stand for Highly Optimized/Organized Tolerance/Tradeoffs), based on insights from biology and engineering. For the Internet, HOT modeling would predict "no ... central hubs and any highly-connected routers lie at the periphery." For security, a HOT model is clearly preferable to a scale-free one, since, as Greene puts it, "if one of those well-connected, outlying routers were taken out, Internet traffic would simply divert to another well-connected router." Whereas in the scale-free Internet, "a targeted attack on a central router could halt virtually all data flow." This is not completely hypothetical: as Greene reports, Doyle and his team have tested their model on Internet2 (an academic subnetwork whose map is known, and which according to Doyle is a "good representation" of the structure of the entire Internet). "The researchers report that their proposed model corresponds well to the structure of Internet2."

Math and the art of mattress flipping. Mattress flipping is one of those household chores that is bothersome because you never know if you are doing it right. Mattress manufacturers recommend periodic flipping for even wear: the four possible combinations of head and foot, top and bottom should receive equal exposure. Ideally there would be a maneuver you can execute each time you flip your mattress such that after four repetitions all four combinations will have been used. Brian Hayes calls such a maneuver a "golden rule" in his treatise on the subject in the September-October 2005 issue of *American Scientist*, and he gives us the bad news: no such golden rule exists.



Mathematically speaking, there are four ways to rotate a mattress so that it ends up aligned with the bed. Hayes uses the symbols I for the Identity rotation (wait until next week) and R, P, Y for the nautical terms Roll, Pitch and Yaw. Image courtesy Brian Hayes.

His argument runs as follows: no matter how creatively you manipulate your mattress, once it's back on the bed you will have performed one of the four operations I, R, P, Y shown in the figure. Each of these operations has the property that if you repeat it, you end up where you started. So you will have missed two of the configurations. Hayes goes on to define the mathematical concept of *group* and to give it content by comparing mattress flipping with another chore: "rotating" (interchanging) the tires on an automobile so that each tire is used, and undergoes wear, in the four different positions. Here there is a "golden rule:" repeating Q (counterclockwise substitution around the outside of the car) four times brings you back to where you started, and each tire will have seen all four positions.

first operation \ second operation	I	R	P	Y
I	I	R	P	Y
R	R	I	Y	P
P	P	Y	I	R
Y	Y	P	R	I

first operation \ second operation	I	Q	Q^2	Q^3
I	I	Q	Q^2	Q^3
Q	Q	Q^2	Q^3	I
Q^2	Q^2	Q^3	I	Q
Q^3	Q^3	I	Q	Q^2

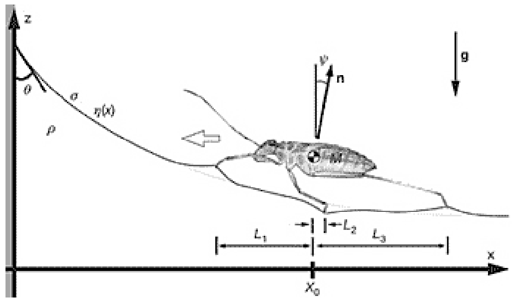
The multiplication tables for mattress flipping (left) and counter-clockwise tire rotation (right). For example, a P (flipping end over end) followed by an R (flipping right over left) has the same effect as a Y (planar rotation by 180 degrees). Each of these tables defines a group with four elements, but the two groups are intrinsically different. Images courtesy Brian Hayes.

The article, available online (www.americanscientist.org/template/AssetDetail/assetid/45938), ends with some fancier material: the complete group of permutations of four objects, and the group of rotations of a cubical mattress.

The math of meniscus mountaineering. Walking on water is a way of life for many species of insects and spiders. But when they need to get onto dry land, they face a problem: surface tension, the same phenomenon that allows these creatures to exist, makes water curve upwards at the shore; the inclined surface marking the edge between wet and dry is called the *meniscus*.

Mesovelia approaches a meniscus. Image from *Nature* **437** 733-736, courtesy John W. M. Bush and David L. Hu.

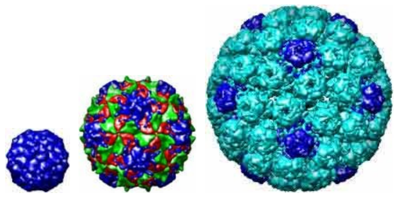
For small insects (say, millimeter-sized) the meniscus appears as a perfectly slippery slope. If they try to walk up, they slide back down. But some species have developed a method that seems like magic: they adopt a special posture and slide *up* the meniscus. David Hu and John Bush of the MIT Mathematics Department have recently worked out the math that makes this possible. Their article, "Meniscus-climbing insects," appears as a Letter in the September 29 2005 *Nature*.



A diagrammatic version of the photograph above, showing the positive and negative meniscus pockets created by *Mesovelia*'s three pairs of legs. Image from *Nature* **437** 733-736, courtesy John W. M. Bush and David L. Hu.

Their analysis is based on the well known observation that "lateral capillary forces exist between small floating objects, an effect responsible for the formation of bubble rafts in champagne and the clumping of breakfast cereal in a bowl of milk." More precisely, they calculate that a body of buoyancy T at distance x from the wall is attracted to the wall by a force $F = ATe^{-Bx}$, where A and B depend on properties of water and on the contact angle θ shown in the diagram. An insect like the water-walker *Mesovelia* faces the meniscus and exploits its three pairs of legs: it pulls up on the surface with the front and rear pairs as it pushes down with the middle pair. Even though the three sets of Ts must add up to something negative (the weight of the insect) the exponential advantage gained by the front legs being closer to the wall will propel the insect forward and up the hill. Where does the work come from? As the authors explain at the end, "by deforming the free surface, the insect converts muscular strain to the surface energy that powers its ascent." Many images and movies (recommended!) available at the project website (www-math.mit.edu/~dhu/Climberweb/climberweb.html).

Virus geometry. A virus is essentially genetic material in a box. The box, or *capsid*, is assembled from specialized proteins called *capsomers*. Watson and Crick had observed in 1956, on topological grounds, that viral capsids could be expected to show the regularities of platonic solids. In fact, icosahedral-type symmetry is the most prevalent.



Satellite Tobacco Mosaic Virus (diameter = 168Å), Type 1 Poliovirus (304Å) and Simian Virus 40 (488Å) have different sizes and capsid structures, but all exhibit icosahedral symmetry. Images from Virus Particle Explorer (VIPER) (viperdb.scripps.edu), a website for virus capsid structures and their computational analysis.

Recent progress in understanding this bias towards icosahedra was reviewed ("Armor-plated Puzzle") by Peter Weiss in the September 3 2005 *Science News*. Weiss first describes research by the UCLA team of Roya Zandi, David Reguera, Robijn Bruinsma, William M. Gelbart and Joseph Rudnick (PNAS **101**, 15556-15560). This team used Monte-Carlo simulations to find locally energy-minimizing configurations of "pentamers" and "hexamers." As Weiss explains it, "They developed a computer model that treated capsomers as malleable disks. ... Then, by having the computer repeatedly shuffle those disks into arbitrary arrangements on a spherical surface, they simulated the formation of millions of hypothetical capsids. ... To explore all possible ratios of pentamers and hexamers, the researchers also programmed into the process random switching of disks between the two types." The lowest energies occurred with arrays of 12 pentamers, surrounded by 0, 20, 30 and 50 hexamers respectively. These corresponded exactly to the prediction, made in 1962 by Donald Caspar and Aaron Klug, of capsids made of 12 pentamers, or 12 pentameters with $20(T-1)$ hexamers, where T is one of the series 3, 7, 13, 19, ... of numbers of the form $h^2 + hk + k^2$; h and k are integers with $(h, k) = 1$.

The Satellite Tobacco Mosaic Virus (12 pentamers), and the Poliovirus (12 pentamers plus 30 hexamers) fall into this classification, but the Simian Virus 40 does not: every one of its 72 capsomers is a pentamer. Weiss explains how Reidun Twarok (York University) read about this problem and saw how her previous work on quasi-crystals could be applied. "The technique employs some mind-bending concepts, such as a six-dimensional lattice based on a hypercube or other building block. Twarock considered lines and planes projecting from such a lattice onto a three-dimensional sphere representing a viral capsid. ... Exploring the expanded portfolio of possible capsid structures that her tiling method had revealed, Twarock found a tile arrangement for a capsid comprising 72 pentamers and no heptamers." This turned out to be exactly the SV-40 structure pictured above. Her work appeared in the *Journal of Theoretical Biology* last year (**226**, 477 - 482), with a more general classification of possible capsid structures available online (arxiv.org/pdf/q-bio.BM/0508015).

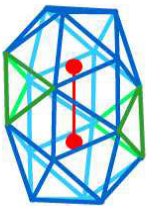
Math and narrative on Mykonos. "Can mathematicians learn from the narrative approaches of the writers who popularize and dramatize their work?" This is the sub-heading on a news feature piece by Sarah Tomlin in the August 4 2005 *Nature*. Tomlin is reporting on a conference held this summer on Mykonos, where a "select group of about 30 mathematicians, playwrights, historians, philosophers, novelists and artists" met to "find a common ground between story-telling and mathematics." The meeting was the brainchild of the poet and novelist Apostolos Doxiadis (*Uncle Petros and the Goldbach Conjecture*), who has formed a foundation (Thales & Friends) dedicated, according to its website (www.thalesandfriends.org), to "bridging the chasm between mathematics and human culture." Among the participants looking for that common ground from the mathematical side of the chasm, Tomlin quotes Timothy Gowers ("Most mathematics papers are incomprehensible to most mathematicians"), Perci Diaconis ("I can only work on problems if there is a story that is real for me") and Barry Mazur ("I don't think I personally understood the problem until I expressed it in narrative terms"). "Mazur," she tells us, "did not find a solution by using the narrative device of a cliff-hanger, but it helped him to frame the question — and that, he argues, may be as important." Mazur also is reported as suggesting "that similar narrative devices may be especially helpful to young mathematicians, who seem particularly poor at explaining their work to others." Tomlin also gives us a sobering quote from Diaconis: "To communicate we have to lie. If not, we're deadly boring."

The math behind "Intelligent Design". H. Allen Orr's "Devolution," subtitled "Why intelligent design isn't," ran in the May 30 2005 *New Yorker* under their Annals of Science rubric. Orr, Professor of Biology at the University of Rochester, examines the most recent instars of the Intelligent Design (I.D.) argument. In particular he mentions the claim that "recent mathematical findings cast doubt on the power of natural selection." This claim, Orr tells us, has been put forward by William A. Dembski, who "holds a Ph.D. in mathematics, another in philosophy, and a master of divinity in theology." Dembski, once on the faculty at Baylor University and now a member of the Center for Science and Theology at Southern Baptist Theological Seminary, uses the "so-called No Free Lunch, or N.F.L. theorems" to attack natural selection. These theorems analyze the efficiency of search algorithms. "Roughly, the N.F.L. theorems prove the surprising fact that, averaged over all possible terrains, no search algorithm is better than any other." Therefore the search algorithm posited by Darwinism (random mutation plus natural selection), looking for the best in the landscape of all possible adaptations, "is no better than blind search, a process of utterly random change unaided by any guiding force like natural selection." So runs the argument. "Since we don't expect blind change to build elaborate machines showing an exquisite coordination of parts, we have no right to expect Darwinism to do so, either." As Orr reports, "Dembski's arguments have been met with tremendous enthusiasm in the I.D. movement. In part, that's because an innumerate public is easily impressed by a bit of mathematics." But Orr mentions recent work showing that the N.F.L theorems "don't hold in the case of co-evolution, when two or more species evolve in response to one another. And most evolution is surely co-evolution. Organisms ... are perpetually challenged by, and adapting to, a rapidly changing suite of viruses, parasites, predators and prey. A theorem that doesn't apply to these situations is a theorem whose relevance to biology is unclear." He ends this discussion by quoting David Wolpert, one of the authors of the N.F.L. theorems, on Dembski's use of those theorems: "fatally informal and imprecise." Wolpert's paper, joint work with William Macready, is available online (www.santafe.edu/research/publications/wplist/1995), as is Wolpert's critique of Dembski's argument.

Archimedes palimpsest update. Capsule history: A 10th-century parchment containing several works of Archimedes (one of them known to us only by its title) was partially erased sometime between the 12th and 14th century and reused as a religious text. The new book, carefully preserved in monasteries, was found in 1906 by J. L. Heiberg, a scholar who recognized the subtext and was able to decipher and publish most of it. The book went out of sight and resurfaced in 1998 when it was sold at auction for $2 million. The collector who bought it has loaned it until 2008 to the Walters Art Museum in Baltimore. There modern techniques (X-ray fluorescence, optical character recognition and multi-spectral imaging) are being used to tease out the maximum possible of Archimedes' barely legible text. The update: Scholars in Baltimore were stymied by four pages which a 20th-century forger had overpainted with pseudo-medieval imagery, presumably to make the book more valuable. One of them, hearing that the ancient ink was iron-based, thought to take those pages to the Stanford Synchrotron Radiation Laboratory, where high-energy X-rays could make the hidden iron atoms fluoresce and give up their information. The result is four superimposed images (both texts, both sides of the parchment) but the message, which deals with floating bodies and the equilibrium of planes, is there for deciphering. This material is taken from a Stanford University news release posted online (www.sciencedaily.com/releases/2005/05/050521154449.htm). by *Science Daily* on May 22 2005.

The release was picked up in the May 19 2005 *Nature* ("Eureka moment as X-rays slice through forgery"). The *Nature* item shows one of the forged overpaintings and gives a glimpse of the SSRL radiograph.

World's largest nano-deltahedron. *Deltahedron* is chemists' name for a polyhedron with all faces triangular. These shapes occur as ions in cluster chemistry. Until recently, the largest one known had twelve lead atoms forming an icosahedral cage enclosing a platinum atom. Earlier this year, Jose Goicoechea and Slavi Sevov (Notre Dame) reported in the *Journal of the American Chemical Society* that they had assembled a deltahedral cage of eighteen germanium atoms around a palladium dimer. The structure is shown schematically below: two Pd-centered 9-atom Ge-clusters (blue) are joined with the interpolation of four (green) non-equilateral faces. The palladium dimer is shown in red, with the two palladium atoms approximately at the foci of the ellipsoidal cage. Goicoechea and Sevov report that the new cluster stays intact in solution. Their work was picked up under the Editor's Choice rubric in the May 20 2005 *Science*.



The structure of $[Pd_2@Ge_{18}]^{4-}$, after Goicoechea and Sevov. The Pd-Pd distance is about 3Å.

Math and the Unicorn Tapestries. Richard Preston's "Capturing the Unicorn," subtitled "How two mathematicians came to the aid of the Met" appeared under the Art and Science rubric in the *New Yorker* for April 11, 2005. It turns out that the Metropolitan Museum had a problem. The Unicorn Tapestries, the crown jewels of the Met's Medieval collection, were taken down for cleaning in 1998, and were photographed then as part of the Museum's high-resolution digital record project. The tapestries — there are six of them and a fragmentary seventh — are typically twelve feet high and somewhat wider. The digital Leica set up to do the job could only capture one 3 by 3-foot square at a time. But assembling the digital files into a coherent image was too large a job for the Museum's computers to handle. The data — more than two hundred CDs — were filed away, and the tapestries reinstalled on the museum walls. Fast forward to 2003, when David Chudnovsky meets a Metropolitan curator at a dinner party. He and his brother Gregory ("The Chudnovsky brothers claim they are one mathematician who happens to occupy two human bodies") soon take on the computing job, which should be a snap for their latest home-built supercomputer (called "the Home Depot thing" or just "It"). But there is a twist: even after geometric transformations have corrected for all possible perspective changes between adjacent frames, the images on the overlaps are hopelessly out of registration: it's as if the tapestry were a living being which had taken a breath between takes. Everything has slightly shifted. Coaxing the overlaps back into registration requires a new "warping" technique, as the Chudnovskys explain it, a 2-dimensional analogue of techniques used in DNA sequencing and speech recognition. The computation is huge: it takes the "Chudnovsky Mathematician" — Preston's coinage — and "It" three months to process just one tapestry, but "The Unicorn in Captivity" is digitally captured in seamless splendor.

The left-hand side shows the overlap between two adjacent frames of the photo mosaic, after all perspective corrections have been made. The right-hand side shows the two images brought into registration by the Chudnovskys' warping technique. Images courtesy Tom Morgan, IMAS.

End of story: One tapestry, apparently, was enough. The brothers have moved on to a bigger project, working on the design of what may be the world's most powerful supercomputer, for "a United States government agency."

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An Interview with Alexei Sossinsky

Professor Sossinsky, can we start this interview with the genesis of your celebrated book "Knots, mathematics with a twist" (KMT)?

Certainly. I wrote the whole book (in French) during the summer of 1998 in my *isba* in a tiny village 120 kilometers from Moscow, without access to any literature on the subject. For me it was more of a linguistic experience than a mathematical one: I had never written any expository mathematical texts in French before, in fact for thirty years I had written almost nothing in that language (one of my mother tongues – I have two). The actual writing process was a return to and an immersion into the French literary language and culture, which the writing process was extricating from the depths of my subconscious. The pleasure of writing, as well as the total absence of math books and journals in my village, resulted in several unfortunate mathematical errors in the original manuscript; these were never corrected – my French editors published the book without sending me the proofs.

In how many languages has it been published?

I know of seven: French, Rumanian, German, Italian, English, Finnish, and Russian. I have heard that a Portuguese edition is planned.

What makes knot theory so popular?

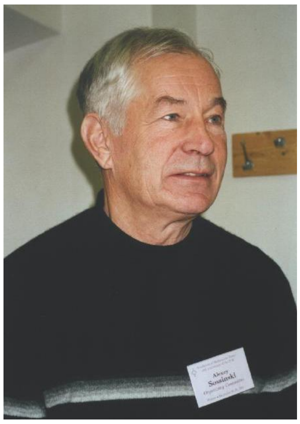
Well, knot theory is very visual, mathematical knots have numerous models in real life (from ropes and cables to DNA), its main problems are simple to formulate but very difficult to solve, it has applications to (or at least deep relationships with) several branches of physics and mathematical analysis, biology, and biochemistry.

Good popularization is difficult and requires a lot of hard work. Why do you think it important enough to dedicate a significant amount of your time and effort to it?

In my opinion, among all scientists, it is the mathematicians who have done the worst job of advertising their field of study. It suffices to note that the overwhelming majority of the general public do not know that there is such a thing as "research mathematics". The excitement and pleasure of mathematical research is therefore worth describing, if only to stimulate the younger generation to try their luck in that field.

KMT is organized historically into eight chapters. Each of the first seven chapters reports on one main breakthrough in knot theory and revolves around a major scientific figure. Is the author's view of mathematical history, as A. Goriely wrote, "one of singular personal achievements by brilliant individuals rather than one of incremental buildup of unostentatious details"?

Yes, I think this is true, and not only of knot theory: great breakthroughs in mathematics usually come from brilliant individuals rather than groups of researchers.



Alexei Sossinsky

It is curious that the birth of knot theory is the result of a scientific flop: the attempt by Lord Kelvin in 1867, to model atoms by knots, which led Tait to establish the first table of alternating knots. What are the central problems in knot theory, nowadays?

At the present time I would put forward two. The first is to find a computer implementable unknotting algorithm: an algorithm which, given a knot diagram (a generic projection of the knot on a plane showing underpasses and overpasses at the crossing points), determines if it can be unknotted (transformed into a plane circle) and, if so, indicates how this may be done. (Unknotting algorithms exist, in particular the famous one due to Wolfgang Haken, but they are much too complicated to be implemented on even the most powerful computers.) The second is to find a complete system of invariants (preferably calculable by computer) that distinguishes knots. (According to a recent book by Serguei Matveev, there is an algorithm that distinguishes knots, but its result is not expressed as an invariant and it is not computer implementable.)

There are a number of strong knot invariants (such as the Alexander, Conway, Jones and HOMFLY polynomials), however they are not complete. But what about the deep and mysterious family of Vassiliev invariants, would you bet on its completeness?

It is amusing that, in the literature, the positive answer to this question is sometimes referred to as the "Vassiliev Conjecture", although Vassiliev himself believes that the answer is negative. So do I, but I have no serious arguments to support that claim.

The last chapter of KMT contains your own predictions and suggestions for future research. Do you still think that coincidences between different fields (for example between the various forms of the Yang-Baxter equation in statistical physics, in quantum physics, in operator theory, in braid and knot theory, or between Feynman diagrams in quantum field theory and key ingredients of the Kontsevich integral for the Vassiliev invariants) are more than just coincidences ?

Yes, I still do. But I must admit that my predictions to that effect (in 1998 I conjectured that a breakthrough in that direction by a then unknown researcher could occur in 2004) were too optimistic. So in the Russian version of the book (which appeared this year), I have changed the date to 2013 (incidentally, according to the Mayas, it will be the end of the world anyway).

So, maybe Lord Kelvin's idea of knots as a unifying physics principle might not be so bad after all ?

Yes, but perhaps at a higher level of sophistication, as some symbiosis between knots and strings (two-dimensional rather than one-dimensional objects), who knows...

In recent years, knot theory has been applied to molecular biology (with the realization that DNA sometimes is knotted). Some mathematicians think that the future of Mathematics is in Biology. What is your own view ?

I prefer the opposite claim: the future of Biology is in Mathematics. But this brash statement, as well as the opposite one, is something of an exaggeration. As an example, let me mention that my colleague, friend, and ex-compatriot Misha Gromov, the great geometer, declared a couple of years ago that he is giving up Mathematics for Biology. But I know that now he is doing some straight mathematics again (on the sly).

Professor Sossinsky, you were born in Paris in 1937, in a family of Russian emigrés. What prompted your parents to leave Russia ? (I read that your maternal grandfather, V. M. Chernov, was — until 1991, with the election of Yeltsin — the only democratic elected president of Russia, only to be overthrown by the Bolsheviks in 1917, less than 24 hours after his election by the short-lived Constituent Assembly).

Chernov was not really the president of Russia, he was only the president of the democratically elected Constituent Assembly (at which his party had won an absolute majority). He was very popular and would have undoubtedly become the President of the country, had the Constituent Assembly adopted a democratic constitution. So the Bolsheviks decided to disperse the assembly and arrest Chernov. But my grandfather, an experienced conspirator since tsarist times, got away. As to my parents, they had no other choice than to leave: my mother, Chernov's daughter, was exiled with her own mother (who had spent several years in a communist prison) in 1923, while my father, a young and dashing cavalry officer in the White Army, emigrated three years earlier, when the reds defeated the whites in the civil war. My parents met in Paris in the mid-twenties.

In 1948, your family moved from Paris to New York. Can you remember how you felt about leaving France for the United States ?

I was very excited and pleased at the prospect of travels and new experiences, I felt no special regrets at leaving Paris at the time.

Please tell us a little bit about your early education. Were you already interested in mathematics as a child ?

I was quite interested in arithmetic in kindergarten, but lost interest in mathematics for many years after I learned and understood the algorithms corresponding to the four arithmetical operations. I was never interested in the magic of numbers, although I remember I was rather excited when I learned the tests for divisibility by 3 and 9 of integers expressed in decimal notation. I went to school in France until the age of ten, then spent two years (fifth and sixth grade) in an American public school. When my parents realized I wasn't learning anything there (except spoken English, or rather American), they transferred me to a private French school, the *Lycée Français de New York*. It is there, in the class of *quatrième*, that I discovered real mathematics: algebra with equations involving variable numbers denoted by letters and the unknown x, plane geometry with axioms and proofs. It was the latter that attracted me most. The French curriculum, created by such great mathematicians and educators as Borel and Hadamard, had not yet been destroyed by Bourbaki, and difficult geometry problems with beautiful solutions, which you were invited to discover on your own, abounded.

What was it about mathematics that attracted you ?

Frankly, I just don't know. It happened suddenly at age 12, and it was love at first sight. Perhaps one factor was that, like many mathematicians, I was a timid child, and math (which came so easily to me) was a means to assert myself in the classroom. Another factor (but I may be inventing this *post factum*) is that mathematics is objective and democratic in the sense that how well you do in it is not a matter of opinion, not something imposed by some authority: if your solution is correct, it is **correct**, and nobody can do anything about it, they can't take it away from you. At 13 or 14 I learned that mathematics was still an open-ended science and decided that I would become a research mathematician. And of course at least as good as Gauss, Galois or Lobachevsky, if not better.

You obtained your BS degree from New York University in 1957. Was there anyone at NYU who was particularly influential ?

Oh yes! And what a person: Jean Van Heijenoort (a living legend, as I was later to discover) brought me back to the road to mathematics, from which for a while I was ready to diverge. What happened was that I was accepted at NYU with a year's credits for my French High school diploma (the *baccalauréat*), but without any credits for introductory calculus and algebra. As the result, I was not allowed to take any serious math courses, because I supposedly did not have the required prerequisites. When, a semester later, I convinced my faculty advisor to let me take some advanced math, the courses were so poor that, although I did very well, I did not find them interesting, and was disappointed and hesitant about opting for a math major. But the next semester I took Van H's (as we all called him) Advanced Calculus course, and I was back on track. I can talk for hours about the late Van H (shot to death at 70 in his bed by a jealous ex-wife), but I will simply refer you to his biography by Anita Feferman entitled *From Trotsky to Gödel*.

I was astonished when I got to know that, in the summer of 1957, you transferred from NYU to Moscow University (third year). This was for sure a very tough decision to make. How did you happen to come to the decision to make the switch from USA to a totally different life in Moscow ?

That's a difficult question (that I have been asked countless times over the years) to which I am not sure I really know the answer. Rationalizing *post factum*, I see three reasons: first, I rapidly realized that in 1957 Moscow was the best mathematical center of the world; second, my ties with Russian language and culture were stronger than those to the US and France (although I was well adapted both to the French and to the American lifestyles); finally, I was politically naive: although strongly anti-communist, I had leftist political views (and still have) and felt, at the time, that the uneducated boor Khrushchev would soon be replaced by an enlightened ruler and a more democratic "socialism with a human face" would prevail.

Do you have any regrets about that decision ?

Surprisingly, no. When Khrushchev was deposed and Brezhnev began re-implementing a harsh totalitarian regime, I had adapted to the scene, belonging to that unique fraternity of the leading mathematicians of Moscow and Leningrad, people who despised the powers to be, but managed to escape political pressure and lived in a close knit intellectual and professional oasis. This period is rather well described in a collection of articles published by the AMS under the evocative title "The Golden Years of Soviet Mathematics".

So, you studied at the Mechanics and Mathematics Department (Mekh-Mat) of Moscow University during its remarkable golden age. Who were the persons most responsible for those golden years at Mekh-Mat ?

Foremost was the President (or Rector, as we say in Russia) Ivan G. Petrovky. An outstanding (although very underrated) research mathematician, he was a strong, dedicated, and extremely ingenious administrator. Petrovsky succeeded in assembling the best mathematicians of the country at Mekh-Mat and created excellent working conditions for them. Although he was not a communist party member (very unusual at the time for a high level administrator), he concentrated a great deal of organizational power in his own hands and used it very efficiently to enrich the university, promote research, and raise the educational level in all the scientific departments (faculties, as we call them). The other key player in the complicated administrative games with (and often against) the party bureaucracy was Nikolai V. Efimov, the renowned geometer, who was the Dean of Mekh-Mat during the "golden years" and whose resignation for political reasons in 1969 marked the end of that extraordinary period. Two other people, both cautious liberals and great scientists, P.S.Alexandrov and A.N.Kolmogorov, were responsible for the scientific policies of Mekh-Mat and played important roles in enhancing the level of research there. Finally, I must mention I.M.Gelfand, whose seminar was arguably the greatest math seminar of all time (both literally – by the number of participants – and figuratively – by the quality and influence of the results discussed there).

Who would you say was the leading mathematician in Moscow at that time ?

Undoubtedly, Kolmogorov.

Who were some of the great students at the time ?

I belong to what I call the generation of 1937 (one of the bloodiest years in Russian history). It includes Manin, Anosov, Sinai, Alexeev, Tikhomirov, Arnold, Kirillov, Fuchs, Tyurina, Novikov, Shiryaev.

In some of your writings, and in one of your answers to a previous question, I noticed some bitter comments on the "new math" and the Bourbakization of high-school curricula. Is Russian mathematics, traditionally, more oriented to applications rather than abstraction ?

Today my strongly negative attitude to the "new math" (and "Bourbakized" high school textbooks) is shared by almost all math educators, and – thank God! – it has disappeared, at least in its original extreme forms, from all curricula. Concerning Russian mathematics, although less formalized than, say, French mathematics, I would not describe it as primarily applications-oriented. All the great Russian mathematicians, from Lobachevsky to Kontsevich via Chebychev, Markov, Alexandrov, Kolmogorov, Gromov, Arnold, Novikov, are famous for their fundamental mathematical theories, and not for work in the applications (if any). On the other hand, none of them considered themselves "pure mathematicians". Kolmogorov went further: for him there was no such thing as "pure" or "applied" mathematics, just good or bad mathematics; math purporting to be "pure", if good enough, eventually finds applications, while good math used to solve a concrete "applied" problem eventually evolves into a significant abstract theory.

You got your PhD at Mekh-Mat in 1966 with a thesis entitled "Multidimensional topological knots". Is it possible to give us some idea of the problems you were dealing with ?

The main result of my PhD thesis may be explained even to the layman: it asserts that any multidimensional knot can be decomposed into a finite composition of "prime factors" (just as any whole number can be decomposed into a product of prime numbers). The proof, unlike the statement of the result, is complicated and uses of some fairly sophisticated machinery involving homotopy groups, homological invariants, spectral sequences, etc.

Who influenced you most at Mekh-Mat ?

My thesis advisor, Lyudmila Keldysh, my fellow student and long time friend Dmitry Fuchs, and Kolmogorov.

What did you do after you obtained your PhD ?

I was offered an assistantship at the Chair of Topology and Geometry, headed by P.S.Alexandrov, and began, under good auspices, what promised to be a successful academic career at Mekh-Mat, one of the best mathematical centers of the world. A few years later, I was appointed associate professor at the same chair. But then serious problems began.

What problems ?

First of all, the "golden era" of Mekh-Mat was dramatically coming to an end. In 1968, the "Letter of the 99", a mild protest of 99 mathematicians against the forcible incarceration of Esenin-Volpin, the mathematical logician and human rights activist, in a psychiatric institution, was used by the authorities as a pretext to replace the liberal administration and party organization of Mekh-Mat by extreme reactionaries implementing the new totalitarian principles put forward by Brezhnev for monitoring "Soviet science" (as opposed to "bourgeois science").

The result of that were the sadly famous entrance examination problems at Mekh-Mat (used there to flunk Jewish applicants), a complete change of hiring policy (only docile people, approved by the party organization, had a chance for a position at Mekh-Mat), the ban on foreign travel (except for people selected by the KGB), and other measures of the same type. Trying to fight for the survival of the university, I.G.Petrovsky died of a heart attack in the waiting room of one of the big party bosses, a great loss for the cause of the liberals, already weakened by the forced resignation of N.V.Efimov from his position as Dean.

My own position became more and more insufferable, the new authorities were annoyed at my popularity with the students and my outspoken criticism of what was happening at the department, and harassed me in various ways...

In 1974 you were forced to resign from Moscow University for political reasons. What did you do after that ?

For a year I was unemployed (making a living by translating math books), until Kolmogorov succeeded in getting me a job as math editor of the famous popular science magazine "Kvant" (Quantum), where I worked for almost 13 years. (Apparently, I was on a KGB blacklist, so that my attempts to find a position more suited for a research mathematician were systematically blocked.)

By that time, you also taught at a rather peculiar institution, sometimes called the "Bella Muchnik University". Could you tell us something about that ?

Bella Muchnik, an alumnus of Mekh-Mat with a PhD, but working as an ordinary school teacher, decided to organize a math circle for students of engineering and other technical schools, who had been flunked at the Mekh-Mat entrance exams for being Jewish, part Jewish, or just too smart. This circle, which had some thirty participants, gathered thrice a week at Bella's tiny apartment and listened to lectures by a number of mathematicians (Sasha Vinogradov, Mitya Fuchs, Sacha Shen, Andrei Zelevinski, Arkady Vaintrob, to name only the first five who come to mind) and to solve problems. I remember that I taught Algebra, and then Calculus on Manifolds to this group of extremely talented and motivated students, some of whom managed to overcome what seemed to be insurmountable difficulties and eventually became research mathematicians, but only after emigrating from Russia. In a rare visit to the Soviet Union, John Milnor actually lectured to them in Bella's apartment, a beautiful talk on topology that I translated into Russian and still vividly remember. Later we managed to give Bella's math circle official status as an extracurricular math seminar at the Gubkin Oil and Gas Institute.

The whole enterprize ended tragically in the summer of 1982 (or was it 1983?) when Bella Muchnik was killed late at night in a deserted street in a traffic "accident" that we all believe was orchestrated by the KGB. One of the instructors (my friend V.Senderov) and one of the students were arrested by that same organization. The "university" did not survive these events.

How did the Independent University of Moscow start ?

The IUM is not really a university, it is a small elite school training future research mathematicians. But it should not be compared to Bella's university: it is a perfectly legal, officially licensed institution, whose creation became possible after *perestroïka* put an end to the Soviet totalitarian regime. It was created in 1992 at the initiative of N.N.Konstantinov (famous for his brilliant organization of math contests and extra-curricular activities outside the official educational establishment) by a group of mathematicians including Arnold, Faddeev, Feigin, Ilyashenko, Kirillov, Khovansky, Vassiliev, and others, in fact practically all the leading mathematicians of my generation or younger still based in Russia at the time. Rather than try to rejuvenate Mekh-Mat, which more than 20 years of mismanagement had been reduced from the leading mathematical center that it was in the 1960ies to a drab institution of average research and educational level, and which was still headed by the same people who were responsible for that sad state of affairs, it was decided to create something new.

Besides the enthusiasm of the people involved, we had nothing – no locale, no means of financial support, really nothing. In the first year we taught in the afternoons in a high school, the use of whose classrooms and main auditorium had been offered to us by a friendly director, to enthusiastic students from Mekh-Mat and other institutions (there were over a hundred in mid-semester at my Geometry course, which I taught in "simple mathematical English"). It was probably the only college ever where the professors had to pay for teaching: the school was provided to us rent-free, but we had to pay for the electricity and the work of the cleaning women.

You can find out about the subsequent history of the IUM, its Math in Moscow program and the closely associated Center of Continuous Mathematical Education (MCCME) on the web site www.mccme.ru/ium.

You have been involved in mathematical competition activities. What are your thoughts and experience concerning mathematical competitions for the young ?

While an undergraduate and graduate student at Moscow U, I was active in the math olympiads on the city, national, and international level. At the time, I was unreservedly in favor of such competitions, in particular as a means of selecting young talents and motivating them to study mathematics. Today, I have some reservations: imagine a fifteen year old Einstein or a Hilbert of the same age in the 1960ies in Russia or, say, Hungary. Interested in mathematics, he would certainly have participated in the local olympiads and ... failed miserably: it is well known that both Hilbert and Einstein were very slow, they totally lacked the competitive spirit and nimbleness of mind needed to succeed in olympiads. Discouraged, they would have abandoned mathematics – Einstein would have become a mediocre violinist, perhaps, and Hilbert, say, a school teacher very unpopular with his pupils. How many potential deep but slow thinkers, such as Hilbert or Einstein, have the olympiads deprived us of?

There is a way out of this incongruity. First, it should be clearly explained to students interested in math that success in olympiads is neither a necessary nor a sufficient condition for becoming a successful mathematician. Second, there should be alternative ways of testing mathematical aptitude, other than assessing problem solving capacity under stressful time limitations. Fortunately, in Russia today there exist other ways of attracting youngsters to mathematics, in particular the Dubna summer school "Contemporary Mathematics" or the remarkable Tournament of Towns Summer School, or the so-called "math battles" and "math regattas". (You can read about these on the MCCME web site). Other important events with the same functions, but aimed at university students rather than high school pupils, are the Budapest Semesters, the MASS program at Penn State and the Math in Moscow program at the IUM.

I know you have been involved both in the MASS and MiM programs. Can you tell us something about them ?

The MASS (Mathematics Advanced Study Semesters) program at Penn State University brings together undergraduates from different universities to study and do mathematics in novel and exciting ways. Besides math courses (on topics not usually included in undergraduate and even graduate curricula), weekly "colloquium talks" of expository nature by leading researchers, research projects (some of which have resulted in publications in respected math journals), the participants have their own seminar, where difficult problems are proposed without any imposed time limits. You can read about this in more detail in the book *Mass Selecta: Teaching and Learning Advanced Mathematics*, ed. by S.Katok, A.Sossinsky and S.Tabachnikov, AMS, 2003. About the Math in Moscow program, aimed mainly at North American students, let me say that it is organized by the IUM along more traditional lines (lecture courses with exercise classes), but the contents of the courses are research-oriented, there are a lot of proofs and problem solving. Details are available at the MCCME web site.

Do you notice any differences in the way people from different cultures do mathematics ?

I am only competent to compare the Russian and French mathematical cultures: I have no first hand familiarity with the British mathematical traditions and with Japanese mathematics, not enough with the German ones, and I am not sure there is such a thing as an American mathematical style. Although the French and Russian way were once very similar (to a great extent the latter was derived from the former), and in both cultures the weekly seminar, headed by a great maestro surrounded by his leading pupils, played a crucial unifying role, now things have been changed ... by Bourbaki. French math is highly formalized, its proponents present their results with great precision, but usually without explaining the motivations, whereas the Russians have a more intuitive, geometric and pragmatic no-nonsense style, closer to the Anglo-Saxon traditions (with which Russia historically had very little interplay). And yet, my feeling is that the accelerated internationalization of mathematics (as the saying goes, an American math department is a place where Russian professors teach Chinese graduate students) is not only removing the existing national barriers, but also progressively erasing the specifically national traits of doing mathematics.

If you had to mention one or two great moments in 20th century mathematics which one(s) would you pick ?

I would mention only one: Gödel's Incompleteness Theorem, arguably the most important scientific achievement of the past century, just as important, to my mind, for philosophy and for computer implementation as it is for mathematics and its foundations. I love to lecture about that theorem, and equally enjoy explaining the three very different proofs of it that I know (Gödel's original direct construction based on "Gödel numberings", the proof using a reduction to an undecidability theorem, which I learned from Kolmogorov, and the beautiful, although not too well known one, due to Chaitin and based on Kolmogorov complexity).

Alexei Sossinsky, lecturing on the Gödel's Incompleteness Theorem, in Luso, September 11, 2005.

You were the first director of the French-Russian Poncelet Mathematics Laboratory (CNRS-IUM) at Moscow. How did that come into being ?

Rather than retell the dull (if successful) story of the lab's creation (all about overcoming bureaucratic obstacles to create a novel form of scientific cooperation), let me tell you why it bears the name of Poncelet. Jean-Victor Poncelet, an alumnus of *École Polytechnique*, was a young *lieutenant du génie* in Napoleon's *Grande Armée* when it invaded Russia in 1812. In one of the skirmishes during the catastrophic winter retreat that followed, Poncelet was wounded and left for dead on the field of battle by his comrades-at-arms. He would have frozen to death if he had not been picked up by local peasants, who cured him of his wounds and surrendered him to Russian military authorities. He was a prisoner of war for three years in Saratov, where he did the research that he is now famous for, revolutionizing the field of projective geometry and later earning the unofficial title of "father of modern algebraic geometry". Back in France, he had a brilliant scientific career (although he never rose to the mathematical heights that he had achieved in Saratov), became a general and the director of *École Polytechnique*.

So we tell the young French researchers whom we hire to work at the lab that we will keep them prisoner in Moscow for at least a year, we expect them to do their best research there, and then they can be sure to have a brilliant career upon their return to France.

At the Poncelet lab you have a project ("Knots and braids") where you use the neologism "statistical topology" to name the area of study. Could you explain it ?

The term was actually coined by my friend and colleague Serguei Nechaev, who works at the laboratory of Statistical Models and Theoretical Physics at Orsay (near Paris) which is associated with the Poncelet lab in the project. It is very easy to explain: it deals with a wide range of objects with nontrivial topology (mostly one-dimensional, e.g. knots, braids, graphs) involved in random processes. A typical problem is to estimate the probability of a random closed curve in space to be knotted.

Outside Mathematics what are your interests ?

Like most mathematicians, I am lover of symphonic and chamber music. Like very many Russian mathematicians, I am (or rather was, the years now take their toll) a great enthusiast for the outdoors: long camping trips, on foot or on skis in the mountains, or on kayaks on white water. My real *violon d'Ingres*, however, is the translation of poetry from Russian to English, something I have done professionally for many years, and which helped me considerable in my early family life (the one US dollars per line paid to me in the 1960ies were an important contribution to the family budget). But more important in my non-mathematical life are my love of tennis (I still play in men's amateur tournaments, despite my age) and of the sea – swimming or surfing in the ocean waves give me the same degree of pleasure as proving a tough and beautiful theorem.

Interview by Jorge Picado (University of Coimbra)

ERCOM (European Research Centres on Mathematics)

by **Manuel Castellet** (Barcelona)

Chairman of ERCOM

The Director of the *Centro Internacional de Matemática* (CIM) is a member of ERCOM. What does it mean? How can CIM benefit from this membership? Shortly, what is ERCOM?

This article has been written, in the fourth year of my ERCOM chairmanship, with the aim to explain what ERCOM is, which are its objectives, how it is structured, which role can play in the European mathematical community.

ERCOM is a committee of the European Mathematical Society created in 1996 consisting of Scientific Directors of European Research Centres in the Mathematical Sciences, or their chosen representatives. Only European centres, predominantly research oriented, with a large international visiting programme and covering a broad area of the Mathematical Sciences are eligible for representation in ERCOM. The eligibility of centres is decided by the EMS Executive Committee.

The aims of ERCOM are to contribute to the unity of Mathematics, from fundamental to applications, with the purposes of constituting a forum for communication and exchange of information and to foster collaboration and co-ordination between the centres themselves and the EMS, to promote advanced research training on a European level, to advise the Executive Committee of the EMS on matters related to activities of the centres, to contribute to the visibility of the EMS, and to cultivate contacts with similar research centres within and outside Europe.

The 25 centres at present represented in ERCOM have different working and organisational structures, they are also different in size and scope, but they can be grouped in three different classes. Centres mainly for meetings, centres without permanent researchers, only visitors and post-docs, and centres with a small permanent staff and a large number of visitors. They broadly cover the European Geographical Area, from Russia and Sweden to Portugal and Israel. A Chairman, at present as Director of the Centre de Recerca Matemàtica, and a Vice-chairman, Kjell-Ove Widman, Director of the Institut Mittag-Leffler, co-ordinate the annual ERCOM meeting and the other initiatives proposed by the ERCOM members. Sir John Kingman, Director of the Isaac Newton Institute, acts as link between ERCOM and the EMS Executive Committee.

Prof. Manuel Castellet talking about ERCOM at the General Assembly of CIM, May 21, 2005.

In mathematical research, exchange of ideas plays a central role and needs a deep human contact. A contact which is the true mathematical laboratory, and, as Friedrich von Siemens said at the end of the nineteenth century, “Laboratories are the fundamental basis of knowledge and power”. The high degree of abstraction of mathematics and the compact way it is presented need direct personal communication, since mathematics is an attempt to develop tools that can be used to achieve a better understanding of the world's measurable aspects and to identify processes that appear in very different natural situations but that are essentially analogous.

In order to foster this deep human contact as well as collaboration and co-ordination between the European research institutes and to provide communication and exchange of information, ERCOM meets yearly, usually in March, in one of the member institutes. Since the 1999 meeting in Cambridge the Administrators have been invited to discuss separately and jointly with the Directors matters of their competence.

Several topics are considered for discussion in the meetings: The European Research Area and the presence of Mathematics in the Framework Programmes of the European Union, the annual call for post-doctoral grants by the European Post-doctoral Institute for the Mathematical Sciences (EPDI), the situation of Mathematics and the mathematicians in Eastern Europe, the INTAS programme, the Digital Math Library, issues arisen from the EMS Executive Committee, etc.

In particular, ERCOM has established an exchange programme for short-term visits of post-doctoral fellows between the centres. It is flexible and capable of adapting to different centres and situations, the main goal being to stimulate the exchange and the increase of knowledge and to create synergies enriching research.

Whereas the economic and social conditions in some countries, including parts of Africa, Asia and the former Soviet Union, make it difficult for graduate students and postdocs to establish international contacts, ERCOM encourages its member centres to institute and seek funds for programmes intended to facilitate visits by young mathematicians from disadvantaged geographical areas. On its web page, ERCOM will create links to the member centre web pages describing such programmes.

The Administrators usually discuss practical matters in their meetings: ways of funding, legal situation of the researchers in the different countries, the electronic poster, the web page, update the information of the centres represented in ERCOM, presence of women in the ERCOM centres... In March 2004 in the Aarhus meeting the following resolution was adopted: "ERCOM encourages its members to take actions to facilitate the presence of women in their scientific activities, and to collect data regarding the number of applications from female researchers received and approved".

Although the EPDI is not an ERCOM activity, I would like to mention it here because of the strong connection between both bodies. The EPDI (European Post-doctoral Institute for the Mathematical Sciences) joins the ERCOM centres of Bûres-sur-Yvette, Bonn, Cambridge, Leipzig, Vienna, Stockholm, Warsaw and Barcelona, plus the Forschungsinstitut für Mathematik in Zürich. It annually offers 5 two-year grants to recent PhD's with the requirement for the fellow to make 6 to 12 month stays at EPDI institutes (or at a Japanese university) conducting scientific achievements related to the areas of interest of the candidates.

ERCOM, as a committee of the European Mathematical Society, wishes and has the capacity to act as one of the EMS means of scientific outreach, fostering the presence of mathematicians in the new emerging multidisciplinary research domains. With this aim a group of research centres represented in ERCOM submitted last year to the European Commission the proposal Shaping New Directions in Mathematics for Science and Society (MATHFSS), as a NEST Support Action through a Co-ordination Action instrument, with the main objective of designing curricula and finding ways to train researchers into emerging specialities of Mathematics, linked especially with System Biology, Neuroscience, Risk Assessment and Data Security, and identifying future research opportunities on the interface between Mathematics and suitable areas of Medicine, Industry and Social Sciences. The proposal has been approved and a contract for two years has been recently signed.

A home page (www.crm.es/ERCOM), from which you can reach that of each ERCOM centre, provides information on the ERCOM initiatives and, in particular, on the MATHFSS project, open positions offered by the centres and scheduled conferences and training courses.

List of centres represented in ERCOM ordered by alphabetical order:

Alfréd Rényi Institute of Mathematics (Budapest, Hungary, www.renyi.hu)

Centre de Recerca Matemàtica (Barcelona, Spain, www.crm.es),

Centre International de Rencontres Mathématiques (Marseille, France, www.cirm.univ-mrs.fr),

Centro di Ricerca Matematica Ennio De Giorgi (Pisa, Italy, www.crm.sns.it),

Centro Internacional de Matemática (Coimbra, Portugal, www.cim.pt),

Centrum voor Wiskunde en Informatica (Amsterdam, Netherlands, www.cwi.nl),

Emmy Noether Research Institute for Mathematics (Ramat Gran, Israel, www.cs.biu.ac.il/∼eni),

Erwin Schrödinger International Institute for Mathematical Physics (Vienna, Austria, www.esi.ac.at),

Euler International Mathematical Institute (St. Petersburg, Russia, www.pdmi.ras.ru/EIMI),

European Institute for Statistics, Probability and Stochastic Operations Research (Eindhoven, Netherlands, www.eurandom.nl),

Forschungsinstitut für Mathematik (Zürich, Switzerland, www.fim.math.ethz.ch),

Institut des Hautes Études Scientifiques (Bûres-sur-Yvette, France, www.ihes.fr),

Institut Henri Poincaré, Centre Émile-Borel (Paris, France, www.ihp.jussieu.fr),

Institut Mittag-Leffler (Stockholm, Sweden, www.mittag-leffler.se),

International Centre for Mathematical Sciences (Edinburgh, United Kingdom, www.icms.org.uk), Isaac Newton Institute for Mathematical Sciences (Cambridge, United Kingdom, www.newton.cam.ac.uk),

Istituto Nazionale di Alta Matematica Francesco Severi (Rome, Italy, www.altamatematica.it),

Lorentz Center (Leiden, Netherlands, www.lc.leidenuniv.nl),

Mathematical Research Institute Universiteit Utrecht (Utrecht, Netherlands, mri.math.uu.nl),

Mathematisches Forschungsinstitut Oberwolfach (Oberwolfach, Germany, www.mfo.de),

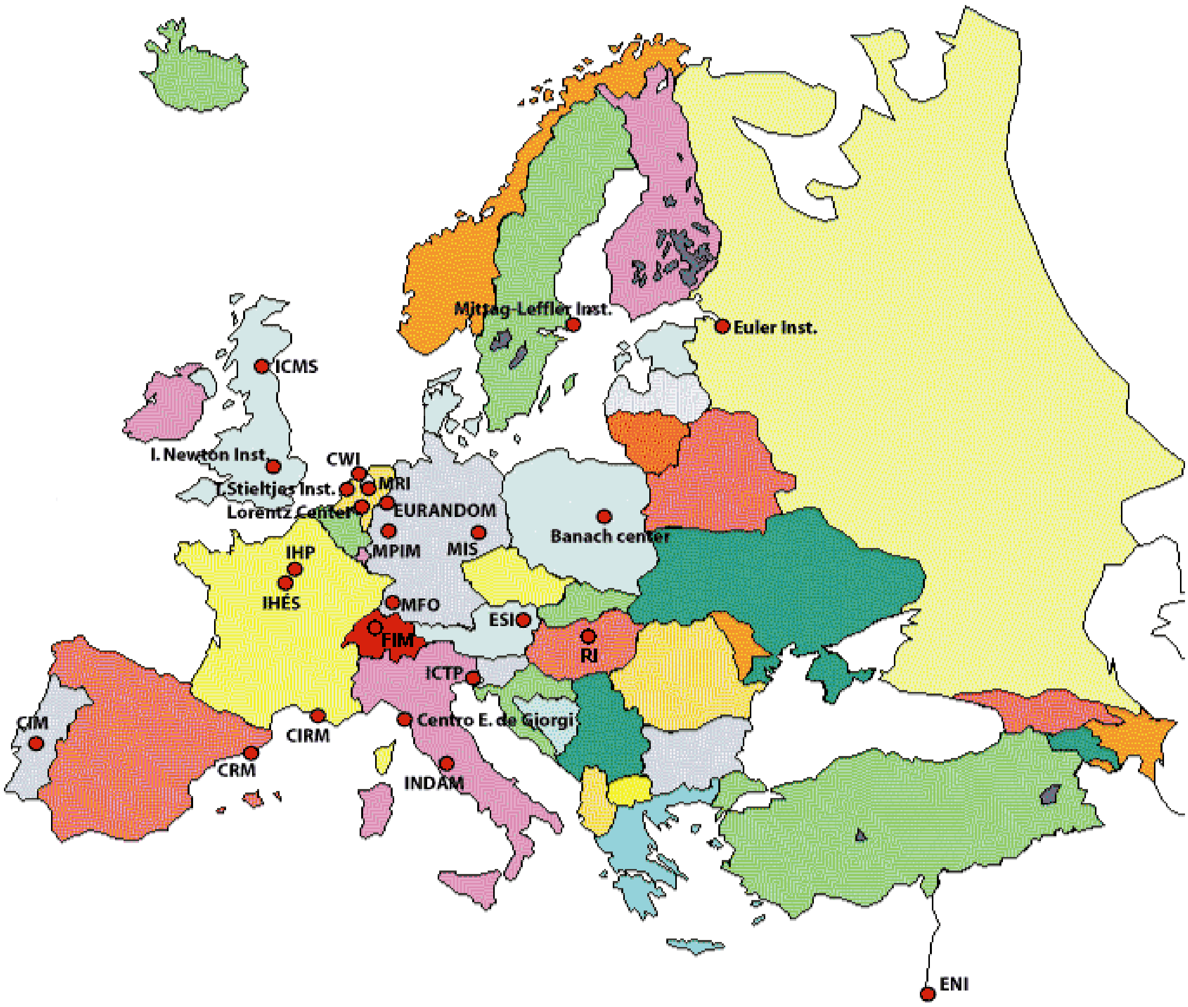
Max-Planck-Institut für Mathematik in den Naturwissenschaften (Leipzig, Germany, www.mis.mpg.de),

Max-Planck-Institut für Mathematik (Bonn, Germany, www.mpim-bonn.mpg.de),

Stefan Banach International Mathematical Center (Warsaw, Poland, www.impan.gov.pl/BC),

The Abdus Salam International Centre for Theoretical Physics (Trieste, Italy, www.ictp.it),

Thomas Stieltjes Institute for Mathematics (Leiden, Netherlands, www.stieltjes.org).



ERCOM centres

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