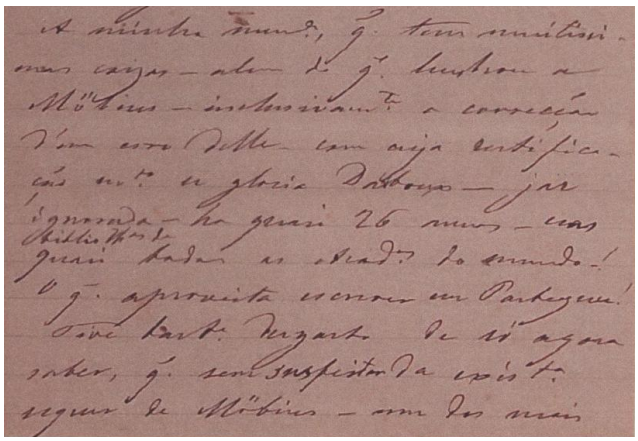
Daniel da Silva

Almost all the results obtained by Darboux were published in my memoir, which has many more things than those obtained by Möbius (...) lays down ignored for almost twenty–six years in the libraries of almost all the Academies of the world. The reward of writing in Portuguese!

This was written in 1877 by Daniel da Silva, a Portuguese mathematician of the nineteenth century with a wide range of interests, from Number Theory to Experimental Physics through Mechanics and Actuarial Sciences, in a letter addressed to da Silva young friend and disciple Gomes Teixeira.



Excerpt from the letter of da Silva to Gomes Teixeira

After a life dedicated to Science (and with a poor health) and despite the fact that he was recognized as a leader of the Portuguese mathematical community, with several honours and prizes, he sadly realized that, although having worked on some important problems of mathematics (so important, in fact, that they attracted the attention of such famous mathematicians as Darboux, as da Silva refers in the above letter) and obtained substantial results on those subjects, his name and his work would be ignored by the international mathematical community and the credits of his discoveries would be given to others.

In this note we will sketch the life and works of this mathematician.

Daniel da Silva

Daniel Augusto da Silva was born in Lisbon, on the 16th of May of 1814. In 1829 he was admitted in the Navy (*Companhia dos Guardas-Marinhas*) becoming a Navy officer in 1833. Still in 1829, he enrolled in the *Academia Real da Marinha* (which was to be transformed later, in 1837, into the Polytechnical School of Lisbon and, in 1911, into the Science Faculty of the University of Lisbon), finishing is course in 1832; that same year he entered the *Academia Real dos Guardas-Marinhas* — the *Escola Naval* (Navy School), after 1845 —, the school that at the time prepared the Navy officers. In 1835 he completed the three years course in that school and, in the following year, moved to the University of Coimbra, at the time the only university in Portugal, to pursue his studies in Mathematics. In 1839 he received a degree in Mathematics. He had been always a distinguished student and received high marks and prizes in both Navy Schools, in Lisbon, and in the University of Coimbra. When much later (in 1878) his friend,

disciple and main biographer Gomes Teixeira became a professor in Coimbra, some old professors could still remember ([9, pag. 188]) the brightness of da Silva as a student.

In 1845, with the transformation of the *Academia Real dos Guardas-Marinhas* into the *Escola Naval*, Daniel da Silva was appointed as a professor there, teaching Mechanics, Astronomy and Optics; later on, in 1848, he became professor of Artillery, Fortification, Geography and Hydrography. In 1865 he retired from the *Escola Naval* and in 1868 he retired from the Navy as a high rank officer (*Capitão de Fragata*).

In 1851, da Silva published his first scientific work, which is also one of his two major works: *Memória sobre a rotação das forças em tôrno dos pontos de aplicação* (Memoir on the rotation of forces around the applications points), published in *Historia e Mem. Acad. Sci. Lisboa* 3 (2^{a} Ser.). In this paper he studied the action of several forces acting on a solid body, namely he searched for equilibrium conditions. This kind of problems in the field of Statics had been previously studied by, among others, Poinsot, in 1803 (whose work da Silva was aware of), Möbius in 1837 and Midding in 1841 (these two authors were unknown to da Silva), and, later on (1877), by Darboux. Da Silva rediscovered, in his work, most of the results of Möbius and Midding and found the results later published by Darboux. An analysis and comparison of these works was done in [10]. It was Darboux's paper, of which da Silva became aware through a review by Moigno in the *Jornal des Mondes*, that prompted his letter to Gomes Teixeira referred to above. Although, during the da Silva lifetime, this work of his had been ignored, due to [10] and Teixeira efforts, the name of da Silva appears in some publications on the history of Statics.

With that work Daniel da Silva was elected a corresponding member of the *Academia Real das Sciencias de Lisboa* in 1851, becoming a full member in 1852 and later on, in 1859, a honorary member.

He published some other works on Mechanics, but soon a different subject called his attention: Number Theory. In 1854 he published his second major work, *Propriedades Gerais e Resolução das Congruências binomiais* (General properties and resolution of binomial congruences). This long paper (163 pages) had not the same destiny of his work in Statics. A detailed analysis of it was published by C. Alasia de Quesada [1, 2]; due to Alasia's papers the name of da Silva appears several times in the classical book of Dickson, [4]. Curiously, Alasia came across this paper by someone completely unknown to him, in an antique bookshop, where he found a complimentary copy of it sent by the author to J. Liouville.

It is in this work that da Silva presents (in Chap. 1), without proof, the sieve formula for the cardinal of set unions, to which is sometimes attached his name (see e.g. [7, page 19]), although the formula had been used by Nikolas Bernoulli in the eighteenth century, and may be even earlier. The novelty with da Silva treatment, as pointed out in [3] (see also [5, 6]), is that da Silva developed, long before Cantor and even Boole, some kind of (naive) set notions and notations. In fact, he considers a collection S of numbers and he denotes by S_a (respectively ${}^aS$) the collection of elements of S that have (resp. do not have) property a. Similarly, $S_{ab\ldots}$ (respectively ${}^{\cdots ba}S$) denotes the collection of elements of S that have (resp. do note have) properties $a, b, \ldots$. Then he writes symbolically

$${}^aS = S - S_a = S[1-_a], \quad {}^{\cdots cba}S = S[1-_a][1-_b][1-_c]\ldots .$$

He denotes by φS the number of elements of a collection S and then he writes:

$$\varphi^{\cdots cba}S = \varphi S[1-_a][1-_b][1-_c]\ldots ,$$

where it is understood that the multiplications in the second member should be done formally, obtaining, in his words, *an additive series and a subtractive series in the second member*. This means that the second member is to be equal to

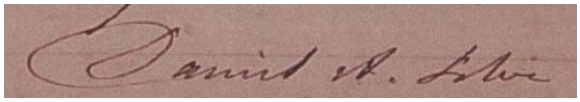
$$\varphi S - (\varphi S_a + \varphi S_b + \varphi S_c \cdots) + (\varphi S_{ab} + \varphi S_{ac} + \varphi S_{bc} + \cdots) - \cdots ,$$

which is precisely the sieve formula.

He applies this formula to the study of certain functions of Number Theory, namely the Euler function φ; he also presents a generalization of the classical Theorem of Euler on the function φ. Chapter 2 is devoted to linear congruences in one and several variables. Some authors, following Gomes Teixeira [9, page 171], refer to this study of systems of linear congruences, thus anticipating the classical results of H. J. Smith of 1861. In fact, although da Silva carefully studies linear congruences of one and several unknowns, for systems with less equations than unknowns, he just mentions that, by elimination procedures, the system can be reduced to one equation. The chapters 3 to 9 are devoted to a detailed analysis of the congruences $x^n \equiv 1 \bmod n$ (primitive roots) and $ax^n \equiv b \bmod n$. There, da Silva studies the existence of solutions, as well as their number, and formulæ and techniques to obtain these solutions. Several of those results, now classic, were new at the time. The last chapter, entitled *applications*, seems to be just a draft of what da Silva intended to do; in fact, in some sections (e.g. *Continued Fractions*) there is nothing but the title! This was due to the severe illness that affected the author in 1854 (probably some kind of mental breakdown) from which da Silva never completely recovered. According to Gomes Teixeira [9], da Silva could not finish the last chapter and the proofreading of the book.

Due to his illness, da Silva published nothing between 1854 and 1866. After that, he published 2 more papers on mechanics (in 1866 and 1873) and several papers

and newspaper articles of some importance on quite different subjects: actuarial science, statistics and demographical studies (see [9, page 174-176]. In 1873 he published his last paper, once again changing his field of interest, now turning to experimental physics; in this paper he studied properties of flames describing several experiences about the most bright part of a flame. Daniel da Silva died in Lisbon, on the 6th of October of 1878.



An interesting aspect of da Silva personality is shown in the letter he wrote to a young 20 year student of the 3rd year of the mathematical course in Coimbra. After hearing, in the mechanics course, the high references the teacher made to da Silva, the student decided to send da Silva a small paper (on continued fractions) he had just published. Daniel da Silva acknowledged the reception of the paper with words of high praise; he further refers his love, passion and hard work in mathematics, his illness, and how difficult was to him, since then, to concentrate in mathematics; he adds that he had a great admiration and interest for everyone who works on mathematics and ends the letter by saying:

To say that I esteem the author of the Memoir I received is much more than a mere compliment; it is just the statement of a necessary condition of my mental organization.

That student was Gomes Teixeira, and this letter was the beginning of a close friendship between the old master and the young mathematician; da Silva had been always very supportive of Gomes Teixeira, a well-known Portuguese mathematician (see [8]), at the beginning of his career.

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