Topics in Nonlinear PDEs

CIM/UC Summer School

Coimbra, Portugal · July 22-27, 2007 · www.cim.pt/pdes07

Scholarships available for PhD students and Post-Docs
Deadline for applications: April 15, 2007

Luis Caffarelli
University of Texas at Austin, USA
Problems and methods involving free boundaries

Charlie Elliott
University of Sussex, UK
Critical state models in superconductivity

Felix Otto
University of Bonn, Germany
Analysis of pattern formation in physical models

Benoit Perthame
Ecole Normale Supérieure, France
Nonlinear PDEs in Biology

Scientific Coordinators

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10.30-11.00: Coffee-break.

11.00-12.30: Felix Otto, *Analysis of pattern formation in physical models*, Part I.

12.30-14.00: Lunch.

14.00-15.30: Charlie Elliott, *Partial differential equations on surfaces*, Part I.

15.30-16.00: Coffee-break.

16.00-17.30: Benoit Perthame, *Adaptive evolution and concentrations in parabolic PDEs*, Part I.

17.40-19.00: Short Communications 1:

- Adrian Tudorascu, Georgia Institute of Technology, USA, *Adhesion dynamics and the pressureless Euler/Euler-Poisson system.*
- Eugen Varvaruca, Department of Mathematical Sciences, University of Bath, UK, *On the existence of extreme waves and the Stokes conjecture with vorticity.*
- Marco Veneroni, Technische Universiteit Eindhoven, The Netherlands, Γ*-limit of an energy functional for diblock copolymers.*
- Arghir Dani Zarnescu, Mathematical Institute, University of Oxford, UK, *Orientable and non-orientable director fields for liquid crystals.*

Day 2, July 24

9.00-10.30: Felix Otto, *Analysis of pattern formation in physical models*, Part II.

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11.00-12.30: Luis Caffarelli, *The obstacle problem for integral diffusions*, Part II.

12.30-14.00: Lunch.

14.00-15.30: Benoit Perthame, *Adaptive evolution and concentrations in parabolic PDEs*, Part II.

15.30-16.00: Coffee-break.

16.00-17.30: Charlie Elliott, *Partial differential equations on surfaces*, Part II.

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- Norayr Matevosyan, University of Vienna, Austria, *Regularity of a free boundary in parabolic problem without sign restriction.*
- Hayk Mikayelyan, Max Planck Institute Leipzig, Germany, *Convexity and regularity of the free boundary for an exterior free boundary problem involving the perimeter.*
- Bjorn Stinner, University of Sussex, Department of Mathematics, UK, *Evolving multiphase systems with surface energy and volume constraints.*

Day 3, July 25

9.00-10.30: Luis Caffarelli, *Problems and methods involving free boundaries*, Part III.

10.30-11.00: Coffee-break.

11.00-12.30: Charlie Elliott, *Partial differential equations on surfaces*, Part III.

12.30-14.00: Lunch.

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9.00-10.30: Luis Caffarelli, *The obstacle problem for integral diffusions*, Part IV.

10.30-11.00: Coffee-break.

11.00-12.30: Benoit Perthame, *Adaptive evolution and concentrations in parabolic PDEs*, Part III.

12.30-14.00: Lunch.

14.00-15.30: Felix Otto, *Analysis of pattern formation in physical models*, Part III.

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- Elio Arenas, Max Planck Institut for Mathematics in the Sciences and Leipzig University, Germany, *Two chemotaxis species.*
- Vincent Calvez, Ecole Normale Supérieure, Paris, France, *Energy methods in chemotaxis models.*
- Tomasz Cieslak, Institute of Mathematics, Polish Academy of Sciences, Poland, *Quasilinear non-uniformly parabolic drift-diffusion equations with the gradient of the drift term bounded and applications.*
- Nataliya Vasylyeva, Institute of Applied Mathematics and Mechanics of NAS of Ukraine, *On smooth solutions in the Hele-Shaw and Stefan problems in the case of nonregular initial data.*

Day 5, July 27

9.00-10.30: Benoit Perthame, *Adaptive evolution and concentrations in parabolic PDEs*, Part IV.

10.30-11.00: Coffee-break.

11.00-12.30: Felix Otto, *Analysis of pattern formation in physical models*, Part IV.

12.30-14.00: Lunch.

14.00-15.30: Charlie Elliott, *Partial differential equations on surfaces*, Part IV.

15.30-16.00: Coffee-break.

16.00-17.20: Short Communications 4:

- Agnese Di Castro, Department of Mathematics, University La Sapienza, Rome, Italy, *Existence results of positive solutions of an anisotropic quasilinear elliptic equation.*
- Olena Domanska, Ivan Franko National University of Lviv, Ukraine, *Nonlinear elliptic equations in unbounded domains.*
- Razvan Iagar, Universidad Autónoma de Madrid, Spain, *Self-similarity and asymptotic analysis for the p-Laplace equation in exterior domains.*
- Hélia Serrano, University of Castilla-La Mancha, Spain, *Homogenization of elliptic equations with oscillatory source terms.*

2 Titles and Abstracts

The obstacle problem for integral diffusions

Luis Caffarelli
University of Texas at Austin, USA

- Monday 23, 9.00—10.30;
- Tuesday 24, 11.00—12.30;
- Wednesday 25, 9.00—10.30;
- Thursday 26, 9.00—10.30.

Plan of the course:

1. Non local diffusions
2. Fractional diffusions as a Dirichlet to Neumann map
3. The obstacle problem for fractional diffusions: optimal regularity, free boundary regularity.

The course will be based on the preprints:

- L. Caffarelli and L. Silvestre, "An extension problem related to the fractional Laplacian".
- I. Athanasopoulos, L. A. Caffarelli, and S. Salsa, "The structure of the free boundary for lower dimensional obstacle problems".
- L. A. Caffarelli, S. Salsa, and L. Silvestre, "Regularity estimates for the solution and the free boundary to the obstacle problem for the fractional Laplacian".

Partial differential equations on surfaces

Charlie Elliott
University of Sussex, UK

- Monday 23, 14.00—15.30;
- Tuesday 24, 16.00—17.30;
- Wednesday 25, 11.00—12.30;
- Friday 27, 14.00—15.30.

In these notes we consider elliptic and parabolic partial differential equations on hypersurfaces Γ in $\mathbb{R}^{n+1}$. Two approaches will be discussed. In the first we use surface gradients to define weak forms of elliptic operators and naturally generate weak formulations of elliptic and parabolic equations on Γ. In the second approach we define an Eulerian level set method for partial differential equations on a hypersurface Γ contained in a domain $\Omega \in \mathbb{R}^{n+1}$. The key idea is based on formulating the partial differential equations on all level set surfaces of a prescribed function Φ whose zero level set is Γ. The resulting equation involves degenerate elliptic operator but can be solved in Ω which although one dimension higher does not explicitly refer to the hypersurface Γ .

In both approaches we formulate a scalar conservation law on stationary and evolving hypersurfaces $\Gamma(t)$ and, in the case of a diffusive flux, derive a surface transport and diffusion equation. Our motivation is to formulate finite element approximations. In the first approach the key idea is based on the approximation of Γ by a polyhedral surface Γ_h consisting of a union of simplices (triangles for $n = 2$, intervals for $n = 1$) with vertices on Γ. A finite element space of functions is then defined by taking the continuous functions on Γ_h which are linear affine on each simplex of the polygonal surface. Our surface finite element method (SFEM) or evolving surface finite element method (ESFEM) is applied to weak forms of the equations. The computation of the mass and element stiffness matrices are simple and straightforward. In the second approach finite element method is applied to the weak form of the conservation equation yielding an Eulerian FEM. The computation of the mass and element stiffness matrices are simple and straightforward.

Numerical experiments are described for several linear and nonlinear partial differential equations. We describe how this framework may be employed in applications.In particular the power of the method is demonstrated by employing it to solve highly nonlinear second and fourth order problems such as surface Allen-Cahn and Cahn-Hilliard equations and surface level set equations for geodesic mean curvature flow.

Analysis of pattern formation in physical models

Felix Otto
University of Bonn, Germany

- Monday 23, 11.00—12.30;
- Tuesday 24, 9.00—10.30;
- Thursday 26, 14.00—15.30;
- Friday 27, 11.00—12.30.

Adaptive evolution and concentrations in parabolic PDEs

Benoit Perthame
École Normale Supérieure, France

- Monday 23, 16.00—17.30;
- Tuesday 24, 14.00—15.30;
- Thursday 26, 11.00—12.30;
- Friday 27, 9.00—10.30.

Living systems are subject to constant evolution through the mutation/selection principle discovered by Darwin. In a very simple and general description, their environment can be considered as a nutrient shared by all the population. This allows certain individuals, characterized by a "physiological trait", to expand faster because they are better adapted to the environment. This leads to select the "best adapted trait"in the population (singular point of the system). On the other hand, the new-born population undergoes small variance on the trait under the effect of genetic mutations. In these circumstances, is it possible to describe the dynamical evolution of the current trait?

We will give a mathematical model of such dynamics, based on parabolic equations, and show that an asymptotic method allows us to formalize precisely the concepts of monomorphic or polymorphic population. Then, we can describe the evolution of the "best adapted trait" and eventually to compute branching points which allows for the cohabitation of two different populations.

The concepts are based on the asymptotic analysis of the scaled parabolic equations. This leads to concentrations of the solutions and the difficulty is to evaluate the weight and position of the moving Dirac masses that desribe the population. We will show that a new type of Hamilton-Jacobi equation with constraints naturally describes this asymptotic. Some additional theoretical questions as uniqueness for the limiting H.-J. equation will also be addressed.

This work is a collaboration with O. Diekmann, P.-E. Jabin, S. Mischler, S. Cuadrado, J. Carrillo, S. Genieys, M. Gauduchon and G. Barles.
Relevant Publications:

- B. Perthame and S. Génieys, “Dynamics of Nonlocal Fisher concentration points: a nonlinear analysis of Turing patterns”.
- G. Barles and B. Perthame, “Concentrations and constrained Hamilton-Jacobi equations arising in adaptive dynamics”.

Short Communications

- *Adhesion dynamics and the pressureless Euler/Euler-Poisson system*

Adrian Tudorascu
Georgia Institute of Technology, USA

We obtain global existence and prove the time regularity of the solution for the pressureless Euler/Euler-Poisson system, and obtain that the velocity satisfies the Oleinik entropy condition, which leads to a partial result on uniqueness. Our approach is motivated by earlier work of Brenier and Grenier who showed that one dimensional conservation laws with special initial conditions and fluxes are appropriate for studying the pressureless Euler system.

- *Existence results of positive solutions of an anisotropic quasilinear elliptic equation*

Agnese Di Castro
Dipartimento di Matematica, Università di Roma La Sapienza, Italy

We study a quasilinear elliptic equation containing a differential operator, in which the partial derivatives are involved with different powers (called, in the literature, the anisotropic pseudolaplacian), and a nonlinear reaction term. The nonlinearity in the reaction term is a power function, whose exponent belongs to the range of the powers arising in the differential operator. The interplay between the different growths produces a convex-concave effect, in such a way that it is possible to prove the existence of two positive solutions (at least) if the nonlinearity is sufficiently strong, or to prove a nonexistence result if the nonlinearity is small.

- *Orientable and non-orientable director fields for liquid crystals*

Arghir Dani Zarnescu
Mathematical Institute, University of Oxford, UK

Uniaxial nematic liquid crystals are often modelled using the Oseen-Frank theory, in which the mean orientation of the rod-like molecules is modelled through a unit vector field n. This theory has the apparent drawback that it does not respect the head-to-tail symmetry in which n should be equivalent to -n, that is, instead of n taking values in the unit sphere S2, it should take values in the sphere with opposite points identified, i.e. in the real projective plane RP2. The de Gennes theory respects this symmetry by working with the tensor Q=s(nn-Id). In the case of a non-zero constant scalar order parameter s the de Gennes theory is equivalent to that of Oseen-Frank when the director field is orientable. We report on a general study of when the director fields can be oriented, described in terms of the topology of the domain filled by the liquid crystals, the boundary data and the Sobole space to which Q belongs (which in turn prevents or allows certain singularities). We also analyze the circumstances in which the non-orientable configurations are energetically favoured over the orientable ones. This is joint work with John Ball. Acknowledgment: this work was supported by EPSRC grant EP/E010288/1.

- *Evolving multi-phase systems with surface energy and volume constraints*

Bjorn Stinner
University of Sussex, Department of Mathematics, UK

Multi-phase systems are considered where the phase interfaces move according to a curvature flow and some of the phases are subject to volume constraints. An approximation of the free boundary problem by a phase field model has been developed resulting in parabolic differential equations with nonlocal terms due to the constraints. Several ideas will be presented to numerically solve the nonlocal equations, and simulations have been performed to study local minima of the system energy. Applications concern Wulff forms, bubble clusters, and tessellation problems.

- *Two chemotaxis species*

Elio Eduardo Espejo Arenas
Max Planck Institut for Mathematics in the Sciences and Leipzig University, Germany

The research of my PhD-thesis deals with the analysis of two chemotactic species, which are interacting via one chemo-attractant. Two equations of cross-diffusion type are coupled with a reaction-diffusion-equation. In the limiting case, where the chemotactic sensitivity of one species may be zero — thus the system would be decoupled — one can construct solutions, where blowup for one species and global solutions for the other species can be observed. A question of interest is now, if and how this situation changes, as soon as both species are subject to chemotaxis. One aim is to completely classify this case in the radial symmetric setting. I am expecting to finish the written part of my PhD-examination by middle of June 2007.

- *On the existence of extreme waves and the Stokes conjecture with vorticity*

Eugen Varvaruca
Department of Mathematical Sciences, University of Bath, UK

We present some recent results concerning singular solutions of the problem of travelling water waves on flows with vorticity. We show that the existence of a wave with stagnation points at its crests follows from the existence of a sequence of regular waves satisfying certain natural bounds. We also show that at a stagnation point about which it is supposed symmetric, the wave profile must have either a corner of 120° or a cusp if the vorticity is everywhere nonnegative, and either a corner of 120°, or a cusp, or a horizontal tangent if the vorticity is everywhere nonpositive.

- *Convexity and regularity of the free boundary for an exterior free boundary problem involving the perimeter*

Hayk Mikayelyan
Max Planck Institute, Leipzig, Germany

Joint work with H. Shahgholian (KTH, Stockholm). We prove that if the given compact set K is convex then a minimizer of the functional

$$I(v) = \int_{B_R} |\nabla v|^p dx + \mathrm{Per}(\{v > 0\}),\ 1 < \infty,$$

over the set $\{v \in H^1_0(B_R)|\ v \equiv 1 \text{ on } K \subset B_R\}$ has a convex support, and as a result all its level sets are convex as well. We derive the free boundary condition for the minimizers and prove that the free boundary is analytic and the minimizer is unique.

- *Homogenization of elliptic equations with oscillatory source terms*

Hélia Serrano
Universidad de Castilla-La Mancha, Spain

The characterization of the effective coefficients coming from the homogenization of second order elliptic equations with non-constant source term is given by means of the joint Young measure associated with relevant sequences, through the Γ-convergence of its associated energies.

- Γ*-limit of an energy functional for diblock copolymers*

Marco Veneroni
Technische Universiteit Eindhoven, The Netherlands

A diblock copolymer is a molecule consisting of two subchains of monomers. We study a functional describing the energy of a two-dimensional periodic domain of diblock copolymers. The functional consists of a term proportional to the measure of the interface between the two monomers and a term given by the 1-Wasserstein distance between them. In analogy with a former application to cellular membranes, we study the second order Gamma limit as a quantity related to the L^2 average of the mean curvature of the interface between the monomers.

- *On smooth solutions in the Hele-Shaw and Stefan problems in the case of nonregular initial data*

Nataliya Vasylyeva
Institute of Applied Mathematics and Mechanics of NAS of Ukraine

The free boundary problems arise under research of spatiotemporal phenomena in diverse areas, including, tumor growth, wound healing, developmental biology, ecology, phase transitions (see, for example, investigations of L.Caffarelli, A.Friedman, J.-F.Rodrigues, J.L.Vazquez, J.M. Elliott, B.V.Bazaliy, J.R.Ockendon, S.Luckhaus, J.R.King, A.A.Lecy et al.). One of the well-known such problems is the classical Stefan problem which is a nonlinear free boundary problem for parabolic equation describing a crystallized process (melting ice, for example). The Hele-Shaw moving boundary problem describes a plane motion of viscous liquid and is a Stefan-like problem for an elliptic equation because of the analogous boundary conditions on a free boundary in these problems. The main assumption of the model is the liquid velocity is proportional to the antigradient of a pressure. We have proved the one-to-one solvability both these problems in the weighted Hölder classes in the case the initial free boundary has the corner points or free and fixed boundary forms corners. The sufficiently conditions on the geometry of initial domain have been obtained in order to the "waiting time" phenomena exists.

- *Regularity of a free boundary in parabolic problem without sign restriction*

Norayr Matevosyan
University of Vienna, Austria

We consider a parabolic obstacle-type problem without sigh restriction on the solution. An exact representation of the global solutions is found. It is proved, without any additional assumptions on a free boundary, that near the fixed boundary where the homogeneous Dirichlet condition is fulfilled, the boundary of the "non-coincidence set" is graph of a Lipschitz function.

- *Nonlinear elliptic equations in unbounded domains*

Olena Domanska
Ivan Franko National University of Lviv, Ukraine

We will speak about boundary problems for static equations having exponential nonlinearities that vary at x and are different with respect to various derivatives. Boundary conditions are mixed. I will show that corresponding boundary problems have unique solution without restrictions on its behaviour and increasing of initial data at infinity. In addition to one-valued solvability of the problem we will go into question on continuous dependence of the solution on initial data.

- *Self-similarity and asymptotic analysis for the p-Laplace equation in exterior domains*

Razvan Gabriel Iagar
Universidad Autónoma de Madrid, Spain

We study the asymptotic behaviour of the general solutions of the parabolic p-Laplace equation in an exterior domain, with compactly supported initial data. Our analysis has two parts: first, to study the outer behaviour, i.e. the behaviour at infinity of the solutions, and second, the inner behaviour, i.e. the behaviour near the holes. We show that the situation is very different whether $N > p$, $N = p$ and $N < p$.

- *Quasilinear non-uniformly parabolic drift-diffusion equations with the gradient of the drift term bounded and applications*

Tomasz Cieslak
Institute of Mathematics, Polish Academy of Sciences, Poland

In my talk I would like to present results concerning the equation

$$u_t = \nabla \cdot (\alpha(u)\nabla u - u\beta(u)\nabla v)$$

in a bounded domain U, under no-flux boundary conditions. $\alpha(u)$ and $\beta(u)$ are C^2, bounded, positive functions such that $\alpha(u) \to 0$ when $u \to \infty$. Thus, the equation is non-uniformly parabolic. We assume ∇v bounded in $L^\infty((0,T) \times U)$ and at least weak solutions exist. Assuming β bounded I will present critical exponents of the decay of α distinguishing between L^∞ a priori estimates of u and the lack of it. I will also present a theorem concerning L^∞ estimates if α and β satisfy $\frac{\beta(u)}{\alpha(u)} \leq M$ for large enough u and some $M > 0$. Such equations appear as the part of the systems that describe chemotaxis phenomenon with volume filling effect, also they arise in semiconductors, electrochemistry or astrophysics. I will present some results on global existence and finite time blow-ups of chemotaxis systems.

- *Energy methods in chemotaxis models*

Vincent Calvez
Ecole Normale Supérieure, Paris, France

The main question arising from chemotaxis models is whether cell density blows up or not in finite time. Particularly in dimension two, this behavior is subject to a mass threshold. Methods based on the corresponding free energy provide a good understanding of such systems. In this talk we present two extensions of the classical linear Keller-Segel model, by changing respectively equations driving the cell density and the chemical potential. For both models we show how the free energy method brings optimal results concerning the prevention of blow-up.

3 Scientific Coordinators

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4 Support

- Fundação Calouste Gulbenkian;
- Fundação para a Ciência e a Tecnologia;
- Centro de Matemática da Universidade de Coimbra.

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6 List of Participants

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