BULLETIN

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15

Contents

Coming Events

Thematic Term on Mathematics and the Environment

COORDINATORS

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DATES

May-July 2004

The **CIM Thematic Term** for 2004 will be about Mathematics and the Environment. The topic could hardly be more timely. The knowledge about the impact of human activities on our planet's ecosystems is nowadays more vital than ever. Increasing human population to the detriment of others, cutting and burning vast areas of forest, polluting soil, air and water, are just few examples of how we humans have altered our environment. Within this Thematic Term we intend to address some of these issues from a mathematical and a physical modelling point of view.

The first event, School and Workshop on Dynamical Systems and Applications, is aimed at consolidating the research activities in Portugal in this area of mathematics fundamental for the understanding of evolution of ecological environments and monitoring of global changes. The Workshop on Forest Fires attempts to promote the communication among researchers with an interest on theoretical modelling of forest fires, in particular on fire front propagation. The third and fourth events, School on Atmospheric Sciences and Climate Dynamics and School and Workshop on Oceanography, Lakes and Rivers, are closely related and, hence, planned to be organized in two consecutive weeks. As their titles indicate, they address probably the most important natural processes for the world's ecosystem, and will touch on issues such as air quality, weather prediction, ocean waves and currents, estuarine dynamics, and avalanches, among others.

The programme of events is the following:

3-8 May: School and Workshop on Dynamical Systems and Applications

Organizers

José Ferreira Alves (Univ. Porto), Marcelo Viana (IMPA, Brazil).

Aims

The main scientific goals of the workshop are: to present recent developments in the field of dynamical systems and its applications, especially in directions relevant to the sciences of the environment; to promote the interdisciplinary character of this field, and explore its connections to other areas of mathematics and science; to help consolidate the research groups in dynamical systems at Portuguese universities.

The event includes an advanced school as well as a research workshop, thus aiming at an audience ranging from doctoral students to active researchers. We expect most of the students to come from European institutions and networks, including a substantial participation of Portuguese students. The workshop covers several very active research topics in dynamics from fundamental aspects to various applications (e.g. in biology, chemistry, weather prediction). We believe it will attract a good number of experts, in mathematics and in experimental sciences.

Dynamical systems was born and developed as an interdisciplinary field, driven by requests from experimental sciences and aiming at providing a conceptual framework for explaining and predicting their observations. Indeed, some of its spectacular advances were prompted by such issues as the behaviour of the solar system (homoclinic phenomena, non-integrability of the equations of motion), the Earth's atmosphere (convection, Lorenz strange attractors), or the evolution of ecological environments (Lotka-Volterra equations, invading species). The sciences of the environment are providing some new and very exciting new challenges, from the modelling of physical and biological systems, to the monitoring of global changes, just to mention a few.

The meeting will bring together a good number of researchers, both young and experienced. It will include advanced courses and research lectures, covering a wide spectrum of subjects, and aimed at a broad audience of doctoral students and researchers interested in this field and its applications. The meeting should also contribute to enhance the visibility of the research currently carried out in Portugal in this field.

The workshop will be held at the Pure Mathematics Department - University of Porto.

Lectures

Title to be announced

José Ferreira Alves, University of Porto, Portugal

Markov partitions for non-uniformly expanding maps

Vítor Araújo, University of Porto, Portugal

Stability of differential equations and the forgotten work of Lyapunov

Luís Barreira, Technical University of Lisbon, Portugal

Genericity and transitivity

Christian Bonatti, University of Dijon, France

Cycles and iterated functions systems

Lorenzo J. Díaz, PUC-Rio de Janeiro, Brazil

Title to be announced

Celso Grebogi, University of São Paulo, Brazil

KAM and rigidity of group actions

Anatole Katok, Penn State University, USA

Rigidity theory for circle homeomorphisms

Konstantin Khanin, Newton Institute, Cambridge, UK

Hamiltonian homeomorphisms of surfaces

Patrice Le Calvez, University Paris-Nord, France

Arithmetics and solutions of cohomological equations for interval exchange maps

Stefano Marmi, Scuola Normale Superiore di Pisa, Italy

The Lorenz attractor revisited

Maria José Pacifico, Universidade Federal do Rio de Janeiro, Brazil

A global view of dynamical systems

Jacob Palis, IMPA, Brazil

Title to be announced

Carles Simó, University of Barcelona, Spain

Hard ball systems and the Lorentz gas

Domokos Szasz, Technical University of Budapest, Hungary

Validated numerics and dynamical systems

Warwick Tucker, University of Uppsala, Sweden

Title to be announced

Marcelo Viana, IMPA, Brazil

Title to be announced

Jean Christophe Yoccoz, Collège de France, France

For more information on this event, please visit the site

http://www.mat.uc.pt/∼tt2004/dynsystems

June 3 - 5: Workshop on Forest Fires

Organizers

Jorge André (Univ. Coimbra), José Miguel Urbano (Univ. Coimbra).

Aims

Along the past century, in many parts of the world, for human and natural causes, forest fires have become an increasing threat to ambient and man. At the broadest scale, forest fires interest researchers from very different areas, such as: forestry, ecology, geography, physics and chemistry, mechanical and chemical engineering, and applied mathematics. At a finer scale, forest fire physics can be considered mainly as a sub-area of fire science, itself a part of combustion science, but has also important intersections with forestry (fuels characterisation) and meteorology (interactions between the fire and the atmosphere, at various spatial and temporal scales). Within the different phenomena that have been studied, the quasi-steady propagation of surface forest fire fronts of low-to-medium intensity is the most developed research subject of forest fire physics, which justifies the emphasis of the workshop. Since the nineties, the two-sided challenge of constructing models describing the behaviour of the fire front that are, on the one hand, physically sound and general, and, on the other hand, potentially applicable on operational grounds, has originated the proposal of a diversity of modelling strategies, each one of them giving rise to some hard mathematical problems. Such strategies are in different stages of theoretical development and antagonize or complement each other in a larger or lesser extent, none of them self-imposing as clearly superior to the others. The choice of the lecturers precisely reflects the desire to encompass the most relevant strategies that have been proposed.

The main goals of the event are:

- to promote the communication (i.e., mutual knowledge, criticisms, possible future synergies respecting results and, above all, strategies of research) among researchers with a common interest and competence on theoretical modelling issues of forest fires, with an emphasis on fire front propagation;
- to introduce to the Portuguese mathematicians the open mathematical and physical research problems brought up by some representative theoretical modelling strategies that are being used to describe the behaviour of forest fire fronts.

The workshop will be held at the Departamento de Engenharia Mecânica da Universidade de Coimbra located in Pólo II.

Lectures

Numerical simulation of wild fires

Terry L. Clark (University of British Columbia, Canada)

What is missing from fire ecology?

Edward A. Johnson (University of Calgary, Canada)

Title to be announced

Olivier Séro-Guillaume (CNRS, France)

Convection in forest fires

Jacques Simon (Université Blaise Pascal/CNRS, France)

Some developments in premixed combustion modeling

Gregory Sivashinsky (Tel-Aviv University, Israel)

For more information about the event, see

http://www.mat.uc.pt/~tt2004/fire

July 12 - 16: School on Atmospheric Sciences and Climate Dynamics

Organizers

Didier Bresch (CNRS/Univ. Joseph-Fourier, France), José Miguel Urbano (Univ. Coimbra), Juha Videman (Instituto Superior Técnico, Lisbon)

Aims

The understanding of the fluid dynamics of the atmosphere and oceans and the development of techniques to simulate weather and climate are among the most important challenges for today's science. To make progress in this field and deepen our understanding of the complex processes that control the climate, the chemistry of the coupled atmosphere-ocean system, and the physics of the upper atmosphere, it is fundamental to intensify interdisciplinary collaborations amongst applied mathematicians and geophysicists.

The main goals of the event are:

- to promote the exchange of ideas among the distinct fields that share a common interest in Atmospheric Sciences, including fluid dynamics, physical oceanography, meteorology and applied mathematics;
- to provide students with a broad overview of this challenging topic.

The school will be held in Lisbon at the Complexo Interdisciplinar of the Instituto Superior Técnico.

Short Courses

Evidence for human influence on climate and implications for climate forecasting

Myles Allen (University of Oxford, UK)

Energy balance models in climate dynamics

Jesus Ildefonso Díaz (Universidad Complutense de Madrid, Spain)

The nonlinear dynamics of large-scale atmospheric flows

Michael Ghil (UCLA, USA and ENS-Paris, France)

Transport, stirring and mixing in atmospheric chemistry and dynamics

Peter Haynes (University of Cambridge, UK)

Modeling ocean mixing

Esteban G. Tabak (Courant Institute, NYU, USA)

For more information about the event, see

http://www.mat.uc.pt/~tt2004/atmosphere

July 19-24: Summer School and Workshop on Oceanography, Lakes and Rivers

Organizers

Didier Bresch (CNRS/Univ. Joseph-Fourier, France), José Miguel Urbano (Univ. Coimbra), Juha Videman (Instituto Superior Técnico, Lisbon)

Aims

Mathematics has always played a fundamental role in the study of oceans and river flows, although these research fields are still perhaps more familiar to oceanographers, geophysicists and environmental engineers than to mathematicians. Nonlinear PDEs are crucial in describing ocean processes such as internal and surface waves, ocean tides and currents, turbulence, changes in salinity and temperature, just to mention a few.

The main goals of the event are:

- to promote the communication and interactions between the specialists working on different frontiers of Oceanography;
- to introduce to the Portuguese applied mathematicians, in particular to graduate and PhD students, the fundamentals, as well as some of the most relevant current problems, of Environmental and Geophysical Fluid Dynamics;
- to provide an opportunity for physical oceanographers and mathematicians to establish contacts and develop common research projects.

The event consists of a four-day summer school and a two-day workshop and will be held in Lisbon at the Complexo Interdisciplinar of the Instituto Superior Técnico.

Short Courses in the Summer School

Turbulent geophysical flows and transport in rotating fluids

Peter Constantin (University of Chicago, USA)

Hydrodynamics of rivers and estuaries

Benoît Cushman-Roisin (Dartmouth College, USA)

Rotating fluids and associated boundary layers

Emmanuel Grenier (ENS-Lyon, France)

Elements of geophysical fluid dynamics

Joseph Pedlosky (Woods Hole Oceanographic Institution, USA)

The Saint-Venant system for shallow water. Derivation from Navier-Stokes and numerical solution

Benoît Perthame (ENS-Paris, France)

Plenary Lectures at the Workshop

Stability of Ekman boundary layers and applications

Benoît Desjardins (École Polytechnique, France)

Avalanches: models and mathematical results

Reinhard Farwig (TU Darmstadt, Germany)

Mathematical and numerical analysis of the primitive equations in oceanography

Francisco Guillén-González (Universidad de Sevilha, Spain)

Adjustment of the global thermohaline circulation to local forcing anomalies

David Marshall (University of Reading, UK)

Bifurcations and pattern formation in Geophysical Fluid Dynamics

João Teixeira (UCAR/NRL, USA)

Turbulence, clouds and climate models

Shouhong Wang (Indiana University, USA)

For more information about the event, see

http://www.mat.uc.pt/~tt2004/ocean

Other CIM events in 2004:

Workshop on Nonstandard Mathematics

5-11 July, 2004

Organizers:

Imme van den Berg, University of Évora, Portugal

Francine Diener, Université de Nice, France

A. J. Franco de Oliveira, University of Évora, Portugal

Vítor Neves, University of Aveiro, Portugal

Keith D. Stroyan, University of Iowa, USA

João Paulo Teixeira, IST, Lisbon, Portugal

Summer School on Mathematics in Biology and Medicine

20-24 September, 2004

Organizers: Jorge Careneiro, IGC, Oeiras, Portugal

Francisco Dionísio, IGC, Oeiras, Portugal

José Faro, IGC, Oeiras, Portugal

Gabriela Gomes, IGC, Oeiras, Portugal

Isabel Gordo, IGC, Oeiras, Portugal

Autumn School and International Conference on Stochastic Finance

20-30 September, 2004

Organizers:

Paulo Brito, ISEG, Lisbon, Portugal

Manuel L. Esquível, New University of Lisbon, Portugal

Maria do Rosário Grossinho, ISEG, Lisbon, Portugal

João Nicolau, ISEG, Lisbon, Portugal

Paulo Eduardo Oliveira, Uni. of Coimbra, Portugal

For more information about this event, see

http://pascal.iseg.utl.pt/∼stochfin2004/

For updated information on these events, see http://www.cim.pt/cimE/eventos04.html.

CIM News

CIM Events for 2005

The CIM Scientific Committee will meet in Coimbra on February 14, to analyze the CIM scientific program for 2005.

CIM proceedings published by Birkhäuser

In 2003, Birkhäuser published the proceedings of the conference "Bifurcations, symmetry and patterns", held in Porto as part of the CIM 2000 Thematic Term on Dynamics, Bifurcation and Biology. The editors are Jorge Buescu, Sofia Castro, Ana Paula Dias and Isabel Labouriau.

For more information, see

http://www.birkhauser.ch/books/math/7020.htm

CIM Publications

CIM has recently added four more items to its series of monographs and volumes of proceedings:

21. P. Quaresma, A. Dourado, E. Costa, J. Félix Costa (editors), *Soft Computing and Complex Systems*, 2003.
22. Alírio E. Rodrigues, Paula de Oliveira, José Almiro Castro, José Augusto Ferreira, Maria do Carmo Coimbra (editors), *Modelling and Simulation in Chemical Engineering*, 2003.
23. Isabel Narra Figueiredo, Luis Filipe Menezes, Juha Videman (editors), *Modelling and Numerical Simulation in Continuum Mechanics*, 2003.
24. Carlos Fernandes, Joaquim Júdice, Carlos Salema (editors), *Mathematical Techniques and Problems in Telecommunications*, 2003.

Research in Pairs at CIM

CIM has facilities for research work in pairs and welcomes applications for their use for limited periods.

These facilities are located at Complexo do Observatório Astronómico in Coimbra and include:

- office space, computing facilities, and some secretarial support;
- access to the library of the Department of Mathematics of the University of Coimbra (30 minutes away by bus);
- lodging: a two room flat.

At least one of the researchers should be affiliated with an associate of CIM, or a participant in a CIM event.

Applicants should fill in the electronic application form

http://www.cim.pt/cim.www/cim_app/application.htm

CIM on the WWW

Complete information about CIM and its activities can be found at the site

http://www.cim.pt

This is mirrored at

http://at.yorku.ca/cim.www/

Combining logic systems: Why, how, what for?

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1 Background

The practical significance of the problem of combining logics is widely recognized, namely in knowledge representation (within artificial intelligence) and in formal specification and verification of algorithms and protocols (within software engineering and security). In these fields, the need for working with several calculi at the same time is the rule rather than the exception. For instance, in a knowledge representation problem it may be necessary to work with temporal, spatial, deontic and probabilistic aspects (e.g., for reasoning with mixed assertions like "with probability greater than 0.99, sometime in the future smoking will be forbidden everywhere"). And in a software verification problem it may be necessary to mix equational, epistemic and dynamic logic features. That is, one needs, at least, to be able to develop theories with components in different logic systems, or, even better, to work with theories defined in the combination of those logic systems (where such mixed assertions are allowed).

Motivated by these applications that require the joint use of several deduction formalisms, the interest in combination of logic systems has recently been growing (as reflected in the series [9, 20, 2, 18, 26, 1]), but the topic is also of interest on purely theoretical grounds. For instance, one might be tempted to look at predicate temporal logic as resulting from the combination of first-order logic and propositional temporal logic. However, the approach will be significant only if general preservation results are available about the combination mechanism at hand. For example, if it has been established that completeness is preserved by a combination mechanism $\bullet$ and it is known that logic system $\mathcal{L}$ is given by $\mathcal{L}' \bullet \mathcal{L}''$, then the completeness of $\mathcal{L}$ follows from the completeness of $\mathcal{L}'$ and $\mathcal{L}''$. No wonder that much theoretical effort has been dedicated to establishing preservation results and/or finding preservation counterexamples about different combination mechanisms. For an early overview of the practical and theoretical issues see also [4].

Several forms of combination have been studied, like product [30, 21, 22, 23], fusion [38, 28, 29, 40, 19], temporalization [12, 13, 41, 14], parameterization [6], synchronization [33] and fibring [15, 16, 3, 17, 34, 42]. Fusion is the best understood combination mechanism. In short, the fusion of two modal systems leads to a bimodal system including the two original modal operators and common propositional connectives. Several interesting properties of logic systems (like soundness, weak completeness, Craig interpolation property and decidability) were shown to be preserved when fusing modal systems (see [28, 27]).

More recently, research has been directed at fibring, a more general combination mechanism proposed by Gabbay [15, 16]. Fibring can be applied beyond the universe of modal systems and captures fusion as a special case. Although well understood at the proof-theoretic level since it was proposed, fibring raised some difficulties at the semantic level [34].

For the sake of simplicity, we adopt here a basic universe of logic systems encompassing only propositional-based systems endowed with Hilbert calculi and ordered algebraic semantics. Nevertheless, this universe is rich enough to illustrate interesting features of fibring and to provide the basis for the combination of systems varying from intuitionistic to many-valued logics (including modal systems as special cases). Those interested in wider universes (encompassing first-order quantification, higher-order features, non truth-functional semantics, non Hilbert calculi, etc) where fibring can still be defined should look instead at [35, 36, 7, 5, 25, 31].

It is straightforward to define fibring in this basic universe. And, with respect to preservation results, we concentrate our attention on finding sufficient conditions only for the preservation of (strong global) completeness. Barring some examples and omitted proofs that can be found in the literature, the following presentation is self contained.

2 Logic systems

A *signature* C is an $\mathbb{N}$-indexed family of countable sets. The elements of each C_k are called *constructors* of arity k.

Let $T(C, \Xi)$ be the free algebra over C generated by Ξ. The *language* $L(C)$ is $T(C, \emptyset)$. We shall consider different signatures but we assume fixed once and for all a set Ξ of propositional variables. Fixed Ξ, the *schema language* $T(C, \Xi)$ is denoted by $sL(C)$.

A *rule* over C is a pair $r = \langle \Theta, \eta \rangle$ where $\Theta \cup \{\eta\} \subseteq sL(C)$. We shall work only with finitary rules, that is, we assume that the set Θ of premises is finite.

An *ordered algebra* over C is a tuple $\mathbf{A} = \langle A, \leq, \top, \cdot_{\mathbf{A}} \rangle$ where $\langle A, \leq, \top \rangle$ is a topped partial order and $\langle A, \cdot_{\mathbf{A}} \rangle$ is an algebra over C.

A *logic system* is a tuple $\mathcal{L} = \langle C, \mathcal{A}, R_\ell, R_{\mathrm{g}} \rangle$ where C is a signature, $\mathcal{A}$ is a class of ordered algebras over C (the models of the system) and both R_ℓ and R_{g} are sets of rules over C. It is common to assume that the set of local rules R_ℓ is included in the set R_{g} of global rules.

As an example consider the following intuitionistic system. The signature contains the usual connectives. The class of models includes every ordered algebra induced by a Heyting algebra (with $a \leq b$ iff $a \wedge_{\mathbf{A}} b = a \sqcap b = a$). The local rules are the usual rules of a Hilbert calculus for intuitionistic propositional logic. Finally, there are no extra global rules. A detailed presentation of intuitionistic logic along these lines can be found in [32].

Consider also the example of the following modal system. The signature contains the usual basic propositional constants and connectives plus the modal operator $\Box$. The class of models includes every ordered algebra induced by a general Kripke structure $\langle W, \mathcal{B}, \rho, V \rangle$ as follows:

- $A = \mathcal{B}$; $a \leq b$ iff $a \subseteq b$; $\top = W$;
- $\pi_{\mathbf{A}} = V(\pi)$;
- $\neg_{\mathbf{A}}(a) = W \setminus a$;
- $\Rightarrow_{\mathbf{A}}(a, b) = (W \setminus a) \cup b$;
- $\Box_{\mathbf{A}}(a) = \{w \in W : w \rho v$ implies $v \in a$ for every $v \in W\}$.

(The notion of general Kripke structure was proposed in [39] in order to obtain a completeness theorem for modal logic.) A more direct approach would be to take as models the ordered algebras induced by modal algebras. The local rules include the classical propositional rules plus the normalization axiom

$$\langle \emptyset, (\Box(\xi_1 \Rightarrow \xi_2)) \Rightarrow ((\Box\xi_1) \Rightarrow (\Box\xi_2)) \rangle .$$

The unique extra global rule is the necessitation rule $\langle \{\xi_1\}, \Box\xi_1 \rangle$.

Many other interesting logics (even many-valued ones like Gödel's and Łukasiewicz's — see for instance [24]) are also logic systems in the sense given above.

Within the context of a logic system, the denotation $[\![\varphi]\!]^\alpha_{\mathbf{A}}$ of a schema formula φ on an ordered algebra $\mathbf{A}$ and for an assignment $\alpha : \Xi \to A$ is easily defined by induction on the structure of φ.

In any given logic system $\mathcal{L} = \langle C, \mathcal{A}, R_\ell, R_{\mathrm{g}} \rangle$ we are able to define the following four consequence operators:

- global entailment: $\Gamma \vDash^{\mathrm{g}} \varphi$ iff, for every $\mathbf{A} \in \mathcal{A}$ and $\alpha : \Xi \to A$, if $\top \leq [\![\gamma]\!]^\alpha_{\mathbf{A}}$ for each $\gamma \in \Gamma$ then $\top \leq [\![\varphi]\!]^\alpha_{\mathbf{A}}$;
- local entailment: $\Gamma \vDash^\ell \varphi$ iff, for every $\mathbf{A} \in \mathcal{A}$, $\alpha : \Xi \to A$ and $a \in A$, if $a \leq [\![\gamma]\!]^\alpha_{\mathbf{A}}$ for each $\gamma \in \Gamma$ then $a \leq [\![\varphi]\!]^\alpha_{\mathbf{A}}$;
- global derivation: $\Gamma \vdash^{\mathrm{g}} \varphi$ iff φ can be derived from Γ using the rules in R_{g};
- local derivation: $\Gamma \vdash^\ell \varphi$ iff φ can be derived from Γ and theorems (formulae globally derived from an empty set of assumptions) using only the rules in R_ℓ.

Observe that in the modal system described above we can globally derive $(\Box\xi_1) \Rightarrow (\Box\xi_2)$ from $\xi_1 \Rightarrow \xi_2$ but we can not do so locally. The distinction between local and global reasoning appeared in the context of modal logic (local means carried out at a single world and global refers to reasoning about all worlds) but can be useful in other universes.

A logic system is said to be *strongly globally sound* when if $\Gamma \vdash^{\mathrm{g}} \varphi$ then $\Gamma \vDash^{\mathrm{g}} \varphi$. And it is said to be *strongly globally complete* when if $\Gamma \vDash^{\mathrm{g}} \varphi$ then $\Gamma \vdash^{\mathrm{g}} \varphi$. When we only consider $\Gamma = \emptyset$ we get the corresponding weak notions. Mutatis mutandis, we define the local versions.

3 Completeness theorem

A logic system is said to be *full* when $\mathcal{A}$ is composed of all ordered algebras over C that fulfill the rules in both R_ℓ and R_{g}. Therefore, every full logic system is (weakly and strongly, locally and globally) sound. A logic system has *verum* if its language contains a theorem that denotes $\top$ in every model.

A logic system is said to be *congruent* when for every Γ closed for global derivation, $c \in C_k$ and $\varphi_1, \ldots, \varphi_k, \psi_1, \ldots, \psi_k \in sL(C)$:

$$\frac{\begin{array}{ll} \Gamma, \varphi_i \vdash^\ell \psi_i & \\ \Gamma, \psi_i \vdash^\ell \varphi_i & i = 1, \ldots, k \end{array}}{\Gamma, c(\varphi_1, \ldots, \varphi_k) \vdash^\ell c(\psi_1, \ldots, \psi_k)}$$

Theorem 3.1 Every full and congruent logic system with verum is strongly globally complete.

The proof is carried out using a Lindenbaum-Tarski construction. A syntactic ordered algebra $\mathbf{A}_\Gamma$ can be built as follows from each Γ closed for $\vdash^{\mathrm{g}}$. First we define a congruence relation over $sL(C)$: $\varphi \cong_\Gamma \psi$ iff $\Gamma, \varphi \vdash^\ell \psi$ and $\Gamma, \psi \vdash^\ell \varphi$. Then, we choose A to be $sL(C)/\cong_\Gamma$. The partial order is defined as follows: $[\varphi]_\Gamma \leq [\psi]_\Gamma$ iff $\Gamma, \varphi \vdash^\ell \psi$. The top $\top$ is the equivalence class of the verum. Finally, for each language constructor, $c_{\mathbf{A}_\Gamma}([\varphi_1]_\Gamma, \ldots, [\varphi_k]_\Gamma) = [c(\varphi_1, \ldots, \varphi_k)]_\Gamma$. Clearly, by construction, we infer that $[\![\varphi]\!]^{\lambda\xi.[\xi]_\Gamma}_{\mathbf{A}_\Gamma} = \top$ iff $\varphi \in \Gamma$ and that $\mathbf{A}_\Gamma$ fulfills the rules of the logic system.

Assume $\Delta \not\vdash^{\mathrm{g}} \epsilon$. We have to show $\Delta \not\models^{\mathrm{g}} \epsilon$. It is sufficient to find an ordered algebra $\mathbf{A} \in \mathcal{A}$ such that $[\![\delta]\!]^{\lambda\xi.[\xi]_\Gamma}_{\mathbf{A}} = \top$ for each $\delta \in \Delta$ and $[\![\epsilon]\!]^{\lambda\xi.[\xi]_\Gamma}_{\mathbf{A}_\Gamma} \neq \top$. Consider $\Gamma = \Delta^{\vdash^{\mathrm{g}}}$. Then, $\mathbf{A}_\Gamma$ globally satisfies each element of Δ (since $\Delta \subseteq \Gamma$) but $\mathbf{A}_\Gamma$ does not globally satisfy ϵ (since $\epsilon \notin \Gamma$). This concludes the proof of the completeness theorem.

Observe that the requirements for completeness are quite weak and usually fulfilled by commonly used logic systems (including those mentioned above as examples). Furthermore, any complete logic system can be made full without changing its entailments. And if verum is not present, it can be conservatively added to the language. But if the system at hand is not congruent, there is nothing we can do within the scope of the basic theory of fibring outlined here.

Note also that through a mild strengthening of the requirements of the theorem we can ensure finitary strong local completeness (see for instance [37]). A similar strong (local and global) completeness theorem is obtained in [42] without extra requirements for local reasoning but assuming a more complex semantics and using a Henkin construction.

4 Fibring

Consider signatures C and C' such that $C'_k \subseteq C_k$ for each $k \in \mathbb{N}$. Given an ordered algebra $\mathbf{A}$ over C, we denote by $\mathbf{A}|_{C'}$ the reduct $\langle A, \leq, \top, \cdot_{\mathbf{A}}|_{C'} \rangle$ of $\mathbf{A}$ by the inclusion (where $\cdot_{\mathbf{A}}|_{C'}$ is the restriction of $\cdot_{\mathbf{A}}$ to C'). Clearly, $\mathbf{A}|_{C'}$ is an ordered algebra over C'.

Given two logic systems $\mathcal{L}' = \langle C', \mathcal{A}', R_\ell{}', R_{\mathrm{g}}{}' \rangle$ and $\mathcal{L}'' = \langle C'', \mathcal{A}'', R_\ell{}'', R_{\mathrm{g}}{}'' \rangle$, their *fibring* $\mathcal{L}' \odot \mathcal{L}'' = \langle C, \mathcal{A}, R_\ell, R_{\mathrm{g}} \rangle$ is as follows:

- $C_k = C'_k \cup C''_k$ for each $k \in \mathbb{N}$;
- $\mathcal{A}$ is the class containing every ordered algebra $\mathbf{A}$ over C such that $\mathbf{A}|_{C'} \in \mathcal{A}'$ and $\mathbf{A}|_{C''} \in \mathcal{A}''$;
- $R_\ell = R_\ell{}' \cup R_\ell{}''$; $R_{\mathrm{g}} = R_{\mathrm{g}}{}' \cup R_{\mathrm{g}}{}''$.

This definition corresponds to the constrained version of fibring (as defined in [34]) since any symbols common to both logic systems will be shared. Unconstrained fibring can be obtained by making sure that no symbols are in both signatures. Fibring can appear as a universal construction in a suitable category of logic systems (as explored in [34] where the categorical approach was important in fine tuning the semantics of fibring).

As a first example of fibring, consider the combination of two modal systems while sharing the propositional connectives. This constrained fibring is equivalent to the fusion of the two given modal systems. The result is a bimodal system.

The combination of a modal system with a relevance system is similar from the point of view of fibring but beyond the scope of fusion. By sharing the propositional connectives we obtain a logic system with a modal box and a relevance implication. For details about relevance logic see for instance [11].

Note that, even when no symbols are shared, fibring may impose unexpected interactions between the logical operations from the two given logics. For instance, consider the unconstrained fibring of classical propositional logic and intuitionistic propositional logic. Unexpectedly, in the resulting logic system the intuitionistic implication collapses into classical implication. In short, in the resulting logic system we have two copies of classical logic. This first example of collapsing was first identified in [10]. Other examples are given in [37] where a relaxed notion of fibring is proposed in order to avoid such collapses.

5 Preservation results

We now turn our attention to transference results. We start by examining if soundness is preserved by fibring. Then we consider completeness. To this end we have to establish the preservation of other interesting properties, namely the metatheorem of deduction.

Theorem 5.1 Soundness is preserved by fibring.

It is straightforward to prove that (strong and weak, global and local) soundness is unconditionally preserved by fibring in the basic universe of logic systems considered here. However, in larger universes things can be more complicated. For instance, when fibring logic systems with quantifiers and using rules with side provisos (like, provided that term θ is free for variable x in formula ξ), soundness is not always preserved [36, 7].

Weak completeness is also not always preserved as shown in [42]. Herein we examine in detail if strong global completeness is preserved when fibring basic logic systems as defined above. Adapting the technique originally proposed in [42], we capitalize on the completeness theorem stated above about such logic systems. That is, when fibring two given logic systems that are full, congruent and with verum (and, therefore, strongly globally complete) we shall try to obtain the strong global completeness of the result by identifying the conditions under which fullness, congruence and verum are preserved by fibring.

Lemma 5.2 Fullness is preserved by fibring.

Lemma 5.3 The result of fibring has verum provided that at least one of the given logic systems has verum.

However, congruence is not always preserved by fibring. Consider the fibring of two logic systems $\mathcal{L}', \mathcal{L}''$ with the following signatures and rules:

$$C_0' = \{\pi_0, \pi_1, \pi_2\} \quad C_1' = \{c\} \quad C_k' = \emptyset \text{ for } k > 1$$
$$R_\ell{}' = \emptyset \quad R_{\mathrm{g}}{}' = \{\langle \{\xi\}, c(\xi) \rangle\}$$
$$C_0'' = \{\pi_0, \pi_1, \pi_2\} \quad C_k'' = \emptyset \text{ for } k > 0$$
$$R_\ell{}'' = R_{\mathrm{g}}{}'' = \{\langle \{\pi_0, \pi_1\}, \pi_2 \rangle, \langle \{\pi_0, \pi_2\}, \pi_1 \rangle\}$$

Clearly, both $\mathcal{L}'$ and $\mathcal{L}''$ are congruent, but their fibring $\mathcal{L} = \mathcal{L}' \odot \mathcal{L}''$ is not congruent. Indeed, consider $\Gamma = \{\pi_0\}^{\vdash^{\mathrm{g}}} = \{c^n(\pi_0) : n \geq 0\}$. So, from Γ, π_1 and π_2 are locally interderivable but, from Γ, $c(\pi_1)$ and $c(\pi_2)$ are not locally interderivable.

Fortunately, it is possible to establish a useful sufficient condition for the preservation of congruence by fibring. A logic system is said to have *implication* if its signature contains a binary connective $\Rightarrow$ fulfilling the following Metatheorem of Modus Ponens (MTMP)

$$\frac{\Gamma \vdash^\ell (\delta_1 \Rightarrow \delta_2)}{\Gamma, \delta_1 \vdash^\ell \delta_2}$$

and the following Metatheorem of Deduction (MTD):

$$\frac{\Gamma^{\vdash^{\mathrm{g}}}, \delta_1 \vdash^\ell \delta_2}{\Gamma^{\vdash^{\mathrm{g}}} \vdash^\ell (\delta_1 \Rightarrow \delta_2)} .$$

When fibring two logic systems with implication while sharing the implication symbol, it is straightforward to verify that the resulting logic system also has implication. Indeed:

Theorem 5.4 The result of fibring has MTMP provided that at least one of the given logic systems has MTMP and the implication symbol is shared.

Theorem 5.5 The result of fibring has MTD provided that both given logic systems have MTD and the implication symbol is shared.

The latter result is a direct corollary of the following fact:

Lemma 5.6 MTD holds in a logic system iff: (i) $\vdash^\ell (\xi \Rightarrow \xi)$; (ii) $\{\xi_1\}^{\vdash^{\mathrm{g}}} \vdash^\ell (\xi_2 \Rightarrow \xi_1)$; and (iii) $\{(\xi \Rightarrow \gamma_1), \ldots, (\xi \Rightarrow \gamma_k)\}^{\vdash^{\mathrm{g}}} \vdash^\ell (\xi \Rightarrow \gamma)$ for each local rule $\langle \{\gamma_1, \ldots, \gamma_k\}, \gamma \rangle$.

A logic system is said to have *equivalence* if it has implication and its signature contains a binary connective $\Leftrightarrow$ fulfilling the two Metatheorems of Biconditionality (relating implication with equivalence) and the Metatheorem of Substitution of Equivalents (MTSE).

Theorem 5.7 A logic system with equivalence is congruent.

When fibring two logic systems with equivalence while sharing the implication symbol as well as the equivalence symbol we obtain a logic system with equivalence. Therefore:

Theorem 5.8 The fibring while sharing implication and equivalence of full logic systems with equivalence and verum is strongly globally complete.

This preservation result is quite useful because many widely used logic systems do have equivalence in the sense above.

6 Final remarks

In this guided tour of the issues raised by the combination of logics we defined fibring in a very simple (yet useful) context and established some interesting transference results. As already mentioned, fibring can and has been defined and analyzed in much more complex situations. Current research is directed at widening the universe where fibring can be defined and at establishing other transference results like sufficient conditions for the preservation of interpolation properties, weak completeness and decidability. Concerning conditions for the preservation of weak completeness, it is still an open problem if the ghost symbol technique (used in [28] for proving the preservation of weak completeness by fusion) can be generalized in order to be used for fibring. With respect to the preservation of interpolation properties, the recent results in [8] seem to provide an appropriate context.

Acknowledgments

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What's New in Mathematics

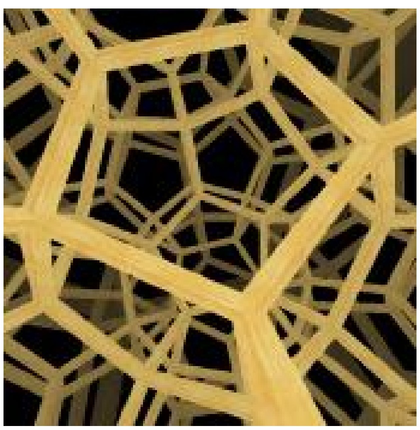
Largest prime yet. "MSU student's prime number largest one yet" is a story by Sharon Terlep in the December 4 2003 *Lansing State Journal*. "Michael Shafer, a 26-year old chemical engineering student, made math history by discovering the largest prime number known." Shafer did it by running a program that "hooked up... more than 200,000 computers worldwide." The program had been running for 19 days when "an alarm sounded letting him know his computers tagged a prime number." The number is a Mersenne prime (of the form $2^p - 1$, where p is prime); in Schafer's case $p = 20,966,011$ and the number itself has over six million digits. According to Shafer, "The number itself really isn't useful. What's more important is what's gone into developing the server and that the programs can get all these computers to work together for a common goal." And: "There may come a time when there's more important research that can harness this technology and use it for something more relevant." Terlep's story is available online; *Largest prime number ever* is found on the *New Scientist* News Service (http://www.newscientist.com/news/news.jsp?id=ns99994438).

Mathe-musical instrument. It's the tritare ("TREE-tar"), it's the invention of two Canadian mathematicians (Samuel Gaudet and Claude Gauthier, both at the University of Moncton), and it may revolutionize music. The story, by Karen Burchard, ran in the November 28, 2003 *Chronicle of Higher Education*. Gaudet and Gauthier are number theorists. In their research on "the odd-number portion of the 'p-series' problem" they came across a class of numbers with symmetries that seemed initially to have potential in engineering but ended up instantiated in a musical instrument "shaped like an inverted Y and equipped with six networks of strings that can produce a range of sounds, from guitarlike musical notes to percussive beats reminiscent of a church bell. If one string is plucked, it vibrates across all three of the fretboards". More details are available from various canadian sources. CBC - New Brunswick shows a photograph of the apparatus. Radio Canada has link to a 4-minute streaming video interview with the two inventors and their instrumentalist (en français. Gaudet: "Tout à coup on a pensé que ça pouvait faire un méchant instrument de musique."). Guitariste.com has some hints about the design (also en français). CBC Arts News has a link to a 1-minute broadcast where you can actually hear the tritare playing in the background.

Who invented the electronic digital computer? It was a professor of math and physics at Iowa State University, and it happened between 1937 and 1941. An article in the Ames, Iowa *Tribune* for November 1, 2003 reports on a meeting of "computer experts from around the world" to celebrate what would have been the 100th birthday of John Vincent Atanasoff who, together with his graduate student Clifford Berry, developed and built the first modern computer. What about ENIAC, you might ask. The Army Research Laboratory webpage is still running *The ENIAC Story* (Martin Weik, 1961) which states "The world's first electronic digital computer was developed by Army Ordnance to compute World War II ballistic firing tables," crediting Dr. John W. Mauchly and J. P. Eckert, Jr., of the University of Pennsylvania, for the original design and quoting from the patent (No 3,120,606) they filed on June 26, 1947. But 12 years after that story was written, a Federal Judge ruled that the ENIAC patents were invalid, and that "Eckert and Mauchly did not themselves first invent the automatic digital electronic computer, but instead derived that subject matter from one Dr. John Vincent Atanasoff". It's not a pretty story, and it's all told on the ISU Dept of Computer Science website.

Auxin and the Fibonacci Numbers. A European team led by Didier Reinhart and Eva-Rachele Pesce of the University of Bern has made great progress towards an understanding of the biochemical basis of phyllotaxis, the regular arrangement of leaves around a plant's stem that leads to spirals with charcteristic mathematical properties. Namely, the numbers of left and right-turning spirals are almost always two consecutive Fibonacci numbers. The team performed an ingenious set of experiments using recently developed mutant strains of Arabidopsis to show that the concentration of the plant hormone auxin and the distribution of primordia (leaf buds) participate in a positive-negative feedback system analogous to "the short-range activator and long-range inhibitor in reaction-diffusion mechanisms." They conclude: "Our model accounts for the reiterativity and the stability of organ positioning." But the way the precise divergence angles are determined (which is where the "golden angle" 137.5° and the Fibonacci numbers enter the picture) is only addressed speculatively. The research is reported in an article in *Nature*, November 20, 2003.

At home in dodecahedral space. The cover story in the October 9 2003 *Nature* is "Dodecahedral space topology as an explanation for weak wide-angle temperature correlations in the cosmic microwave background." Dodecahedral space was invented a hundred years ago by Henri Poincaré -he used it as a counterexample to an early version of his famous conjecture. You make a 3-dimensional space with no boundary by taking a solid dodecahedron and identifying opposite sides (after a rotation by $\pi/5$). If you are living in this space you don't feel any boundaries: as you cross one of the original faces you re-enter, slightly rotated, from the opposite side. This should feel perfectly natural, because the authors of the article (Jean-Pierre Luminet, Jeffrey Weeks, Alain Riazuelo, Roland Lehoucq and Jean-Philippe Uzan) give evidence to show that dodecahedral space may in fact be the shape of the universe we live in.



The view in dodecahedral space (if the framework of the docecahedron is visible). Adjacent cells are just the cell you're in, seen from different points. A spherical wavefront will intersect with itself in "circles in the sky." If detected, these would give an experimental confirmation of the theory. Three dodecahedra fit together evenly around an edge only if the space is positively curved. In physical terms, this means a value strictly greater than 1 for the mass-energy density parameter ?0, another point subject to experimental test. Image courtesy Jeff Weeks, used with permission.

The evidence comes from the spectrum of the temperature fluctuations on the microwave sky ("the waves from the Big Bang"). The data from the Wilkinson Microwave Anisotropy Probe reveal that the lowest-mode observable vibration (the quadrupole) is "only about one-seventh as strong as would be expected in an infinite flat space". The team calculated the spectrum of dodecahedral space, which "depends strongly on the assumed mass-energy density parameter Ω_0". They observe that for $1.012 < \Omega_0 < 1.014$ the values for both the quadrupole and the next-lowest mode (the octopole) give good matches to the experimental numbers from WMAP, while their range for Ω_0 falls "comfortably within WMAP's best-fit range of $\Omega_0 = 1.02 \pm 0.02$". Numbers from upcoming experiments including the Planck Surveyor should determine ?0 within 1"Finding $\Omega_0 < 1.01$ would refute the Poincaré space as a cosmological model, while $\Omega_0 > 1.01$ would provide strong evidence in its favour".

Infinite Wisdom, a piece by Erica Klarreich in the August 30 2003 *Science News*, surveys some recent work on the continuum hypothesis. Klarreich starts with a review of Cantor's proof that the set **R** of real numbers is strictly larger, in a precise sense, than the set **Z** of integers. In this connection she shows Helaman Ferguson's clever visualization of Cantor's diagonal argument:

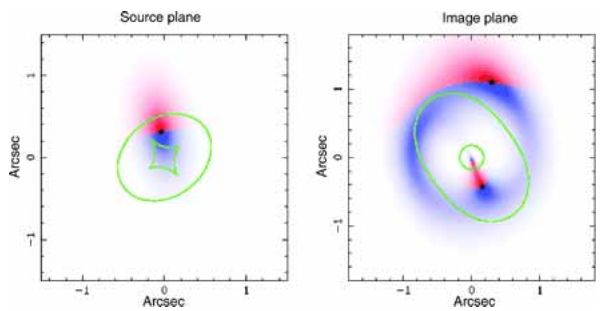
Cantor's Flickering Diagonal. The left half of this stereo pair shows the beginning of an enumeration of the real numbers between 0 and 1. The top line represents the start of the binary expansion of the first number on the list (white=0, black=1). The next line corresponds to the second number on the list, and so on. The right half is identical, except that each diagonal element (the first digit of the first number, the second digit of the second number, and so on) has been reversed: changed from white to black or from black to white. When the two images are fused, the reversed diagonal flickers in and out. The reversed diagonal is the binary expansion of a real number that cannot occur on the original list. Since this will happen for any list, the construction shows that there is no way of listing the real numbers between 0 and 1. Image courtesy Helaman Ferguson, used with permission.

The Continuum Hypothesis is the statement that there is no intermediate size: there is no set with strictly more elements than **Z** and strictly fewer elements than **R**. The truth or falsity of this hypothesis was number one on Hilbert's 1900 list of important unsolved problems. Klarreich continues with the history of the hypothesis, and of its relation to the standard axioms of set theory. She surveys the work of Kurt Gödel (1938) and Paul Cohen (1963): "Put together, those two results indicate that it's impossible either to prove or to disprove the continuum hypothesis using the standard axioms." Which brings us to Hugh Woodin

(U.C. Berkeley) and his recent work on the characterization of an axiom which could be added to the standard set and which would "answer all questions up to the level of the hierarchy that the continuum hypothesis concerns–the realm of the smallest uncountably infinite sets." Woodin calls such an axiom "elegant." Rather than try to construct such an axiom, "Woodin has proved –apart from one missing piece that must still be filled in– that elegant axioms do exist and, crucially, that every elegant axiom would make the continuum hypothesis false." Two survey papers by Woodin are available online: The continuum hypothesis, part I (http://www.ams.org/notices/200106/fea-woodin.pdf) and part II (http://www.ams.org/notices/200107/fea-woodin.pdf).

A heap of trouble. The July 3 2003 issue of *Nature* has a news feature by George Szpiro entitled "Does the proof stack up?" The topic is the fate of the research paper describing Thomas Hales' five year old proof of the Kepler Conjecture: the optimal way to pack equal spheres is the face-centered cubic arrangement used by grocers to stack oranges. The proof was unusual in that, after "reducing the infinite number of possible stacking arrangements to 5,000 contenders," it relied on a computer program to calculate the density of each arrangement; thereby verifying that face-centered cubic was the densest. Nevertheless Robert MacPherson (IAS, Princeton) asked Hales and his graduate student collaborator Sam Ferguson to submit their manuscript to the Annals of Mathematics. Understanding the complexity of the project, he named a team of twelve referees. But the referees have given up. Checking all three gigabytes of code, inputs and outputs turned out to be more than twelve humans could handle. So the paper is to be published with "a cautionary note ... stating that proofs of this type ... may be impossible to review in full." Hales is unhappy and has started a project to use computers to check every line of his proof; he estimates 20 person-years of work to carry it through.

Gravitational caustics. In the May 2 2003 *Science* a 7-member team led by Chris Carilli (National Radio Astronomy Observatory) published "A Molecular Einstein Ring: Imaging a Starburst Disk Surrounding a Quasi-Stellar Object." The QSO in question is PSS J2322+1944; images both in the Infrared (CO emission) and at 1.4 GHz show the "Einstein ring" diagnostic of "strong gravitational lensing by an intervening galaxy." In the absence of information about that particular lens, the team worked from a better known one and experimented with "various source configurations" until they could get a close match to the observed pattern. The model they derive is illustrated here.



"A gravitational lens model for the CO emission in PSS J2322+1944. ...The left panel presents the source plane distribution, corresponding to the true (i.e., undistorted by lensing) morphology of the system. The image plane distribution is presented in the right panel, corresponding to the observed morphology after being distorted by the gravitational lens. The pointlike QSO is represented by a black asterisk in the left panel and by two black asterisks in the right panel. The green solid lines are the caustics and critical lines in the source and image planes, respectively ... . The CO emission is modeled as an inclined disk ($i = 60^o$) around the QSO, and the north and south parts of the disk are color-coded red and blue, respectively, corresponding to different velocity regions on opposite sides of the QSO." (Image ©2003 Science, used with permission).

The paper argues from this reconstruction of the source that it must in fact be a star-forming galaxy.

An Interview with William R. Pulleyblank

Many of us have no idea as to how is the research environment in a private laboratory like the IBM T.J. Watson Research Center. Could you start by telling us about this research environment, in particular the one in the Department of Mathematical Sciences?

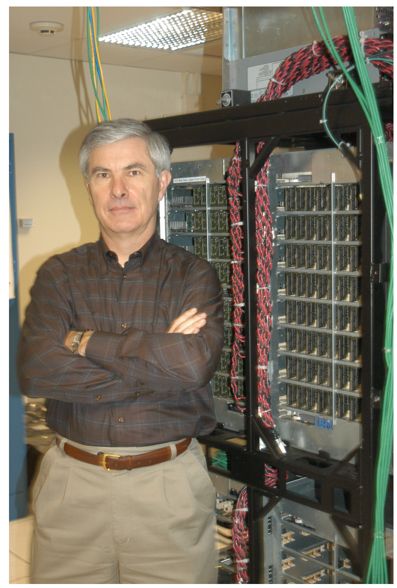
There is probably as much difference between different industrial research laboratories as there is between different universities. IBM Research has consistently had a mission that combined carrying out a top level scientific research agenda with the desire to make the results relevant to the corporation. In some ways, the Mathematical Sciences Department operates like a university department. We write and referee papers, edit journals and present papers at conferences. Some department members teach courses and supervise graduate students at nearby universities, for example, Columbia, NYU, Yale and MIT. However, there are significant differences. Many of the problems we work on come from IBM customers and other units within IBM. Often we are able to apply our work directly to real world problems. In addition, we always have the possibility of seeing the results that our research realized in the form of products. We do get pretty excited when this happens.

I recently heard a biographer of T. J. Watson (the father) emphasizing the importance of Research in the early days of the IBM company. Do you also think that research has played a vital role in the long success of the company?

Absolutely. When Lou Gerstner, our previous Chairman, formulated the principles that he wanted to guide the company, the first was "at our core, we are a technology company". He was a strong supporter of Research, as is our present chairman, Sam Palmisano. There is a feeling here that the things we do really have a chance to have an impact on the company and on our customers. It is very energizing.

In particular, how do you envision the Department of Mathematical Sciences thirty years from now? Will research staff there continue to do basic research and prove theorems?

I hope so. The model of combining serious mathematics with doing things that have the potential to positively affect the company has been remarkably successful, and robust. Examples range from devices like the Trackpoint, to software systems like OSL, to inventing new algorithms for digital half-toning. At the same time, there has been a remarkable collection of papers and books written by department members. The legacy of current and former department members like Shmuel Winograd, Mike Shub, Roy Adler, Alan Hoffman, Ellis Johnson, Ralph Gomory and Herman Goldstine sets quite an example!



William R. Pulleyblank

Now, let us focus on your career. In 1990 you moved from the University of Waterloo in Canada to the Watson Research Center. Would you like to comment on those times? What was the driving force that made you move?

In 1987, I was awarded an NSERC Industrial Research Chair in Optimization and Computer Applications, in part funded by CP Rail. This gave me a chance to expand the applied part of my research program, and also started me thinking about what I would do for the next 25 years of my career. In the spring of 1989, Ellis Johnson called and supposed that I would not move, but wanted suggestions for a possible successor to himself as Manager of the Optimization Center. It got me thinking about alternatives and I came here for a visit. I soon realized that IBM Research would be an excellent place to work on applied problems. Also, earlier in my career I had worked for IBM as a systems engineer. I had always had a high regard for the company, and the Watson Research Center had always seemed to me to be an exciting place.

So, after a lot of discussion with my wife, Diane, we decided to give it a shot and here we are.

How did your research program change as result of moving to the industry?

It evolved. I have always been an interactive mathematician, enjoying working with co-authors. Here I had the chance to work with people like John Tomlin, John Forrest and Ellis Johnson. I became very interested in computational questions. I was granted my first patent ever, jointly with John Tomlin and Alan Hoffman — an application of the Koenig edge coloring theorem to make certain matrix computations much more efficient.

A few years later you were chairing the Department of Mathematical Sciences in the Watson Research Center. How do you describe the leadership skills required for this job in comparison to those needed for a similar academic position?

The scope of a department Director's job is in some ways similar in scale to that of a dean. Tom Brzustowski when he was Provost at the University of Waterloo, described the faculty at the university as "800 small businessmen sharing a library and a parking lot". Today, he would probably add a computer network to the list. IBM is a much more hierarchical organization. When someone becomes a manager, it is not assumed that it is a temporary, three to five year, assignment. A department Director has a responsibility to generate funding for the department as well as to make sure that the careers of department members are progressing satisfactorily. In addition, it is important to understand and be able to present all the work done in a department. This has really encouraged me to broaden my outlook. For example, I am sure that I know much more Computational Biology now than I ever would have learned in a university mathematics department.

Do you think that an academic training can position one better for the industry than the other way around? I mean, do you think that someone with a career in the industry would have had a more difficult time chairing an academic department?

I believe that some of the skills needed for success in an industrial research position can be learned on the job. However, I think that the only way to understand what it takes to carry out a serious research agenda is to do it oneself. I think it is feasible for a person who has worked in an industrial research lab like IBM to be quite successful in a university environment — there are several examples I can think of who have made this switch. However, I have not seen many people who have not got a research background being very successful running a research department in industry or at a university.

How did you find time to write a book during those years? Has it payed off?

Ron Graham once, when asked a similar question, said that his secret is that every day contains 24 hours! You can do a lot of things if you decide to do so. In the case of our best seller Combinatorial Optimization (number 158,831 on the Amazon best seller list!), the big thing was the co-authors. Bill Cook and I launched it one night at Oberwolfach. We began by getting a group of luminaries to contribute comments for the back of the dustcover. Our plan was to write the index next, because then, we thought, it would be simple to write the book — just see what page we were on, check in the index, and see what had to go there. Later Bill Cunningham and Lex Schrijver joined the project. It was really interesting working through this material, that we all really loved, trying to combine our different pedagogic approaches.

I enjoyed doing it — I learned a lot and am very satisfied with the end result. However, I still earn more from my day job than from my book royalties.

Could you also tell us a bit about the Deep Computing project which you are currently coordinating? Has it been a rewarding experience for you? How will it impact IBM's future development policies?

The Deep Computing Institute at IBM Research was formed following our second chess match with Gary Kasparov in 1997 (which IBM's Deep Blue won 3.5 to 2.5!). The challenge was to see what we could do to take the ideas and apply them to a much broader set of problems. The idea of combing large amounts of computation and data to solve business and scientific decision problems is very broad, and the challenge has been to make it concrete. The breadth of topics — from simulation to optimization to data mining to advanced computation has been extremely interesting and has, I believe, led to some interesting research. For example, one of the projects I am currently leading is to construct BlueGene — the largest supercomputer in the world (by a large margin).

Let us now backtrack to the old days in Waterloo. It always impressed me in Waterloo the existence of a School of Mathematics, consisting of different departments. Did you see it as positive too?

The University of Waterloo has always been a very successful and innovative institution. In the sixties, the university decided to focus on mathematics, engineering, and an emerging discipline — computer science. It pioneered coop education in Canada. The idea of creating a Faculty of Mathematics, including pure and applied mathematics, statistics, computer science and

"Combinatorics and Optimization" had very positive consequences. There were stresses and conflicts that had to be resolved, but it seemed easier because of the common background of so many of the faculty members. And, it was really fun being bigger than Engineering!

How exciting was to do combinatorics in Waterloo? Who had a greater impact on you? Do you miss those times?

It was wonderful. I spent two and a half years at Waterloo as a PhD student and nine years there as a faculty member. I had the great fortune to be part of an extraordinary group of researchers in C&O. Jack Edmonds was a huge influence and we were all inspired by being able to work around Bill Tutte. I really enjoyed the time I spent as Managing Editor of Journal of Combinatorial Theory–B with Bill, Adrian Bondy and U.S.R. Murty. One of the exciting things was the set of visitors continually passing through. I also really enjoyed working with some of the young pups — we had quite a few Bruces at Waterloo — ranging from PhD students to Full Professors. They were an amazing bunch of colleagues. Somehow the group of students, postdocs and faculty members formed an amazingly homogeneous group of researchers. The thing that mattered most was the mathematics — everything else existed to support that.

We talked about mathematics in Canada and I don't want to miss this opportunity to ask you to compare the pre-college mathematical education in Canada to the one in the United States.

Clearly there is a huge variety within both countries. I do think that the Canadian system has been more strongly influenced by the British, or European model, and we expect students to take significant responsibility for their own programs and activities. The American system has huge diversity — ranging from top tier research schools to nurturing educational environments. Top schools in both countries are very competitive with each other.

Also, do you think graduate programs in US are stronger than in Canada, especially when it comes to applied mathematics and connection to the industry?

I like the practice of including external examiners on PhD committees in Canada. I believe that it raises the standard of the doctoral program and ensures a high quality of result. I think that the NSERC funding programs have been remarkably effective in supporting a broad base of graduate research. However the much larger size of the United States educational enterprise does result in a huge variety of opportunities.

Both systems work — top graduates from both systems carry out excellent research programs and have great careers.

It is time to end this interview with your future projects. What do you have in hands for the next years?

The big thing right now is building BlueGene — a single computer with about as much power as the total of the world's 500 largest machines today. This includes hardware, software and finding ways to construct applications that can exploit this machine. We should be able to solve some pretty big optimization problems very quickly!

How and when would you like to end your career...?

I don't think of my career ending as much as changing focus. There are still many things that I have not had time to do yet — understand quantum mechanics, the proof of Fermat's Last Theorem, and how the human cell translates DNA into proteins. I'd also like to finish some of the novels that I have started. And, I've still got a long way to go before Eric Clapton will consider me a rival on blues guitar.

Interview by Luís Nunes Vicente (Univ. of Coimbra)

W. R. Pulleyblank chaired the Department of Mathematical Sciences of the IBM T. J. Watson Research Center from 1994 to 2000. He is currently directing the Deep Computing Institute of this Center.

He held faculty positions in the University of Calgary (1974-1981) and in the University of Waterloo (1982-1991), before moving to IBM Corporation in 1991.

Bill Pulleyblank is one of the authors of the book Combinatorial Optimization, John Wiley and Sons, 1998, and the author of more than seventy research papers in this field. He has served on an extensive number of external and editorial boards.

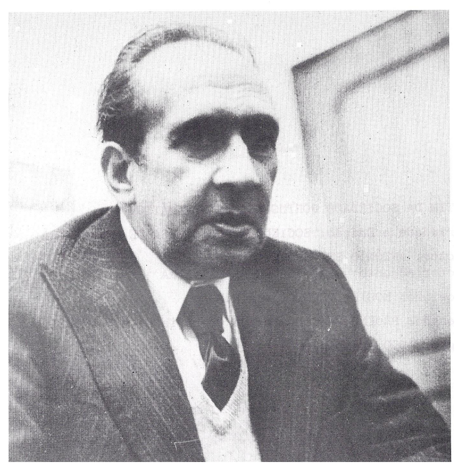
Ruy Luis Gomes - A Portuguese Mathematician in Recife[1]

The Institute for Physics and Mathematics of Recife University (presently the Pernambuco Federal University, UFPe)

In early February 1953, Prof. Alfredo Pereira Gomes, at the invitation of then University Rector Prof. Joaquim Amazonas, came to Recife to establish a Mathematics Department in the Philosophy, Science and Humanities School, newly created at the UFPe. This was an attempt at following the example of the University of São Paulo (USP), which in the 30s had invited foreign professors, mostly Italians, to create departments in the basic sciences in its Philosophy, Science and Humanities School, keeping at the same time the tradition of teaching the exact sciences in the Engineering School. Shortly afterwards, and before the start of the academic year, Prof. Manoel Zaluar Nunes, at the indication of Pereira Gomes, came from Lisbon to Recife. At the beginning of the academic year, Pereira Gomes and Zaluar Nunes were also hired, at the suggestion of its dean and Mathematics professor Newton Maia, by the Engineering School, which traditionally had better students in the exact sciences, specially Mathematics.

In the Philosophy, Science and Humanities School, Pereira Gomes taught courses on Mathematical Analysis, including the courses on Differential and Integral Calculus, while Zaluar Nunes taught the Geometry courses. This effort led to the creation of the Mathematics degree. In the Engineering School, on the other hand, Zaluar was charged with the courses on Numerical Analysis and Probability, while Pereira Gomes had the Calculus courses.

The most important fact, meanwhile, was the announcement by Luis Freire, a Physics professor in the Engineering and Chemistry Schools, that in 1954 an Institute for Physics and Mathematics (IFM) would be created at the UFPe, where extra-curricular activities on Mathematics and Physics would be carried out, in complement to those in the official program, as post-graduate studies to enhance the scientific training of young graduates and assistants. The IFM had the support of the recently established (1951) National Council for Research (CNPq), based in Rio de Janeiro, and its Director would be Luis Freire, with Newton Maia, Pereira Gomes and Zaluar Nunes as Mathematics professors, the latter also charged with the acquisition of the first collection of books for the Library, which he later managed. The first book donation by the CNPq, in 1953, consisted of 38 crates of books and journals, and the first book was registered on September 19, 1953.



Ruy Luis Gomes

In the first years of its existence, IFM was visited by renowned mathematicians, mainly French, who gave a series of lectures at the invitation of Pereira Gomes. Among them we should mention Arnaud Denjoy, of the Institut de France, Roger Godement, of the University of Paris, and François Bruhat, of the University of Nancy. The latter two produced lecture notes (on Differentiable Manifolds and Lie Algebras, Lie Groups and Applications, respectively) which were published as

[1] Previously published, in Portuguese, in the Proceedings of "Um dia com o Centro de Estudos Matemáticos do Porto", Centro de Matemática da Universidade do Porto, 2001. We thank CMUP and the author for their permission to publish this article in the CIM Bulletin.

numbers 2 and 3 of the series "Textos de Matemática", initiated by Pereira Gomes in 1955 with a course on Linear and Multilinear Algebra (the second part of the course on Multilinear Algebra was not published). Numbers 4 and 5 of Textos de Matemática, both by S. S. Chern, were titled "Differentiable Manifolds" and "Complex Manifolds", respectively.

In the beginning of 1960, UFPe hired, at the suggestion of Pereira Gomes, the Portuguese mathematician José Cardoso Morgado Júnior, who would play an important role, for 14 years, in the Mathematics Department of the Philosophy, Science and Humanities School, and above all in the IFM, where he was the supervisor of Wolmer Vasconcelos and Aron Simis, today professors at Rutgers University and at UFPe, respectively, and well-known experts on Commutative Algebra. At that time, the activity of the Portuguese mathematicians in Recife was already getting the attention of other academic centers, so that Pereira Gomes was a member of the organizing committee of the first and second Brazilian Mathematical Colloquia, held in Poços de Caldas (Minas Gerais) in 1957 and 1959, respectively. In 1960, the CNPq made Pereira Gomes a three-year member of the Governing Board of the Institute for Pure and Applied Mathematics (IMPA), and in 1961 Pereira Gomes and José Morgado were designated for the Advisory Board of the Central Mathematics Institute in the recently established University of Brasília. As a consequence, the IFM's connections with the various mathematical centers in Brazil became stronger, which facilitated the coming to Recife of many mathematicians visiting other centers. As a witness to this fact we have the publication of number 6 of "Textos de Matemática", whose authors were: Leopoldo Nachbin, from the IMPA, Luiz Mendonça de Albuquerque, of the University of Montpellier, Charles Ehresmann, from the University of Paris, and Frederico Pimentel Gomes, from the University of São Paulo. In the following years, Laurent Schwartz and François Trèves came to give talks, in both cases after visiting IMPA. Still in 1960, Leopoldo Nachbin gave a three months course in Recife, leading to volume 7 of "Textos de Matemática", on the Haar Integral, later translated into English and published by Van Nostrand.

The arrival of Ruy Luis Gomes in Recife

In February 1962, Professor Ruy Luis Gomes came to Recife at the invitation of Pereira Gomes. He had been at the Universidad Nacional del Sur, in Bahia Blanca, Argentina, where he was a colleague of his old friend of the University of Porto, Professor António Aniceto Monteiro. As we shall see, his 12-year presence in Recife had a decisive importance for the future of the Mathematics Department of the UFPe (the name given, from 1968 onwards, to the IFM's Mathematics Section), as well as for the Physics Department. According to Ruy Luis Gomes' plan of work within the UFPe, his main activities in the university should be to do research, to run a seminar at the IFM, and to teach a course in Mathematics in the Philosophy, Science and Humanities School. I was his first graduate student (and, for a few months, the only one) at the IFM. At the time, I was in the fifth and last year of the Engineering School, and I had already decided to do a PhD in Mathematics abroad. With the help of Leopoldo Nachbin, I tried to persuade the board of the Engineering School to authorize me to spend the 1962 year at the IMPA, in Rio de Janeiro, returning in the middle and in the end of the year to do my final exams. The School's resistance to my request led me to give up that plan, and I stayed in Recife. After Pereira Gomes' departure to France, in April 1962, to become a professor at the University of Nancy, Professor Ruy Luis Gomes became my supervisor. I remember that, at his suggestion, I settled in his office, in a desk next to his own, and so we began talking about Mathematics almost daily, with me often preparing talks about subjects chosen by him. This was a rich and important experience in my mathematical training, which lasted until my departure to the USA in September 1963, to study for a doctorate at the New York University. That way, I also had the chance to know and enjoy other aspects of Ruy Luis Gomes' fascinating personality: his admirable militancy and his ethical view of politics, his irreprehensible character, his commitment to the mathematical training of young people, his contagious enthusiasm in persuading us of the need to establish a quality Mathematics center in northeastern Brazil, his optimistic attitude towards life and his deep humanism. Naturally, Ruy Luis Gomes immediately concerned himself with the IFM. Together with José Morgado, he took over the series Textos de Matemática from volume 14 onwards, and a little later they created two new series, "Notas e Comunicações de Matemática" and "Notas de Curso", the first aiming at the preliminary publication of research articles and the second at advanced lecture notes. These new series quickly started circulating both in Brazil and abroad, and were used in exchanges to improve the IFM's Library. For several years, we exchanged our series with, for example, the journal Annales de l'Institut Fourier, published in Grenoble, France.

During my stay in New York, from September 1963 to December 1967, new students, specially from Engineering, began receiving the influence of Ruy Luis Gomes. In that period, he taught Mathematical Physics to a group of five students who later got their PhDs in Physics in centers in southern Brazil or abroad, and who eventually became the founding group of the UFPe's Physics Department. The recognition of this influence is found in the main conference room at the Physics

Department, which is called Ruy Luis Gomes Auditorium.

Throughout the years, a considerable effort was made to develop and improve the quality of the Library of the Mathematics Department, not just in books but also in journals and collections, so that it could become, as it did, one of the best in the country. Professor Ruy Luis Gomes always paid attention to this matter and, with his prestige, he managed to obtain substantial funding, both from Brazilian agencies and from foreign institutions such as the Gulbenkian Foundation (that's why our Library carries its name).

The UFPe Master course in Mathematics, another initiative by Ruy Luis Gomes and José Morgado, started in 1967. In 1970, the CNPq recognized the UFPe as a Center of Excellence for the Master course in Mathematics; in 1979, the Master course was accredited by the Federal Education Board. The first assessment by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES), carried out in 1980, gave the Master course the A mark, the highest possible, and this mark has been maintained ever since. From 1967 to December 1999, 133 Master students have completed their courses, more than 30 of which went on to do their PhDs, 22 abroad.

In April 1974, for reasons known to all, Professors Ruy Luis Gomes and José Morgado returned to Portugal to work at the University of Porto, leaving us with the huge responsibility of continuing the dream that they, and their Portuguese predecessors, Pereira Gomes and Zaluar Nunes, initiated in Recife, the dream of building a high level Mathematics center in the northeast of Brazil.

The PhD program, started in 1984, was accredited in 1991. 16 candidates have already finished the program, and their theses have been published in international journals. The graduate and research activities in Mathematics can count on one of the best libraries in the country (the third largest, after the ones in the USP and the IMPA), with more than 35.000 volumes. There are 274 journal subscriptions in the areas of Mathematics, Computer Science and Statistics. The teaching staff in the Mathematics Department is composed of 29 professors, 25 of which have a doctoral degree, obtained in the best European and American centers, while the others have Master degrees.

Those who knew Ruy Luis Gomes remain grateful for the great work he carried out at the Mathematics Department of the UFPe, and, above all, they keep the same regard for the exemplary citizen that he was. Alongside the political activism he demonstrated so well all his life, Professor Ruy Luis Gomes was a militant for Science, in particular Mathematics, a not too common fact even among great mathematicians. Wherever he went, he played a role in creating or consolidating teaching and research centers, he stimulated the training of young researchers, he fought to keep the best people in the universities.

Because of all this, the IFM of the UFPe, while regretting the deep sorrow of Professor Ruy Luis Gomes in having to leave his beloved country and his many friends, was really the main beneficiary of that separation, taking advantage, to the profit of Brazil, of his valuable knowledge and character.

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