AN INTERVIEW WITH E. C. ZEEMAN

As part of my homework for this interview I read a portuguese translation of the interview you gave to Lewis Wolpert for BBC, Radio 3^{31} Some of the questions I am going to formulate are based on that interview and I want to express my debt to him.



E. C. Zeeman

Professor Zeeman, at 7 you were fascinated when your mother showed you how to solve a problem using the unknown x. I'm sure that during your mathematical career some of the results you proved must have given you a similar feeling.

Which were the peaks of your research?

When my mother showed me at the age of 7 how to use x for an unknown it was a revelation to me. However, I think the feeling of revelation that you get when someone reveals something to you is different from the feeling of exhibiting that you get when you discover something

for yourself. Revelation can be wonderful, but exhilaration can be even better!

I can distinctly remember a few revelations such as understanding limits rigorously for the first time (and hence calculus), or understanding the complex numbers as the algebraic closure of the reals, or using groups and fields to show the insolubility of the quintic, or proving the knottedness of knots, or understanding Newton's proof of elliptic orbits, and much later realising that Newton's equations are contained in the symplectic structure of a cotangent bundle, or understanding Mather's proof of Thom's theorem on elementary catastrophes.

When I began proving my own theorems each one seemed the best at the time, but in retrospect I suppose I am particularly fond of having unknotted spheres in 5-dimensions, of spinning lovely examples of knots in 4-dimensions, of proving Poincaré's Conjecture in 5dimensions, of showing that special relativity can be based solely on the notion of causality, and of classifying dynamical systems by using the Focke-Plank equation. And amongst my applications of catastrophe theory I particularly liked buckling, capsizing, embryology, evolution, psychology, anorexia, animal behaviour, ideologies, committee behaviour, economics and drama.

When he introduced you as the 1992/93 Johann Bernoulli Lecturer, Floris Takens mentioned that you served as a flying officer in R. A. F. during World War II. I presume it must have been between high school and university.

What are your recollections of that experience and how did it affect your mathematical path?

I served in the Royal Air Force during the war from 1943 to 1947 (between the ages 18 - 22). I was a navigator on bombers, trained for the Japanese theatre, but that was cancelled because they dropped the atomic bomb a week before we were due to fly out. Since the death rate was 60% in that theatre it probably saved my life, but at the time I was disappointed not to see action, although relieved not to have to bomb Japan, the land of my birth.

The air force was a rewarding experience, a breath of

 $^{^{31}}$ E. C. Zeeman's interview was part of a set of interviews which were later published in a volume. Such a volume was translated into portuguese and published with the title "Uma Paixão pela Ciência" by Edições Salamandra, with Lewis Wolpert and Alison Richards as authors.

Another interview with E. C. Zeeman, conducted by Steen Markvorsen, was published in the EMS Newsletter in December 1998.

freedom that allowed my self-esteem to recover from the prison of boarding school. It enabled me to realise that I loved mathematics, and wanted to do that more than any other career. I was unashamedly happier my first day back as a student than my last day as an officer in the air force. Of course by then I had forgotten all my mathematics, and so it set me back 5 years in my mathematical career, but then who cares now that I am 75 and still at it. I am grateful to the air force for providing an opportunity for personal development, and for enabling me to laugh at myself slightly as an academic ever since.

And yet "They (the problems one sets about solving) are rarely solved", I quote from the interview mentioned at the beginning.

Were there problems of which the solution eluded you? Do you still think about them from time to time?

Of course the solutions to many problems have eluded me, and I still think about them from time to time. A good mathematician probably has 25 failures to each success. The important thing is that new ideas keep coming.

One of my favourite failures is the 3-dimensional Poincaré Conjecture, which I spent the first year of my research thinking about, and which is still unsolved today. Another little hobby is to try and rediscover Fermat's own original proof of his last theorem, at least for n = 3, without using complex numbers (which he is unlikely to have used). I have done half of it.

At the moment I am busy trying to unfold some difference equations in higher dimensions using alien techniques from dynamical systems, algebraic geometry and number theory. Last month I managed to prove a theorem that I conjectured 25 years ago about Eudoxus' theory of proportion. I suspect that Eudoxus was able to take ratios of ratios, which Euclid was not able to do in Book 5 (nor in Book 6, in spite of Definition 5, which is a later blemish added by other writers) because he had fouled up Eudoxus' beautiful abstract approach by, ironically, introducing the Euclidean algorithm too soon.

"Among students the good ones are automatically good and it is not possible to improve the bad ones' performance". You are talking about Maths students. I agree with you and it brings to my mind the following question.

What do you think of Mathematical Education as a scientific discipline?

There are two different meanings to the word "discipline". The first meaning is my definition of an academic discipline as a corpus of works of genius that a student can study without the interference of the lecturer. In this sense mathematics is a discipline, as are also physics, chemistry, biology, literature, etc. But mathematical education is not.

This became sharply clear to me once at Warwick. Each year the Mathematics Institute there runs a year-long symposium, with some 80 long-term visitors, in topics like topology, groups, dynamical systems, algebraic geometry, etc. One year we debated whether to run a symposium on mathematical education, and tried out a pilot week to examine the potential, but it transpired that there was not enough material: it was not an academic discipline.

On the other hand vocational apprenticeship to the profession of mathematical teaching needs discipline if the student is to master the necessary techniques. And such discipline needs to be taught, needs specialists to teach it, and needs to be supported by research on curriculum reform and the analysis of learning techniques.

"Deep down I am a geometer and geometry is very clear. ... Proofs are rigorous and very satisfying from the aesthetic viewpoint". This is something you said. On the other hand René Thom is well known for statements such as "I do not think that a mathematician's vocation is to prove theorems" or "One can always find imbeciles to prove theorems". Such different approaches to Mathematics and however parts of your works overlap very significantly.

How did that come about? It would be unthinkable to interview you and not bring up Catastrophe Theory...

Thom is quite witty, and he occasionally talks rubbish because he loves being provocative. At the same time he is the greatest genius I have had the privilege to know well. He is the fountainhead of many wonderful ideas. Sometimes he does not bother to be rigorous, nor to get down to the nitty-gritty of proofs, whereas I do. I like to rework and repolish a proof until it is in its simplest rigorous form.

Thom occupies a position halfway between mathematics and philosophy. He was reluctant to get his hands dirty predicting experiments, lest the potential failure of those predictions detracted from the purity of his theory. He quoted the unfortunate example of D'Arcy Thompson who got all his theoretical ideas right but all his experimental predictions wrong, and said he did not want to be caught the same way.

I, on the other hand, occupied a position halfway between mathematics and science. I wanted to get my hands dirty, and make predictions, and get the experimentalists to test them, because I knew that the scientific community would never take a theory seriously unless it was capable of being tested experimentally. And I was gratified that several of my predictions were confirmed. Some were refuted, and others remain to be tested.

Since we occupied different positions Thom and I complemented each other. We met over the mathematics and the theory in between, and our collaboration turned out to be very fruitful.

In the 60's when you were in your early forties you founded the Mathematics Institute and Research Centre at Warwick. It must have meant a lot of paper and administrative work and surely it must have affected your mathematical output.

Do you have any regrets?

It is true that the founding of the Mathematics Institute and Research Centre at Warwick was a big administrative load, that prevented me from doing much research in topology during the first 5 years 1964 - 69 (while I was 39 - 44). But I certainly had no regrets, because founding Warwick was one of my best and most rewarding achievements. And it made me into a much broader mathematician. During 1968/9 I learnt all about dynamical systems by running a symposium on it for a year, with many of the world leaders including Smale and Thom coming for long periods. Then in 1969/70 I had the good fortune to spend a sabbatical year with the latter at the IHES in Paris, where I learnt all about catastrophe theory. So I was very fortunate to get in on the ground floor of such beautiful new subjects.

Your mathematical career has been showered with lots of awards and other forms of recognition: A knighthood, an F. R. S. fellowship, the Senior Whitehead prize, a Forder lectureship, book dedications....

Was it important for you to have achieved such a recognition?

Of course I was very pleased to receive such recognitions, although I never set out to achieve them - I merely did what I liked best in teaching and research. The awards proved useful in that they enabled me to go ahead and do further things.

I was elected a Fellow of the Royal Society primarily for my work in geometric topology, which helped to resuscitate that subject in the 60's, and partly for my work in dynamical systems and catastrophe theory. The Whitehead Prize and Forder Lectureship were for both research and teaching. I attach great importance to teaching, and at Warwick I insisted that it should be given as much importance as research, which is one of the reasons why the Warwick Mathematics Institute remains so robust today. I was given the Royal Society Faraday Medal for my contributions to the public understanding of science, in particular for giving the Royal Institution Christmas Lectures in 1978, out of which grew the Mathematics Masterclasses for 13-year-olds (which have now been flourishing for 20 years and have spread to 50 centres around the country). My knighthood was probably for four things: my research, founding Warwick, creating masterclasses, and heading an Oxford College.

Your wife is a jeweller (I think she even coined the term "bracelet" in "bracelet umbilic"), you are now in Portugal to give a talk in connection with a video of which the title is "Geometry and Perspective"...

Are you interested in Art? Do you have a favourite painter, a favourite sculptor? I'm tempted to mention Barbara Hepworth or Henri Moore but that is a bit too obvious perhaps...

My wife Rosemary is indeed a jeweller and makes beautiful very feminine enamelled jewelry. Although she has never been a mathematician, yet she loved geometry at school, and so I try to explain geometrical things to her from time to time.

I coined the term "umbilic bracelet" when I tried to explain to her the natural stratification of the 4dimensional space of real cubic forms in two variables. The elliptic and hyperbolic umbilics form the two open strata, and are separated by the parabolic umbilics, which form a codimension-1 stratum, which is a cone on the bracelet; the bracelet itself being a bundle over S^1 with fibre a triangular hypocycloid and group Z_3 .



Yes, I am very interested in art. My favourite painters are from the Renaissance: Masaccio, Giovanni Bellini, Piero della Francesca, Botticelli, Leonardo, Filippino Lippi and Raphael; and (later) Vermeer, Ingres, Velasquez and Turner. Favourite sculptors include the Pisanos, Donatello, Michelangelo and Rodin, as well as individual pieces of sculpture like Djhutmose's unfinished quartzite head of Queen Nefertiti from Armana (the Cairo one rather than the Berlin one), the Egyptian wooden harp head from the Louvre, Myron's Diskobolos and Greek wrestlers from the 5th century BC, the Winged Victory of Samothrace, and (more modern) Boccioni's "Unique forms of continuity in space", Duchamp-Villon's "The great horse", Teddy Hutton's "Pregnant Woman", and Makonde sculptures from Tanzania and Mozambique. Modern painters I like include Rodolfo de Sanctis, Gordon Onslow-Ford, Edith Smith, Joe Brotherton, Peter Edwards and Picasso (although some of his work is junk). Your suggestions of Barbara Hepworth and Henry Moore have topological appeal but they do not make my spine tingle or move me to tears, as do the sculptures listed above.

(Questions and picture by F. J. Craveiro de Carvalho)

Sir Erik Christopher Zeeman is one of the great XXth century mathematicians. His university studies were at Christ's College, Cambridge and he also received his PhD from Cambridge.

Professor Zeeman spent most of his career in Cambridge, Warwick (where he founded the Mathematics Department and Research Centre) and Oxford.

His election to the Royal Society of London in 1975, the *Senior Whitehead Prize* in 1982, the first *Forder Lectureship* of the London Mathematical Society in 1987 and the Royal Society's *Faraday Medal* in 1988 are some of the honours he received. He was also knighted in 1991.

Professor Zeeman is the author of the video *Geometry and Perspective* based on the *Royal Institution Christmas Lectures* he gave in 1978.