An Interview with Hans Triebel

Hans, you are immediately recognized in the mathematical analysis community by your expertise within the theory of function spaces, your books on the subject serving as unavoidable reference. Can you tell us a little about how this interest started ?

Thank you for the compliments, António, you are very kind. Indeed, you may be surprised that this interest developed somehow by chance.
My Ph.D. project, supervised by Professor Maier in Jena, was concerned with Lamé's differential equation, that is, complex function theory. Later an elder colleague recommended me Sobolev's book from 1950, which I studied with great interest since I was always fascinated both by mathematics and physics.
In the past it was also quite usual in an academic career in the former GDR (East Germany) to go for one year abroad, but the difference was that there was not a big choice. Certainly one could apply for one or the other university in the East, mainly within the Soviet Union, but the decision was made somewhere else by the authorities. In my case it finally turned out to be a rather lucky circumstance to send me to Leningrad (now St. Petersburg) though this has not been my first choice. So before leaving to Leningrad I polished up my Russian learned at school – but even then it was not so easy at the beginning. Later my pronunciation improved such that people did not immediately recognise me as a foreigner, but at the beginning . . . you may recall that in 1963/64 when I came to Leningrad, less than 20 years had passed since the end of World War II. In other words, there still lived many people who had suffered from the Germans, especially in this town. So I was a bit afraid when I arrived, but my experience was that I was met with a kind reception. What concerns Leningrad university, I had no direct personal contact there, I mainly worked on my own and read a lot of books. But I enjoyed the very active and inspiring atmosphere due to many great mathematicians working there. In particular, I had the great pleasure to attend lectures by Birman on functional analysis, spectral theory, quantum mechanics – he really was an impressive lecturer. His main concern was at that time applications to the spectral theory of partial differential equations, using methods from functional analysis. So to study function spaces was a natural task in this direction. Other people working there included, of course, Solomyak, but also Uraltseva and Ladyzhenskaya who's seminar I attended. Later, back in Jena, I read Nikol'skiĭ's book from 1969 . . . but at that time I had already started working on function spaces myself.

Hans Triebel (photo by Alexandre Almeida, used with permission).

Occasionally, when talking about function spaces with other people, I have heard them wondering about the reason the letter F is used for the so-called Triebel-Lizorkin (or Lizorkin-Triebel) spaces ...

Honestly, there is no mystery at all about this letter (in contrast to other spaces and their letters which caused longstanding stories and discussions afterwards). As I occasionally explained, it was the first 'suitable', i.e., free letter when I needed one for the new spaces and invented this symbol around 1970. I even had some concern that it may cause confusion with Fréchet spaces, but it turned out later that this was not the case. As far as I remember, the symbol made its first official appearance in two of my papers in 1973 (it always took very long to get all the necessary permissions to publish some paper abroad).

Before choosing function spaces, you had to choose mathematics as a subject to study. Was it already your childhood dream to become a (famous) mathematician ?

Not at all! I really liked all the science subjects in school, mainly mathematics, physics and chemistry. So when it came to choose a subject to study I hesitated what to take. But then someone suggested that I should try physical chemistry since this was expected to have a bright future soon — and shared the advantage to combine at least two of my favourite subjects. As recommended, I applied for physical chemistry in Jena. As

a second choice only I had named mathematics. Unfortunately my application was not approved probably because of a rather bad evaluation of my insufficiently developed socialistic personality. Nevertheless the university in Jena invited me to follow some other career: they offered me a place to study mathematics and physics to become a teacher. Though I really never wanted to teach at schools, I accepted this offer since I was told that at the beginning many lectures are the same for diploma students and teacher students in mathematics and physics and changing after a while would be much easier. This was in fact the case and I followed both, mathematics and physics, almost up to the end in a parallel way. Apart from very few tasks at the end I could have also completed physics with a diploma like mathematics, but for some reason I only did it in mathematics.

I was looking in the Mathematics Genealogy Project and found out that you are a mathematical descendant of Gauss and Weierstrass. What does it mean to you ?

Nothing particular, I would say. I was amused when I discovered it first time – and I enjoy to point it out to my Ph.D. students and their Ph.D. students that they now enter the famous descendant line of Gauss and Weierstrass in $n+1$st, $n+2$nd generation with my humble person in between.

You obtained your Ph.D. from the University of Jena in 1962 and I have always known of your name in connection with this same university. Apart from the one year abroad which you have already mentioned, have you been there all this time, and if yes, for what reason ?

An academic career in GDR times was in some sense very much different from what you would expect nowadays – and what all my younger Ph.D. students experience now. Almost everything was more restricted, not only publication in 'Western' journals as already mentioned, and – of course – going abroad for research stays or only to take part in conferences was very difficult. But also life was more steady and more regulated at that time, so for many reasons it was not easy and also not usual to move too often. Apart from the year in Leningrad I worked a year outside of the university in a company after I had received my diploma. Otherwise I followed the academic career in Jena. Only at the beginning of the 1970s I really thought of leaving Jena for many different reasons, including professional ones. Finally I decided otherwise and stayed there until my retirement some years ago. But as you know, António, we have so many fine, well-equipped spaces with sufficiently many dimensions, what influence should a three- or four-dimensional world in a medium-sized town like Jena have then in the end?

With S.M. Nikolskii in May 2005 (Moscow), at the Conference celebrating his 100th birthday (photo by Alexandre Almeida, used with permission).

Can you tell us about mathematicians that have influenced you most ? Also some that you interacted with in some crucial moments in the development of the theory of function spaces.

Apart from Birman and Solomyak who I met first during my time in Leningrad, I would name here S.G. Krejn. I think it was at the mathematical congress in Moscow 1966 where I first get to know him. Another colleague that influenced and motivated my studies essentially at some time is certainly Jaak Peetre from Lund. As far as I remember we first talked in Berlin in 1969, where I really understood some advantage of Besov spaces (defined by differences) showing up as interpolation spaces from Sobolev spaces. Later in Lund he directed my interest to the book of Stein from 1970, which also had consequences on my further studies in function spaces. Indeed, in 1975 when I stayed for some time in the Banach Center in Warsaw I really considered to change subjects and turn to the theory of relativity where I already lectured about in Jena. This fascinated me very much – and, in addition, I thought that I am finished at some level with function spaces. I had completed my habilitation thesis about function spaces and nonlinear analysis, had already become a professor at the age of 34. So I thought it a good opportunity to concentrate on something else. But just in Warsaw I read some papers from Fefferman and Stein about the Fourier analytical approach to function spaces ... and this convinced me that research in function spaces is

not outdated. As you know yourselves, António, there is still a lot to do and, even worse or better, there are so many new surprising connections to other areas, not only of analysis, and further ideas, open questions that serve as source of Ph.D. projects, admit to write papers and books, collaborate with colleagues ...

You are always full of new ideas in your research. Is there some recipe we can learn about? How do your new ideas usually come?

Unfortunately I do not have a special method or secret that I could share with you. Ideas come from time to time, I rather have the belief of a sea of potential thoughts and ideas, that only partly and occasionally become more detailed and visible. Certainly essential from my point of view is to read a lot of specific literature, I always consumed various monographs and papers, but also textbooks. Moreover, I meanwhile get many questions, whether in seminars or at conferences, that initiate further conclusions or interesting questions. Nowadays I even receive many emails with more or less tricky problems. But I would not say that I systematically search for new ideas, they rather come to me sporadically.

Giving a lecture during the OTFUSA Conference held in Aveiro in July 2005.

I think one of the first impressions people have when meeting you is that you are a very happy person, always willing to play with words or with unusual (or even common) situations. Together with your easiness in getting a good laugh and the expressive way you put in teaching, maybe this is one of the explanations for the huge number of Ph.D.'s that you have supervised: the number 36 is impressive, and still growing. Do you have a secret recipe for this?

Sorry, but I have to disappoint you again: there is no special trick at all. Even worse, I never really propagated fascinating Ph.D. topics in order to attract especially good students, they rather came by themselves and asked for something to concentrate on. Of course, when I was very much involved in teaching duties I knew many students – and they knew me. So it was easier to come in contact and to promote some of them. Later, in particular when we had the graduate school in Jena, there sometimes appeared the phenomenon that young students were directly sent to me from their supervisors abroad, in order to do a Ph.D. in Jena under my supervision, sometimes already with some special interest and well-prepared mathematical knowledge.

Let us still talk about this graduate school in mathematics, which you have had in Jena, already for some years. Can you tell us how this works? ... This is a topic of special interest nowadays for Portuguese universities, because there has been a trend to set such doctoral programs, though not always backed up with the funds necessary to support students!

We hosted two graduate schools during the last 15 years: the first one with the title 'Analytic and stochastic structures and systems' lasted for the maximal number of years from 1992 until 2002. At that time usually around 10 professors of a faculty (or different faculties) submitted an application and described some topic of joint interest which was wide and promising enough to admit sufficiently many Ph.D. projects and further research, but should also be concentrated enough to have a substantial kernel of collaboration within the different research groups. In the lucky case it is then approved for 3 years and this procedure can be repeated twice at most. The final year is then given to complete the last projects. In our case we had grants for 12 Ph.D. students and 2 Postdoc positions (per three year period), that is, the Ph.D. grants were given usually for 2+1 year, the Postdoc position for one or two years. Students had to apply and were chosen by this small group of professors forming the graduate school in view of their submitted documents and a talk before the audience. In addition to the personal grants for the graduate students (around 1000 Euros at that time, as far as I remember) we received extra money to invite guests, to finance a small separate special library, to support research and conference stays of the students in a modest way, and to organise two workshops or conferences per year. For a long time I was the speaker of this graduate school which was the first mathematical one within the former GDR territory and the first at all that was installed in Thuringia, the federal state Jena belongs to. Apart from the convenient situation to have Ph.D. positions at all and to have some money to spend for conferences, guests and books, the main advantage was in my opinion the uncomplicated and direct administration with short connections between all the people involved, Ph.D. students as well as professors. Our graduate school really worked successfully, almost all Ph.D. theses could be completed.
There existed a second graduate school from 2002 un-

til 2006 in our faculty, this time in combination with applied mathematics and computer science, called 'Approximation and algorithms'. It followed essentially the same scheme.

Would you like to comment about other avenues that your research has taken, besides the concern with the function spaces? I'm thinking, in particular, that for quite some time the underlying domains which you were considering were smooth ones, and afterwards, maybe during the 1990s, you started to systematically consider irregular, even fractal, sets.

The close connection to fractal geometry turned out within the Ph.D. project of Heike Winkelvoss (who, by the way, also had a grant from the graduate school we talked about before). At that time the atomic decomposition in function spaces was already available and sufficiently developed to serve as building blocks also for spaces on fractals. This localised description fits pretty well to the nature of fractals, or, more precisely, d-sets and generalisations like h-sets, which were investigated by Michele Bricchi, another of my Ph.D. students living on a grant from the graduate school. Similarly other areas like wavelet theory entered function spaces scene whenever needed and appropriate. Of course, these extensions to the theory of function spaces are very much welcome.

And what about outside mathematics? Is there anything – certainly less interesting than function spaces – that you enjoy doing when not concerned with mathematics?

Well, it is not very exciting, I confess: it is again reading what I like. In particular, I am more and more interested in historical topics, especially linked to mathematics or physics. I am fascinated by the way in which scientists and science developed in the past. You may imagine me sitting in my garden, reading and reading – and the only witnesses for this picturesque scene are brave birds, shy deer and old trees ...

Which mathematicians do you admire particularly? Do you have a favourite mathematician from before the 20th century? And from the 20th century?

Certainly Archimedes, Pythagoras and Riemann. Concentrating on the last century, then I would first mention Einstein, especially how he came from special to general relativity. Secondly, there is, of course, David Hilbert who can be seen in some sense as successor of Pythagoras in his approach of assumptions and proofs. One of his great credits may be the idea to mathematise physics by models. Finally, related to my field of analysis, let me refer to Sobolev and Laurent Schwartz.

If you had to mention one or two great moments in 20th century mathematics which ones would you pick?

Probably one should allude to the proof of Fermat's Last Theorem by Andrew Wiles here, and to the contribution to the continuum hypothesis by Paul Cohen. But related to my field of research, this is doubtless the discovery of distributions by Laurent Schwartz. One can read in his memories that in the beginning the mathematical society behaved rather hostile against this new ideas, or better to say, the general opinion was split: a smaller part of his colleagues regarded this approach to be ingenious, whereas the majority thought it too simple to be useful and far-reaching. But they were wrong obviously. Nowadays, this theory well-equipped with the tools of Fourier analysis, essentially included and further developed within the concept of function spaces, becomes more and more the language of numerical analysis, too. It took some time until Laurent Schwartz became famous for his discovery.

One of your former students once told me that you know exactly where things should lead to in your area of research and that you have a program to get there. I myself can testify that you have strong feelings about the truth or falsity of some conjectures. Would you like to share with us some clues about important results in your areas of interest that should be possible to prove in the near (or not so near) future?

You are very kind, thank you. But thinking about it, yes, I guess you are right, there are very rare occasions when I was mistaken in my assumptions. The reason might be, that I have a certain feeling for the topography of the territory of function spaces. So I rather have the idea to inspect hidden caves, whether they are promising or boring. There is some inner voice which usually prevents me from falling into a trap, that is, I better circumvent dangerous parts of this area. Sometimes I find something what I have not looked for, this may lead to a Ph.D. topic or a paper afterwards, but not always. In such cases I collect these pieces of new ideas in some small booklet. I see myself strolling around on my own, sometimes listening to music by Bach during these walks ... But to avoid misunderstanding, I do not systematically dig and find new plants in this function space territory, I rather feel like promenading in a fog of thoughts and ideas which only by chance get caught by me. In other words, I cannot predict what I will find next – or what you asked me about future developments. Probably we should meet in some years again and then I will review and honestly tell you what important results could be proved in the past.

Interview by António Caetano (University of Aveiro) and Dorothee Haroske (University of Jena)

Hans Triebel (born February 7, 1936 in Dessau) has retired from Friedrich-Schiller-University Jena (Germany) in 2001, where he was Professor (Chair) in Analysis for more than 30 years, after earning there his Ph.D. (1962) and Habilitation (1966). He also served as Dean of its Faculty of Mathematics and Computer Science for the period 1990-93 and as Speaker of the Graduate College "Analytic and Stochastic Structures and Systems" between 1993 and 2002.

Professor Triebel has written more than 170 papers in the areas of Function spaces, Functional Analysis, Interpolation Theory, Partial Differential Equations and Fractal Analysis and has 13 titles in the list of written textbooks and monographs (one further addition to this collection being in preparation at this moment). Perhaps he is best known by the series of books he has written on the Theory of Function Spaces and its relations with other parts of Mathematical Analysis. He also served as editor of 7 volumes of Proceedings, belongs to the editorial boards of 7 international journals in mathematics and, as yet, supervised 36 Ph.D. theses.

He was a corresponding member of the Academy of Sciences of the former GDR between 1978 and 1987 and a full member of the same Academy between 1987 and 1992. Since 1993 he is a regular (full) Member of the Academy of Sciences of Berlin-Brandenburg. He was awarded, in 1983, the National Prize (of third order) of the former GDR for Science and Technology and, in 1990, a D.Sc.h.c. by the University of Sussex at Brighton (UK).