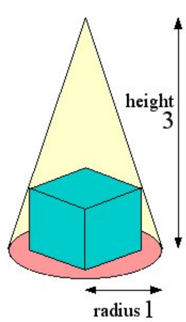
What's New in Mathematics

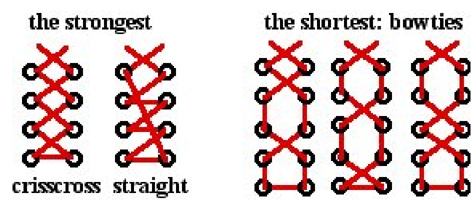
The Putnam in *Time*. "Crunching the Numbers" is the title of a piece by Lev Grossman, in the December 23 2002 *Time* magazine, about the William Lowell Putnam Mathematical Competition. "Every year," it begins, "on the first Saturday in December, 2,500 of the most brilliant college students in North America take what may be the hardest math test in the world." Grossman gives a quick survey of the history of the exam, a summary of the daunting statistics ("the median score on last year's test was 1 point. Out of a possible 120.") and a *Time*-like glimpse of its mystique ("think of it as a coming-out party for the next generation of beautiful minds"). He interviews Leonard Klosinski (Santa Clara; the competition director), Richard Stanley (coach of the MIT team) and Kevin Lacker, one of last year's winners, who remarks: "Doing well on the Putnam and doing good math research are two different tasks that take two different kinds of intelligence."

The piece includes a sample problem, labeled "An Easy One." "A right circular cone has a base of radius 1 and a height of 3. A cube is inscribed on the cone so that one face of the cube is contained in the base of the cone. What is the length of an edge of the cube?" Check *Time* for the answer.



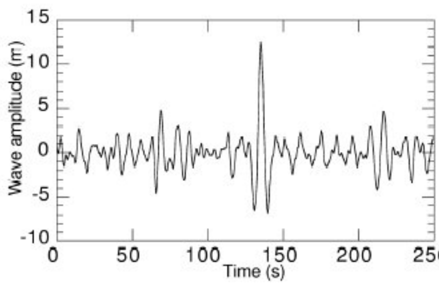
Too much pi? Under the title "How to Slice the Pi Very, Very Thin," the December 7, 2002 *New York Times* ran an AP dispatch from Tokyo reporting on the calculation of π to 1.24 trillion places, "six times the number of places recognized now." A ten-person team led by Yasumasa Kanada broke the trillion-place barrier with the help of a Hitachi supercomputer at the Information Technology Center of Tokyo University. The report quotes David Bailey (Lawrence Berkeley Lab): "It's an enormous feat of computing, not only for the sheer volume, but it's an advance in the technique he's using. All known techniques would exceed the capacity of the computer he's using." Which is, we are told, two trillion calculations a second. Note that light travels .15 mm in one two-trillionth of a second. This must be a very small or very parallel computer.

The best ways to lace your shoes has been worked out by Burkard Polster, a mathematician at Monash University (Victoria, Australia). His report, in the December 5 2002 *Nature*, was picked up in the December 10 *Boston Globe* (via Reuters) and in *Time* magazine for December 23.



The best way to lace depends on your criteria, but in all allowable lacings each eyelet is connected to at least one eyelet on the opposite side. The strongest lacings with n pairs of eyelets are the "crisscross" (when the ratio h of vertical eyelet spacing to horizontal is below a certain value h_n) and the "straight" (when h is greater than h_n). The shortest lacings are the "bowties". There is only one minimal bowtie lacing when n is even, but there are $(n+1)/2$ when n is odd. The shortest "dense" lacing (no vertical segments) is the crisscross.

Freak waves. BBC Two, on November 14, 2002, aired a program on this phenomenon and its recent mathematical analysis. Freak waves, also "rogue waves," "monster waves," are extraordinarily tall and steep waves that appear sporadically and wreck havoc with shipping. One is suspected to have washed away the German cargo München which went down with all hands in the midst of a routine voyage in 1978. More recently, the cruise ship Caledonian Star was struck by a 30m wave on March 2, 2001. The standard analysis of ocean waves predicts a Gaussian-like distribution of heights; extreme heights, although possible, should be very rare - a 30m wave is expected once in ten thousand years, according to the BBC. But these waves occur much more frequently than predicted. The program focused on new methods of analysis, and on the work of the mathematician A. R. Osborne (*Fisica Generale*, Torino). Osborne has applied the inverse scattering transform, which he describes as "nonlinear Fourier analysis," to the time series analysis of wave data. He conducted simulations using the nonlinear Schrödinger equation and found near agreement with the standard analysis, except that "every once in a while a large rogue wave rises up out of the random background noise." His paper, available online, gives an example of such a simulation:



Time series of a random wave train showing the appearance of a large rogue wave with height 20m occurring at 140 seconds.

Mathematical oncology. "Clinical oncologists and tumor biologists posess virtually no comprehensive model to serve as a framework for understanding, organizing and applying their data." This statement is featured in a box at the top of Robert A. Gatenby and Philip K. Maini's "Concepts" piece in the January 23 2003 *Nature*. They point out that despite the glut of publication (over 21000 articles on cancer in 2001) oncology has not been pursuing "quantitative methods to consolidate its vast body of data and integrate the rapidly accumulating new information." The explanations they suggest are mostly cultural. For example: "... medical schools have generally eliminated mathematics from admission prerequisites ..." They also blame "those of us who apply quantitative methods to cancer" for not having "clearly demonstrated to our biologist friends a dominant theme of modern applied mathematics: that simple underlying mechanisms may yield highly complex observable behaviors." An illustration from Wolframscience.com drives home the point. They end with an apology for mathematical modeling, showing how a verbal schema may be be enriched and strengthened by incorporation into a mechanistic and quantitative model which can handle, through computation, properties such as stochasticity and nonlinearity which cannot be handled by verbal reasoning alone. "As in physics, understanding the complex, nonlinear systems in cancer biology will require ongoing, interdisciplinary, interactive research in which mathematical models, informed by extant data and continually revised by new information, guide experimental design and interpretation."

4 log 3 - a new cosmic constant? John Baez (UC Riverside) has a "news and views" piece in the February 13 2003 *Nature* entitled "The Quantum of Area?". We start by asking whether black holes have a discrete spectrum of energy levels. According to Baez, a complete answer would require an understanding of "how quantum mechanics and general relativity fit together – one of the great unsolved problems in physics." But two completely different ways of guessing have recently come to the same answer: the spectrum of discrete energy levels is related to the surface area of the black hole, and the quantum of surface area is exactly 4 times the natural logarithm of 3 times the Planck area (which itself is about 10^{-70} m^2). The "surface" is actually the event horizon - "the closest distance an object can approach a black hole before being sucked in," so it is an imaginary boundary, but nevertheless acts in many ways "like a flexible membrane," and has a geometry of its own: it is flat except at points where it is punctured by one of the "threads" postulated by loop quantum gravity theory. Recent work by Shahar Hod (Hebrew University), Olaf Dreyer (Penn State; available online at http://arxiv.org/list/gr-qc/0211) and Lubos Motl (Harvard; available online at http://arxiv.org/list/gr-qc/0212) relates to earlier research by Hawking, Ashketar and Baez himself.

The Poincaré Conjecture. The *New York Times*, in their Science section for April 15, 2003, ran a piece by Sara Robinson entitled "Celebrated Math Problem Solved, Russian Reports." The problem is the 100-year-old Poincaré Conjecture; the Russian is Grigory Perelman of the Steklov Institute in St. Petersburg. As Robinson describes it, Perelman is claiming even more: a proof of a conjecture due to William Thurston, that "three-dimensional manifolds are composed of ... homogeneous pieces that can be put together only in prescribed ways." The Poincaré Conjecture, about the possible topology of a three-dimensional manifold in which every loop can be shrunk to a point, follows because now it would be known what possible geometric structure such a manifold could have. Robinson comments briefly on the method of proof. There is a natural way for the geometry of a manifold to evolve in time: this is the Ricci flow, "an averaging process used to smooth out the bumps of a manifold and make it look more uniform." Its application to Thurston's geometrization conjecture was pioneered by Richard Hamilton (now at Columbia) and carried out in full, we hope, by Perelman. Robinson remarks on the interesting parallels between Perelman's odyssey and that of Andrew Wiles (who recently proved Fermat's Theorem) and also on Perelman's eligibility, if his proof sustains scrutiny, for one of the Clay Mathematical Institute's million-dollar prizes. The *Times* picked up the story again in the "Week in Review" section on Sunday, April 20: "A Mathematician's World of Doughnuts and Spheres," by

George Johnson. "Poincaré proof adds up to potential payday" is the tack *Nature* chose to follow in a News in Brief item (April 24, 2003). The math got mangled: "Closed two-dimensional surfaces without holes can be transformed onto the surface of a sphere, and Henri Poincaré suggested that similar surfaces with higher dimensions should also transform back to spheres." But they did give a link to one of Perelman's preprints.

"The Superformula". *Nature* Science Update ran a piece on April 3, 2002 by John Whitfield: "Maths gets into shape." Whitfield was reporting on an article by Johan Gielis (Nijmegen) in the March 2003 American Journal of Botany in which Gielis proposes his superformula ("A generic geometric transformation that unifies a wide range of natural and abstract shapes"). The superformula, in slightly different notation, is the following polar equation:

$$r(\varphi) = f(\varphi)(|A\,cos M|^p + |B\,sin M|^q)^{-1/n}, \qquad (4)$$

which, for various values of the parameters A, B, M, p, q, n and various choices of the function $f(\varphi)$ does in fact give a wide variety of interesting shapes. Whether this mathematical unity is of any botanical significance is harder to see. Whitfield quotes Ian Stewart (Warwick): "I'm not convinced ... , but it might turn out to be profound if it could be related to how things grow" as is the case, for example, with D'Arcy Thompson's explanation of the logarithmic spiral in mollusk shells. Gielis' position, as quoted by Whitfield: "Description always precedes ideas about the real connection between maths and nature." A botanical Kepler awaiting his Newton. Meanwhile, Gielis has applied for a patent on his discovery: Methods and devices for synthesizing and analyzing patterns using a novel mathematical operator, USPTO patent application No. 60/133,279 (1999).

Math in *Nature*. The May 15 2003 issue of *Nature* has at least three articles with interesting mathematical aspects.

* Astronomy. "Chaos-assisted capture of irregular moons" is a comparative study of the irregular moon systems of the gas giants Jupiter and Saturn. Irregular moons have highly inclined orbits (but never more than 55 degrees) with respect to the planet's equatorial plane. Their motion may be prograde, counterclockwise when viewed from above, like our Moon and Jupiter's Galilean moons, or retrograde. In fact in the Jupiter system, the retrogrades outnumber the progrades 26 to 6. The authors study the 3-dimensional circular restricted three-body problem, focussing on the Sun-Jupiter-moon system. They use a Monte Carlo simulation to show how, in phase space, "the chaotic layer selects for the sense of the angular momentum of incoming and outgoing particles," i.e. sorts them into prograde and retrograde. (Authors: S. A. Astakhov, A. D. Burbanks, S. Wiggins, D. Farrelly)

* Econophysics. "A theory of power-law distributions in financial market fluctuations" sets up a model to explain the empirical probabilities:

$$\begin{aligned} P(|r_t| > x) &\sim x^{-3} \\ P(V > x) &\sim x^{-1.5} \\ P(N > x) &\sim x^{-3.4} \end{aligned}$$

where r_t is the change of the logarithm of stock price in a given time interval Δt (for a given stock), V is trading volume and N is the number of trades. The model "is based on the hypothesis that large movements in stock market activity arise from the trades of large participants." (Authors: X. Gabaix, P. Gopihrishnan, V. Plerou, H. E. Stanley).

* Neurophysiology. In "Attractor dynamics of network UP states in the neocortex" the authors report that in analyzing the dynamics of spontaneous activity of neurons in the mouse visual cortex, they detected "synchronized UP state transitions" occurring in "spatially organized ensembles involving small numbers of neurons." (UP is short for the membrane potential depolarized state). They argue that the these synchronized transitions, or 'cortical flashes,' are dynamical attractors, and that "a principal function of the highly recurrent neocortical networks is to generate persistent activity that might represent mental states." (Authors: R. Cossart, D. Aronov, R. Yuste)

The Poincaré Conjecture (cont.) The recent developments were also covered by *Science*, in an April 18 2003 piece by Dana Mackenzie whose title, "Mathematics World Abuzz Over Possible Poincaré Proof," correctly suggests his Variety-style approach to the subject. "Furthermore, what was to keep the surgeries, like plastic surgeries on a Hollywood star, from going on endlessly?" Nevertheless Mackenzie gives the best layman's guide so far to the history of the problem and to Perelman's innovations. An excellent presentation, ending in a lovely quote from Bennett Chow (UCSD): "It's like climbing a mountain, except in the real world we know how high the mountain is. What Hamilton did was climb incredibly high, far beyond what anyone expected. Perelman started where Hamilton left off and got even higher yet - but we still don't know how high the mountain is." *Nature* came back to the story, after last month's "News in Brief" item, with a more elaborate, and mathematically substantial, report by Ian Stewart (May 8, 2003). This account, also excellent, is complementary to Mackenzie's: they emphasize different aspects of the problem and of the putative solution.