AN INTERVIEW WITH ALEXEI SOSSINSKY

Professor Sossinsky, can we start this interview with the genesis of your celebrated book "Knots, mathematics with a twist" (KMT)?

Certainly. I wrote the whole book (in French) during the summer of 1998 in my isba in a tiny village 120 kilometers from Moscow, without access to any literature on the subject. For me it was more of a linguistic experience than a mathematical one: I had never written any expository mathematical texts in French before, in fact for thirty years I had written almost nothing in that language (one of my mother tongues – I have two). The actual writing process was a return to and an immersion into the French literary language and culture, which the writing process was extricating from the depths of my subconscious. The pleasure of writing, as well as the total absence of math books and journals in my village, resulted in several unfortunate mathematical errors in the original manuscript; these were never corrected – my French editors published the book without sending me the proofs.

In how many languages has it been published?

I know of seven: French, Rumanian, German, Italian, English, Finnish, and Russian. I have heard that a Portuguese edition is planned.

What makes knot theory so popular?

Well, knot theory is very visual, mathematical knots have numerous models in real life (from ropes and cables to DNA), its main problems are simple to formulate but very difficult to solve, it has applications to (or at least deep relationships with) several branches of physics and mathematical analysis, biology, and biochemistry.

Good popularization is difficult and requires a lot of hard work. Why do you think it important enough to dedicate a significant amount of your time and effort to it?

In my opinion, among all scientists, it is the mathematicians who have done the worst job of advertising their field of study. It suffices to note that the overwhelming majority of the general public do not know that there is such a thing as "research mathematics". The excitement and pleasure of mathematical research is therefore worth describing, if only to stimulate the younger generation to try their luck in that field.

KMT is organized historically into eight chapters. Each of the first seven chapters reports on one main breakthrough in knot theory and revolves around a major scientific figure. Is the author's view of mathematical history, as A. Goriely wrote, "one of singular personal

achievements by brilliant individuals rather than one of incremental buildup of unostentatious details"?

Yes, I think this is true, and not only of knot theory: great breakthroughs in mathematics usually come from brilliant individuals rather than groups of researchers.



Alexei Sossinsky

It is curious that the birth of knot theory is the result of a scientific flop: the attempt by Lord Kelvin in 1867, to model atoms by knots, which led Tait to establish the first table of alternating knots. What are the central problems in knot theory, nowadays?

At the present time I would put forward two. The first is to find a computer implementable unknotting algorithm: an algorithm which, given a knot diagram (a generic projection of the knot on a plane showing underpasses and overpasses at the crossing points), determines if it can be unknotted (transformed into a plane circle) and, if so, indicates how this may be done. (Unknotting algorithms exist, in particular the famous one due to Wolfgang Haken, but they are much too complicated to be implemented on even the most powerful computers.) The second is to find a complete system of invariants (preferably calculable by computer) that distinguishes knots. (According to a recent book by Serguei Matveev, there is an algorithm that distinguishes knots, but its result is not expressed as an invariant and it is not computer implementable.)

There are a number of strong knot invariants (such as the Alexander, Conway, Jones and HOMFLY polynomials), however they are not complete. But what about the deep and mysterious family of Vassiliev invariants, would you bet on its completeness?

It is amusing that, in the literature, the positive answer to this question is sometimes referred to as the "Vassiliev Conjecture", although Vassiliev himself believes that the answer is negative. So do I, but I have no serious arguments to support that claim.

The last chapter of KMT contains your own predictions and suggestions for future research. Do you still think that coincidences between different fields (for example between the various forms of the Yang-Baxter equation in statistical physics, in quantum physics, in operator theory, in braid and knot theory, or between Feynman diagrams in quantum field theory and key ingredients of the Kontsevich integral for the Vassiliev invariants) are more than just coincidences?

Yes, I still do. But I must admit that my predictions to that effect (in 1998 I conjectured that a breakthrough in that direction by a then unknown researcher could occur in 2004) were too optimistic. So in the Russian version of the book (which appeared this year), I have changed the date to 2013 (incidentally, according to the Mayas, it will be the end of the world anyway).

So, maybe Lord Kelvin's idea of knots as a unifying physics principle might not be so bad after all?

Yes, but perhaps at a higher level of sophistication, as some symbiosis between knots and strings (two-dimensional rather than one-dimensional objects), who knows...

In recent years, knot theory has been applied to molecular biology (with the realization that DNA sometimes is knotted). Some mathematicians think that the future of Mathematics is in Biology. What is your own view?

I prefer the opposite claim: the future of Biology is in Mathematics. But this brash statement, as well as the opposite one, is something of an exaggeration. As an example, let me mention that my colleague, friend, and ex-compatriot Misha Gromov, the great geometer, declared a couple of years ago that he is giving up Mathematics for Biology. But I know that now he is doing some straight mathematics again (on the sly).

Professor Sossinsky, you were born in Paris in 1937, in a family of Russian emigrés. What prompted your parents to leave Russia? (I read that your maternal grandfather, V. M. Chernov, was — until 1991, with the election of Yeltsin — the only democratic elected president of Russia, only to be overthrown by the Bolsheviks in 1917, less than 24 hours after his election by the short-lived Constituent Assembly).

Chernov was not really the president of Russia, he was only the president of the democratically elected Constituent Assembly (at which his party had won an absolute majority). He was very popular and would have undoubtedly become the President of the country, had the Constituent Assembly adopted a democratic constitution. So the Bolsheviks decided to disperse the assembly and arrest Chernov. But my grandfather, an experienced conspirator since tsarist times, got away. As to my parents, they had no other choice than to

leave: my mother, Chernov's daughter, was exiled with her own mother (who had spent several years in a communist prison) in 1923, while my father, a young and dashing cavalry officer in the White Army, emigrated three years earlier, when the reds defeated the whites in the civil war. My parents met in Paris in the midtwenties.

In 1948, your family moved from Paris to New York. Can you remember how you felt about leaving France for the United States?

I was very excited and pleased at the prospect of travels and new experiences, I felt no special regrets at leaving Paris at the time.

Please tell us a little bit about your early education. Were you already interested in mathematics as a child?

I was quite interested in arithmetic in kindergarten, but lost interest in mathematics for many years after I learned and understood the algorithms corresponding to the four arithmetical operations. I was never interested in the magic of numbers, although I remember I was rather excited when I learned the tests for divisibility by 3 and 9 of integers expressed in decimal notation. I went to school in France until the age of ten, then spent two years (fifth and sixth grade) in an American public school. When my parents realized I wasn't learning anything there (except spoken English, or rather American), they transferred me to a private French school, the Lycée Français de New York. It is there, in the class of quatrième, that I discovered real mathematics: algebra with equations involving variable numbers denoted by letters and the unknown x, plane geometry with axioms and proofs. It was the latter that attracted me most. The French curriculum, created by such great mathematicians and educators as Borel and Hadamard, had not yet been destroyed by Bourbaki, and difficult geometry problems with beautiful solutions, which you were invited to discover on your own, abounded.

What was it about mathematics that attracted you?

Frankly, I just don't know. It happened suddenly at age 12, and it was love at first sight. Perhaps one factor was that, like many mathematicians, I was a timid child, and math (which came so easily to me) was a means to assert myself in the classroom. Another factor (but I may be inventing this post factum) is that mathematics is objective and democratic in the sense that how well you do in it is not a matter of opinion, not something imposed by some authority: if your solution is correct, it is **correct**, and nobody can do anything about it, they can't take it away from you. At 13 or 14 I learned that mathematics was still an open-ended science and decided that I would become a research mathematician. And of course at least as good as Gauss, Galois or Lobachevsky, if not better.

You obtained your BS degree from New York University in 1957. Was there anyone at NYU who was particularly influential?

Oh yes! And what a person: Jean Van Heijenoort (a living legend, as I was later to discover) brought me back to the road to mathematics, from which for a while I was ready to diverge. What happened was that I was accepted at NYU with a year's credits for my French High school diploma (the baccalauréat), but without any credits for introductory calculus and algebra. As the result, I was not allowed to take any serious math courses, because I supposedly did not have the required prerequisites. When, a semester later, I convinced my faculty advisor to let me take some advanced math, the courses were so poor that, although I did very well, I did not find them interesting, and was disappointed and hesitant about opting for a math major. But the next semester I took Van H's (as we all called him) Advanced Calculus course, and I was back on track. I can talk for hours about the late Van H (shot to death at 70 in his bed by a jealous ex-wife), but I will simply refer you to his biography by Anita Feferman entitled From Trotsky to Gödel.

I was astonished when I got to know that, in the summer of 1957, you transferred from NYU to Moscow University (third year). This was for sure a very tough decision to make. How did you happen to come to the decision to make the switch from USA to a totally different life in Moscow?

That's a difficult question (that I have been asked countless times over the years) to which I am not sure I really know the answer. Rationalizing post factum, I see three reasons: first, I rapidly realized that in 1957 Moscow was the best mathematical center of the world; second, my ties with Russian language and culture were stronger than those to the US and France (although I was well adapted both to the French and to the American lifestyles); finally, I was politically naive: although strongly anti-communist, I had leftist political views (and still have) and felt, at the time, that the uneducated boor Khrushchev would soon be replaced by an enlightened ruler and a more democratic "socialism with a human face" would prevail.

Do you have any regrets about that decision?

Surprisingly, no. When Khrushchev was deposed and Brezhnev began re-implementing a harsh totalitarian regime, I had adapted to the scene, belonging to that unique fraternity of the leading mathematicians of Moscow and Leningrad, people who despised the powers to be, but managed to escape political pressure and lived in a close knit intellectual and professional oasis. This period is rather well described in a collection of articles published by the AMS under the evocative title "The Golden Years of Soviet Mathematics".

So, you studied at the Mechanics and Mathematics De-

partment (Mekh-Mat) of Moscow University during its remarkable golden age. Who were the persons most responsible for those golden years at Mekh-Mat?

Foremost was the President (or Rector, as we say in Russia) Ivan G. Petrovky. An outstanding (although very underrated) research mathematician, he was a strong, dedicated, and extremely ingenious administrator. Petrovsky succeeded in assembling the best mathematicians of the country at Mekh-Mat and created excellent working conditions for them. Although he was not a communist party member (very unusual at the time for a high level administrator), he concentrated a great deal of organizational power in his own hands and used it very efficiently to enrich the university, promote research, and raise the educational level in all the scientific departments (faculties, as we call them). The other key player in the complicated administrative games with (and often against) the party bureaucracy was Nikolai V. Efimov, the renowned geometer, who was the Dean of Mekh-Mat during the "golden years" and whose resignation for political reasons in 1969 marked the end of that extraordinary period. Two other people, both cautious liberals and great scientists, P.S.Alexandrov and A.N.Kolmogorov, were responsible for the scientific policies of Mekh-Mat and played important roles in enhancing the level of research there. Finally, I must mention I.M.Gelfand, whose seminar was arguably the greatest math seminar of all time (both literally – by the number of participants – and figuratively – by the quality and influence of the results discussed there).

Who would you say was the leading mathematician in Moscow at that time?

Undoubtedly, Kolmogorov.

Who were some of the great students at the time?

I belong to what I call the generation of 1937 (one of the bloodiest years in Russian history). It includes Manin, Anosov, Sinai, Alexeev, Tikhomirov, Arnold, Kirillov, Fuchs, Tyurina, Novikov, Shiryaev.

In some of your writings, and in one of your answers to a previous question, I noticed some bitter comments on the "new math" and the Bourbakization of high-school curricula. Is Russian mathematics, traditionally, more oriented to applications rather than abstraction?

Today my strongly negative attitude to the "new math" (and "Bourbakized" high school textbooks) is shared by almost all math educators, and – thank God! – it has disappeared, at least in its original extreme forms, from all curricula. Concerning Russian mathematics, although less formalized than, say, French mathematics, I would not describe it as primarily applications-oriented. All the great Russian mathematicians, from Lobachevsky to Kontsevich via Chebychev, Markov, Alexandrov, Kolmogorov, Gromov, Arnold, Novikov,

are famous for their fundamental mathematical theories, and not for work in the applications (if any). On the other hand, none of them considered themselves "pure mathematicians". Kolmogorov went further: for him there was no such thing as "pure" or "applied" mathematics, just good or bad mathematics; math purporting to be "pure", if good enough, eventually finds applications, while good math used to solve a concrete "applied" problem eventually evolves into a significant abstract theory.

You got your PhD at Mekh-Mat in 1966 with a thesis entitled "Multidimensional topological knots". Is it possible to give us some idea of the problems you were dealing with?

The main result of my PhD thesis may be explained even to the layman: it asserts that any multidimensional knot can be decomposed into a finite composition of "prime factors" (just as any whole number can be decomposed into a product of prime numbers). The proof, unlike the statement of the result, is complicated and uses of some fairly sophisticated machinery involving homotopy groups, homological invariants, spectral sequences, etc.

Who influenced you most at Mekh-Mat?

My thesis advisor, Lyudmila Keldysh, my fellow student and long time friend Dmitry Fuchs, and Kolmogorov.

What did you do after you obtained your PhD?

I was offered an assistantship at the Chair of Topology and Geometry, headed by P.S.Alexandrov, and began, under good auspices, what promised to be a successful academic career at Mekh-Mat, one of the best mathematical centers of the world. A few years later, I was appointed associate professor at the same chair. But then serious problems began.

What problems?

First of all, the "golden era" of Mekh-Mat was dramatically coming to an end. In 1968, the "Letter of the 99", a mild protest of 99 mathematicians against the forcible incarceration of Esenin-Volpin, the mathematical logician and human rights activist, in a psychiatric institution, was used by the authorities as a pretext to replace the liberal administration and party organization of Mekh-Mat by extreme reactionaries implementing the new totalitarian principles put forward by Brezhnev for monitoring "Soviet science" (as opposed to "bourgeois science").

The result of that were the sadly famous entrance examination problems at Mekh-Mat (used there to flunk Jewish applicants), a complete change of hiring policy (only docile people, approved by the party organization, had a chance for a position at Mekh-Mat), the ban on foreign travel (except for people selected by the

KGB), and other measures of the same type. Trying to fight for the survival of the university, I.G.Petrovsky died of a heart attack in the waiting room of one of the big party bosses, a great loss for the cause of the liberals, already weakened by the forced resignation of N.V.Efimov from his position as Dean.

My own position became more and more insufferable, the new authorities were annoyed at my popularity with the students and my outspoken criticism of what was happening at the department, and harassed me in various ways...

In 1974 you were forced to resign from Moscow University for political reasons. What did you do after that?

For a year I was unemployed (making a living by translating math books), until Kolmogorov succeeded in getting me a job as math editor of the famous popular science magazine "Kvant" (Quantum), where I worked for almost 13 years. (Apparently, I was on a KGB blacklist, so that my attempts to find a position more suited for a research mathematician were systematically blocked.)

By that time, you also taught at a rather peculiar institution, sometimes called the "Bella Muchnik University". Could you tell us something about that?

Bella Muchnik, an alumnus of Mekh-Mat with a PhD, but working as an ordinary school teacher, decided to organize a math circle for students of engineering and other technical schools, who had been flunked at the Mekh-Mat entrance exams for being Jewish, part Jewish, or just too smart. This circle, which had some thirty participants, gathered thrice a week at Bella's tiny apartment and listened to lectures by a number of mathematicians (Sasha Vinogradov, Mitya Fuchs, Sacha Shen, Andrei Zelevinski, Arkady Vaintrob, to name only the first five who come to mind) and to solve problems. I remember that I taught Algebra, and then Calculus on Manifolds to this group of extremely talented and motivated students, some of whom managed to overcome what seemed to be insurmountable difficulties and eventually became research mathematicians, but only after emigrating from Russia. In a rare visit to the Soviet Union, John Milnor actually lectured to them in Bella's apartment, a beautiful talk on topology that I translated into Russian and still vividly remember. Later we managed to give Bella's math circle official status as an extracurricular math seminar at the Gubkin Oil and Gas Institute.

The whole enterprize ended tragically in the summer of 1982 (or was it 1983?) when Bella Muchnik was killed late at night in a deserted street in a traffic "accident" that we all believe was orchestrated by the KGB. One of the instructors (my friend V.Senderov) and one of the students were arrested by that same organization. The "university" did not survive these events.

How did the Independent University of Moscow start?

The IUM is not really a university, it is a small elite school training future research mathematicians. But it should not be compared to Bella's university: it is a perfectly legal, officially licensed institution, whose creation became possible after perestroika put an end to the Soviet totalitarian regime. It was created in 1992 at the initiative of N.N.Konstantinov (famous for his brilliant organization of math contests and extra-curricular activities outside the official educational establishment) by a group of mathematicians including Arnold, Faddeev, Feigin, Ilyashenko, Kirillov, Khovansky, Vassiliev, and others, in fact practically all the leading mathematicians of my generation or younger still based in Russia at the time. Rather than try to rejuvenate Mekh-Mat, which more than 20 years of mismanagement had been reduced from the leading mathematical center that it was in the 1960ies to a drab institution of average research and educational level, and which was still headed by the same people who were responsible for that sad state of affairs, it was decided to create something new.

Besides the enthusiasm of the people involved, we had nothing – no locale, no means of financial support, really nothing. In the first year we taught in the afternoons in a high school, the use of whose classrooms and main auditorium had been offered to us by a friendly director, to enthusiastic students from Mekh-Mat and other institutions (there were over a hundred in midsemester at my Geometry course, which I taught in "simple mathematical English"). It was probably the only college ever where the professors had to pay for teaching: the school was provided to us rent-free, but we had to pay for the electricity and the work of the cleaning women.

You can find out about the subsequent history of the IUM, its Math in Moscow program and the closely associated Center of Continuous Mathematical Education (MCCME) on the web site www.mccme.ru/ium.

You have been involved in mathematical competition activities. What are your thoughts and experience concerning mathematical competitions for the young?

While an undergraduate and graduate student at Moscow U, I was active in the math olympiads on the city, national, and international level. At the time, I was unreservedly in favor of such competitions, in particular as a means of selecting young talents and motivating them to study mathematics. Today, I have some reservations: imagine a fifteen year old Einstein or a Hilbert of the same age in the 1960ies in Russia or, say, Hungary. Interested in mathematics, he would certainly have participated in the local olympiads and ... failed miserably: it is well known that both Hilbert and Einstein were very slow, they totally lacked the competitive spirit and nimbleness of mind needed to succeed in olympiads. Discouraged, they would have abandoned mathematics – Einstein would have become

a mediocre violinist, perhaps, and Hilbert, say, a school teacher very unpopular with his pupils. How many potential deep but slow thinkers, such as Hilbert or Einstein, have the olympiads deprived us of?

There is a way out of this incongruity. First, it should be clearly explained to students interested in math that success in olympiads is neither a necessary nor a sufficient condition for becoming a successful mathematician. Second, there should be alternative ways of testing mathematical aptitude, other than assessing problem solving capacity under stressful time limitations. Fortunately, in Russia today there exist other ways of attracting youngsters to mathematics, in particular the Dubna summer school "Contemporary Mathematics" or the remarkable Tournament of Towns Summer School, or the so-called "math battles" and "math regattas". (You can read about these on the MCCME web site). Other important events with the same functions, but aimed at university students rather than high school pupils, are the Budapest Semesters, the MASS program at Penn State and the Math in Moscow program at the IUM.

I know you have been involved both in the MASS and MiM programs. Can you tell us something about them?

The MASS (Mathematics Advanced Study Semesters) program at Penn State University brings together undergraduates from different universities to study and do mathematics in novel and exciting ways. Besides math courses (on topics not usually included in undergraduate and even graduate curricula), weekly "colloquium talks" of expository nature by leading researchers, research projects (some of which have resulted in publications in respected math journals), the participants have their own seminar, where difficult problems are proposed without any imposed time limits. You can read about this in more detail in the book Mass Selecta: Teaching and Learning Advanced Mathematics, ed. by S.Katok, A.Sossinsky and S.Tabachnikov, AMS, 2003. About the Math in Moscow program, aimed mainly at North American students, let me say that it is organized by the IUM along more traditional lines (lecture courses with exercise classes), but the contents of the courses are research-oriented, there are a lot of proofs and problem solving. Details are available at the MC-CME web site.

Do you notice any differences in the way people from different cultures do mathematics?

I am only competent to compare the Russian and French mathematical cultures: I have no first hand familiarity with the British mathematical traditions and with Japanese mathematics, not enough with the German ones, and I am not sure there is such a thing as an American mathematical style. Although the French and Russian way were once very similar (to a great extent the latter was derived from the former), and in

both cultures the weekly seminar, headed by a great maestro surrounded by his leading pupils, played a crucial unifying role, now things have been changed ... by Bourbaki. French math is highly formalized, its proponents present their results with great precision, but usually without explaining the motivations, whereas the Russians have a more intuitive, geometric and pragmatic no-nonsense style, closer to the Anglo-Saxon traditions (with which Russia historically had very little interplay). And yet, my feeling is that the accelerated internationalization of mathematics (as the saying goes, an American math department is a place where Russian professors teach Chinese graduate students) is not only removing the existing national barriers, but also progressively erasing the specifically national traits of doing mathematics.

If you had to mention one or two great moments in 20th century mathematics which one(s) would you pick?

I would mention only one: Gödel's Incompleteness Theorem, arguably the most important scientific achievement of the past century, just as important, to my mind, for philosophy and for computer implementation as it is for mathematics and its foundations. I love to lecture about that theorem, and equally enjoy explaining the three very different proofs of it that I know (Gödel's original direct construction based on "Gödel numberings", the proof using a reduction to an undecidability theorem, which I learned from Kolmogorov, and the beautiful, although not too well known one, due to Chaitin and based on Kolmogorov complexity).



Alexei Sossinsky, lecturing on the Gödel's Incompleteness Theorem, in Luso, September 11, 2005.

You were the first director of the French-Russian Poncelet Mathematics Laboratory (CNRS-IUM) at Moscow. How did that come into being?

Rather than retell the dull (if successful) story of the lab's creation (all about overcoming bureaucratic obstacles to create a novel form of scientific cooperation),

let me tell you why it bears the name of Poncelet. Jean-Victor Poncelet, an alumnus of École Polytechnique, was a young lieutenant du génie in Napoleon's Grande Armée when it invaded Russia in 1812. In one of the skirmishes during the catastrophic winter retreat that followed, Poncelet was wounded and left for dead on the field of battle by his comrades-at-arms. He would have frozen to death if he had not been picked up by local peasants, who cured him of his wounds and surrendered him to Russian military authorities. He was a prisoner of war for three years in Saratov, where he did the research that he is now famous for, revolutionizing the field of projective geometry and later earning the unofficial title of "father of modern algebraic geometry". Back in France, he had a brilliant scientific career (although he never rose to the mathematical heights that he had achieved in Saratov), became a general and the director of *École Polytechnique*.

So we tell the young French researchers whom we hire to work at the lab that we will keep them prisoner in Moscow for at least a year, we expect them to do their best research there, and then they can be sure to have a brilliant career upon their return to France.

At the Poncelet lab you have a project ("Knots and braids") where you use the neologism "statistical topology" to name the area of study. Could you explain it?

The term was actually coined by my friend and colleague Serguei Nechaev, who works at the laboratory of Statistical Models and Theoretical Physics at Orsay (near Paris) which is associated with the Poncelet lab in the project. It is very easy to explain: it deals with a wide range of objects with nontrivial topology (mostly one-dimensional, e.g. knots, braids, graphs) involved in random processes. A typical problem is to estimate the probability of a random closed curve in space to be knotted.

Outside Mathematics what are your interests?

Like most mathematicians, I am lover of symphonic and chamber music. Like very many Russian mathematicians, I am (or rather was, the years now take their toll) a great enthusiast for the outdoors: long camping trips, on foot or on skis in the mountains, or on kayaks on white water. My real violon d'Ingres, however, is the translation of poetry from Russian to English, something I have done professionally for many years, and which helped me considerable in my early family life (the one US dollars per line paid to me in the 1960ies were an important contribution to the family budget). But more important in my non-mathematical life are my love of tennis (I still play in men's amateur tournaments, despite my age) and of the sea – swimming or surfing in the ocean waves give me the same degree of pleasure as proving a tough and beautiful theorem.

Interview by Jorge Picado (University of Coimbra)