What's New in Mathematics

Making wavelets in the art world. "Computers confront the art experts" is a piece by Philip Ball posted on November 22, 2004 at Nature's online site `news@nature.com`. Ball's subheadline reads: "Automated method seems to spot forgeries as well as a connoisseur does." The forgeries are in paintings, and the method in question, devised by Hany Farid, Dan Blackmore and Siwei Lyu of Darmouth University, uses wavelet analysis to characterize the "hand" of an artist. Ball describes it as follows: "They scan a picture at high resolution and then use the wavelet technique to decompose the picture into sets of vertical, horizontal and diagonal lines. [...] From the mathematical distribution of lines, the researchers calculate a set of numbers that characterizes each picture. These numbers can be regarded as coordinates in a multi-dimensional space, like a grid reference. If two images share similarities in their use of line, the points in space defined by their coordinates will lie close together, even if the scenes depicted are totally different." When the Dartmouth team tried their technique on a set of 13 landscapes (genuine and imitation) by Brueghel, "the points corresponding to the eight pictures deemed to be authentic all sat together in a cluster, and the fakes were further away." Then they turned to the Madonna and Child with Saints, attributed to the Italian Renaissance painter Perugino, in the collection of the Hood Museum at Dartmouth. There are four Saints in the picture, so six heads in all. Applying their analysis to the heads, Farid and his collaborators found no fewer than four painters at work. Three of the heads are by three different painters; the three others are by a fourth, "perhaps Perugino himself," as Ball puts it. Article available online.

World Renowned Geometer S.-S. Chern Dies. Shiing-Shen Chern, one of the outstanding mathematicians of the 20th century, passed away in Tianjin, China, on Friday, December 3, 2004, at the age of 93. S.-S. Chern was one of the creators of modern differential geometry as it is known today. Fifty years ago, his global viewpoint, emphasizing relations with topology, was revolutionary. One of his early successes was his elegant proof of the Chern-Gauss-Bonnet theorem, which, together with concepts such as Chern classes and Chern-Simons invariants, made a lasting imprint on the subject. S.-S. Chern was the founding director of the Mathematical Sciences Research Institute in Berkeley. After holding professorial positions at the University of Chicago and the University of California, Berkeley, he returned to China in his retirement and founded the Nankai Institute of Mathematics. He received the first Shaw Prize in 2004. Read more about Chern in an interview that appeared in *Notices of the AMS*.

"Quantum decoys create uncrackable code" by Mark Buchanan, is the title of a short article in the November 13, 2004, *New Scientist*. It describes a recent breakthrough in quantum cryptography by researchers at the University of Toronto. One reason quantum codes were initially thought to be so powerful is that eavesdropping would disturb the photons used to carry the messages and therefore could be detected. Then researchers found a way whereby an eavesdropper could cover up his or her tracks. The new research shows how a message-sender can send out decoy photons that would foil an eavesdropper.

"A fractal life". Interview with Benoit Mandelbrot. Interviewed by Valerie Jamieson. *New Scientist*, 13 November 2004. No mathematical object has become so well known among the general public as has the fractal. Benoit Mandelbrot coined the term and was the first to systematically explore this geometric phenomenon. In the interview, he talks about his views on the popularity of fractals, his background growing up in France, and the influence of his uncle, Szolem Mandelbrojt, who was a mathematician at the Collège de France in Paris. He also talks about his latest research on the concept of "negative dimension" and on the dynamics of financial markets. His book *The (Mis)Behavior of Markets: A fractal view of risk, ruin and reward*, written with Richard Hudson, came out in 2004.

Gödel's Theorem on ABC News. John Allen Paulos, in his "Who's Counting" column on the *ABC News* website, posted "Complexity, Randomness and Impossible Tasks" on November 7, 2004. Paulos starts with algorithmic complexity. He invites us to compare the two sequences

(A) 001001001001001001001001001001001001001001...

(B) 100010110110110001010110010111010010111010...

and asks: "Why is it that the first sequence of 0's and 1's ... is termed orderly or patterned and the second sequence random or patternless?" As a quantitative answer, he proposes Greg Chaitin's definition: *The (algorithmic) complexity of a sequence of 0's and 1's is the length of the shortest computer program that will generate the sequence.* The program for (A) could be "print 0,0,1, repeat." But for (B) it is quite possible that the only way to generate the sequence is to spell it out: "print 1,0,0,0,1,0,1,1,..." Paulos continues: "We define a sequence to be random if and only if its complexity is (roughly) equal to its length; that is, if the shortest program capable of generating it has (roughly) the same length as the sequence itself." If the only way to generate (B) is to spell it out, (B) is a random sequence. Next Paulos introduces us to the "Berry sentence," which he attributes to Bertrand Russel, 1908. *Find the smallest whole number that requires, in order to be specified, more words than there are in this sentence.* "The paradoxical nature of the task becomes clear when we realize that the Berry sentence specifies a particular whole number that, by its very definition, the sentence contains too few words to specify." Now the big step: "What yields a paradox about numbers can be modified to yield mathematical statements about sequences that can be neither proved nor disproved." A formal mathematical system can be encoded as a computer program P. As P runs, it generates all the possible theorems which can be proved in that system. "Now we ask whether the system is complete. Is it always the case that for a statement S, the system either proves S or it proves its negation, $\sim S$?" Paulos explains Chaitin's argument, which involves imagining the the following Berry-like task for the program: *Generate a sequence of bits that can be proved to be of complexity greater than the number of bits in this program.* "The program P cannot generate such a sequence, since any sequence that it generates must, by definition of complexity, be of complexity less than P itself." It follows that "statements of complexity greater than P's can be neither proved nor disproved by P." This is Chaitin's new, quantitative twist on Gödel's Theorem. Since any formal mathematical system has a certain finite complexity, it must allow statements which can be neither proved nor disproved, "a limitation affecting human minds as well as computers."

"How Strategists Design the Perfect Candidate" is the title of a piece by Mark Buchanan in the *Science* issue of 29 October 2004. With the United States presidential election on the horizon, writer Mark Buchanan interviewed some political analysts about mathematical models they use to analyze how voters choose candidates. An often-used model depicts voters "as abstract points in a 'policy space'"; a politician can optimize his or her standing with a set of x voters by choosing a point equidistant from all x. (On the other hand, a politician may have a better chance of executing his or her preferred policies – if elected – by moving away from this central location.) A second model, presented by University of California, San Diego, physicist David Meyer, reflects voters inconsistent preferences, implying that a politician's best strategy is "roughly speaking, to be as inconsistent as the voters," according to Buchanan. Given that closer races resulting from candidates' use of the above models – and that unpopular third-party candidates can heavily influence an election – better voting systems were also discussed. These included eliminating the Electoral College in favor of popular voting, an option that seems unlikely to Eric Maskin of the Institute for Advanced Study in Princeton, New Jersey, since it would necessitate a constitutional amendment. What is clear is that there are no simple answers. As political scientist Larry Bartels of Princeton University says, "the surprising reality is that we still understand relatively little about how presidential campaigns affect the vote."

"What Makes an Equation Beautiful", New York Times, 24 October 2004. In a column in *Physics World* magazine, philosopher and historian Robert P. Crease asked readers which equations they considered to be the greatest. He got 120 responses proposing 50 different equations. This article discusses Crease's experiment and also provides readers with a nice context to appreciate the power of mathematical equations. The top vote-getters were Maxwell's equations for electromagnetism and Euler's equation, $e^{i\pi} + 1 = 0$. A list of 18 other winners is given in a sidebar. Most of the equations relate to physics, but the Pythagorean theorem and the Riemann zeta function made it onto the list.

IT and the Riemann Hypothesis. "What is the Riemann Hypothesis and why Should I Care?" is the provocative title of a piece by Robin Bloor posted at `IT-Director.com` on October 5, 2004. The site "provides IT decision makers with a one stop source of all current IT news, information, analysis and advice." (IT = Information Technology). Naturally, there is no attempt at a correct statement of the Riemann Hypothesis ("Without bothering to state the details, it is a proposed formula that calculates the number of primes less than a given number") but the reason why IT decision makers might be concerned is the "worrying predictions that if the Riemann Hypothesis is confirmed mathematically, then most of the encryption schemes we use in commerce and government will suddenly be vulnerable ...” together with news of its possible confirmation by Louis de Branges and perhaps by others. The risk for IT is “if the mathematics surrounding the solution reveals quicker ways to factorize numbers. Actually even then it will only matter if it reveals much quicker ways to factorize numbers.” Because public-key cryptography “is based on the product of prime numbers. The fundamental idea is that it is very difficult to find factors for a number that is created by multiplying two prime numbers together. It’s easy to multiply the numbers together but very difficult to find out what they were if you’re only given the result.” But not to worry: “Strange as it may seem (if you never studied Mathematics) there are mathematical relations that can be used to create encryption that can be proved to be unbreakable. The real problem is that we founded the early encryption on a technique that wasn’t provably unbreakable.” Bloor’s piece is available online.

The Math of Medical Marriages. “Tweaking the Math to Make Happier Medical Marriages” is the title of a piece by Sara Robinson in the August 24 2004 *New York Times*. “Medical marriages” refers to the process by which the National Residency Match Program assigns medical students to residency positions. Residency Match uses an algorithm that turns out to be equivalent to the “marriage algorithm” devised in 1962 by the mathematicians David Gale and Lloyd Shapley, who proved that it converges. Here is how Robinson explains the algorithm:

- “Each boy ranks all the girls in order of his preference, and each girl does the same. Then, each boy asks his first choice for a date. Each girl with one or more offers dates her favorite and says “no” to the rest.
- In the next round, the boys who were rejected move on to their second-choice girl. The girls again date their favorites, possibly throwing over their date from the earlier round for someone better.
- Continuing in this way, the mathematicians showed, the dating frenzy eventually subsides into a stable situation where each girl has only one boy, and there is no boy and girl who prefer each other to the people they are dating. That is, every time a boy does not get his first choice, he has no hope of getting anything better. Each of the girls he prefers is paired with someone she prefers to him. The same is true for a girl.”

The *Times* diagrams a 3×3 example in which Adam and Eve, Romeo and Juliet, Tristan and Isolde end up paired even though Isolde was only Tristan’s second choice to start with, and he was her third. [An interesting point about this algorithm, unfortunately obscured by the *Times* presentation (boys ask girls in the text, girls ask boys in the diagram) is that it *favors the askers*. The simplest example is with two boys and two girls.

- Suppose Romeo ranks Juliet #1 and Isolde #2, while Tristan ranks Isolde #1 and Juliet #2. And suppose Juliet ranks Tristan #1 and Romeo #2, while Isolde ranks Romeo #1 and Tristan #2.
- Following the *Times* text, Romeo invites Juliet and Tristan invites Isolde. Each girl has only one offer, and has to take it, so Romeo and Tristan get their wish, but Juliet and Isolde do not.
- Following the *Times* diagram, Juliet invites Tristan and Isolde invites Romeo; now the girls get their heart’s desire, and the boys do not.]

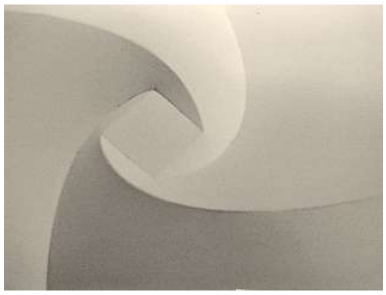
Hospitals used to do the asking. Even though cases like this are very rare (a 1996 analysis, available online, by August Colenbrander, MD, from which the example above was taken, estimates that in the residency match the chance of a discrepancy is less than 0.1%) the algorithm has been reversed since 1996 to make the students the askers. [See also Mathias Lindemann’s The Stable Marriage Problem, available online.] The algorithm is in the news because it is suspected of allowing hospitals to underpay residents.

Bird Logic. “Bigger than” is a transitive relation: if X is bigger than Y, and Y is bigger than Z, then we can infer, without comparing X to Z, that X is bigger than Z. “Pinion jays use transitive interference to predict social dominance,” by Guillermo Paz-y-Miño and three collaborators (*Nature*, August 12, 2004) shows how some birds apply this principle to their pecking order. The experimental setup involved three initially isolated groups of jays; in each group a dominance hierarchy had established itself: $A > B > C > D > E > F$, $1 > 2 > 3 > 4 > 5 > 6$ and $P > Q > R > S$. In a typical run, bird-3 (the “observer”) would watch, on three consecutive days, bird-2 defer to bird-B. Then bird-3 and bird-B were placed in a competitive encounter. If birds can use transitive inference then bird-3, having seen its dominator bird-2 defer to bird-B, should infer that B is its superior. And in fact: “During the first minute of the first encounter, observers displayed subordinance levels that were nearly four times as high as those of controls.” Controls would have watched similar displays involving two birds with which they were not acquainted. Pinyon jays are "among the most social of North American corvids." They also are better than their less gregarious cousins the scrub jays, at applying transitive inference in experiments involving colored markers. The authors' closing remark: "This work ... supports the hypothesis that social complexity provided a crucial context for the evolution of cognitive abilities."

Calculus, the play (New End Theatre, London, until August 24) is reviewed in the August 12 2004 *Nature* by Philip Ball. The play, written by Carl Djerassi, "centres on the deliberations of a Royal Society committee appointed in 1712 to pronounce on the priority issue." The issue being whether or not Leibnitz had plagiarized Newton's discovery of calculus. Newton appears in a play within a play which allows his interlocutor "to anticipate the audience's dismay (and indeed I sensed such a response) at having to hear about the calculus." Ultimately, Ball finds that "there is just not quite enough at stake here to sustain the drama." He adds parenthetically: "I did, however, enjoy the portrayal of the eminent French mathematician Abraham de Moivre as a gluttonous reprobate." *Calculus* was also reviewed online by Rachel Thomas in *+plus* magazine.

Happy 100th Birthday, Henri Cartan! Legendary French mathematician Henri Cartan turned 100 on July 8th. The son of Élie Cartan and a major figure in 20th century mathematics, Henri Cartan made outstanding contributions to several fields of mathematics. He led the famed "Cartan Seminar" in Paris and is also well known for his 1956 book Homological Algebra, written jointly with Sammy Eilenberg. On June 28, his birthday was celebrated in the Journée Cartan, held at his home institution, the École Normale Supérieure in Paris. In addition, the International Mathematical Union has issued a resolution congratulating Cartan. Read more about Henri Cartan in the *Notices of the AMS*: "Happy 100th Henri Cartan!" and "Interview with Henri Cartan".

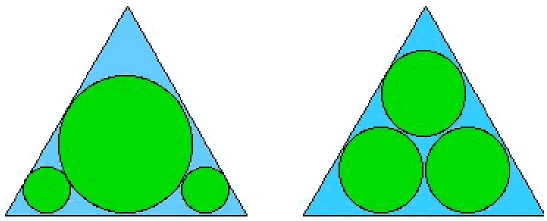
Mathematical Origami. "Cones, Curves, Shells, Towers: He Made Paper Jump to Life" is a piece by Margaret Wertheim in the June 22 2004 *New York Times*. She writes about David Huffman, a computer scientist who died in 1999, and his work on mathematically informed origami.



As the image above exemplifies, Huffman's specialty was folds along curves. He wanted to be able "to calculate precisely what structures could be folded to avoid putting strain on the paper." Huffman, who is best known for the "Huffman codes" he discovered as an MIT graduate student, is also "a legend in the tiny world of origami sekkei," or computational origami. He published only one paper on the subject but his models and his notes are being carefully studied by today's mathematical paper-folders. Wertheim quotes Robert Lang: "he anticipated a great deal of what other people have since rediscovered or are only now discovering. At least half of what he did is unlike anything I've seen." And Michael Tanner, who says that what fascinated Huffman above all else "was how the mathematics could become manifest in the paper."

Local Boy Makes Good?

- The *Zaman Daily Newspaper* (Istanbul) online edition ran a dispatch dated May 18, 2004 from Omer Oruc in Izmir. "200 Year Old Math Problem Solved" is the headline; the story tells how Mustafa Tongemen, a retired mathematics teacher, has solved "a math problem brought forward by the Italian mathematician, Malfatti, in 1803," after working on the problem two hours a day for seven years. "Malfatti's problem aims to extract three circular vertical cylinders from a triangular vertical prism made of marble, with the least material loss." In fact, there are two non-equivalent problems, initially confused by Malfatti. The problem just posed, which Conway calls problem (1), and the problem of inscribing three mutually tangent circles in a triangle, problem (2).



In an equilateral triangle, in fact, the incircle and two smaller inscribed circles give a larger area than the three mutually tangent circles. Malfatti solved problem (2), which, as is clear from the image on the Zaman website, is the problem Mustafa Tongeman actually addressed. The much more difficult problem (1) was only solved in 1992. See the *Historia Mathematica Mailing List Archive* for a summary of the history and the source of the equilateral counterexample.

- The Purdue University *Purdue News* ran "Purdue mathematician claims proof for Riemann Hypothesis" on June 8, 2004. "Louis de Branges ... has posted a 124-page paper detailing his attempt at a proof on his university Web page. While mathematicians ordinarily announce their work at formal conferences or in scientific journals, the spirited competition to prove the hypothesis – which carries a $1 million prize for whoever accomplishes it first – has encouraged de Branges to announce his work as soon as it was completed." The jury is out on this one.

- On June 22, 2004, the *Daily Star* (Beirut) ran May Habib's online story: Has local mathematician proven the '5th Postulate?' "Rachid Matta, a Lebanese mathematician and engineer, claims to have proven Euclid's parallel theorem – a theorem that has remained unproved since Euclid wrote it in 300 BC and one that many of the world's greatest minds have deemed improvable. If verified, Matta's work could have an enormous impact on mathematics because both elliptical and hyperbolic geometry – branches of geometry that violate the parallel theorem – would be discredited." We are told that Matta has spent 10 years working on this problem.

Mind, Music and Math. From the desk of *New York Times* cultural critic Edward Rothstein comes "Deciphering the Grammar of Mind, Music and Math" (June 19, 2004). The piece, under the Connections rubric, is a meditation on the nature of musical intelligence, in the light of recent work on the neural concomitants of musical perception and on the differences between the way the brain processes speech sounds and music. Rothstein emphasizes the "unique" power of music: how, even if completely unfamiliar music is heard in a locked room, where there is no reference to the outside world, "it can teach itself. Gradually, over repeated hearings [...] music shows how it is to be understood. [...] Sounds create their own context. They begin to make sense. [...] Music may be the ultimate self-revealing code." He goes on: "This is one reason that connections with mathematics are so profound. Though math requires reference to the outside world, it too proceeds by noting similarities and variations in patterns, in contemplating the structure of abstract systems, in finding the ways its elements are manipulated, connected and transformed. Mathematics is done the way music is understood." The moral of the story: "This means that music can be fully understood only by maintaining access between the room and the world: neither can be closed off."

The Pythagorean Theorem of Baseball has just been simplified. This news from the web-based *Science Daily* for March 30, 2004. The original PTOB is due to the baseball statistician and connoisseur Bill James. It estimates a team's winning chances in terms of two numbers: R_s, the number of runs scored, and R_a, the number of runs allowed. The formula is

$$P = \frac{R_s^2}{R_s^2 + R_a^2}.$$

Suppose that in 12 games your team scored 72 hits and allowed 64 hits. The PTOB gives $P = 0.56$ so it should have won 7 games and lost 5. If it won more than 7, it is "overperforming," if less, then "underperforming." This is supposed to help in predicting future performance. The simplification is due to Michael Jones and Linda Tappin (Montclair State University). Their formula runs $P = 0.5 + \beta(R_s - R_a)$, where β is a constant, "chosen to give best results" for each season, ranging between 0.0053 and 0.0078 and averaging 0.0065. For your hypothetical team the streamlined formula with $\beta = 0.0065$ gives $P = 0.55$ and leads, after round-off, to the same prediction as the PTOB. (*Science Daily*'s βs were off by a factor of 10). [Linearizing the PTOB about the equilibrium $R_s = R_a$ gives $P \sim 0.5 + (R_s - R_a)/(R_s + R_a)$.]

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