

CIM Workshop on PDE's, Numerical Simulation and Applications

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CIM (International Center for Mathematics)

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Best Western Hotel D. Luís, Coimbra, Portugal

December 13-15, 2007

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1 Conference Programme

Thursday, December 13

12.30: Registration

13.00: Lunch

15.00 - 15.45: Juha Kinnunen (Helsinki Univ Technology), *On the definition and properties of superparabolic functions;*

15.50 - 16.35: José Francisco Rodrigues (Univ Lisboa), *The nonlinear n -membranes evolution problem with time-dependent Dirichlet data;*

16.40: Coffee break

17.00 - 17.45: Pablo Pedregal (Univ Castilha-la-Mancha), *Optimal internal stabilization of the linear system of elasticity;*

17.50 - 18.10: Margarida Baía (Inst. Superior Técnico, Lisboa), *Characterization of two-scale Young-measures and applications to homogenization;*

18.15 - 18.35: Filipe Oliveira (Univ Nova de Lisboa), *A note on a Benney-type system.*

Friday, December 14

09.15 - 10.00: Isabel Figueiredo (Univ Coimbra), *Applications of PDE's in piezoelectricity and in endoscopic image processing;*

10.05 - 10.25: Sylvain Ervedoza (Univ Versailles), *Observability of time discrete schemes;*

10.30 - 10.50: Tommaso Leonori (CMUC, Coimbra), *The boundary behaviour of blow-up solutions related to a stochastic control problem with state constraint;*

10.55: Coffee break

11.20 - 12.05: José M. Arrieta (Univ Complutense, Madrid), *Reaction-diffusion equations and perturbation of the domain;*

12.10 - 12.30: Fernando Miranda (Univ Minho, Braga), *Nonlinear Maxwell systems in mathematical problems of electromagnetism;*

12.35 - 12.55: Ercília Sousa (Univ Coimbra), *The influence of boundary conditions on the numerical solution of convection-diffusion problems;*

13.00: Lunch

15.00 - 15.45: Liviu Ignat (Univ Autónoma, Madrid), *A nonlocal convection-diffusion equation;*

15.50 - 16.35: Pedro Freitas (FMH-UTL and GFM, Lisboa), *Eigenvalues of the Laplacian: geometric aspects, explicit expressions and numerics.*

16.40: Coffee break

20.00: Workshop Dinner

Saturday, December 15

09.15 - 10.00: David Arcoya (Univ Granada), *Some results on quadratic quasilinear equations with general singularities;*

10.05 - 10.50: Karine Beauchard (ENS Cachan), *Large time asymptotics for partially dissipative hyperbolic systems;*

10.55: Coffee break

11.20 - 12.05: Diogo Gomes (Instituto Superior Técnico, Lisboa), *Error estimates for the approximation of the effective Hamiltonian;*

12.10 - 12.55: Luis Escauriaza (UPV/EHU, Bilbao), *Hardy's uncertainty principle and Schrödinger evolutions.*

13.00: Lunch

2 Titles and Abstracts — Invited Communications

Some results on quadratic quasilinear equations with general singularities

David Arcoya
Univ Granada

We study the existence and nonexistence of solution for problems whose model is $-\Delta u + \frac{|\nabla u|^2}{u^\gamma} = f(x)$ in Ω , $u = 0$ on $\partial\Omega$, where $\gamma > 0$ and $f \in L^1(\Omega)$ satisfying $\text{ess inf } \{f(x) : x \in \omega\} > 0$, $\forall \omega \subset\subset \Omega$. These results are part of a joint work with José Carmona, Tommaso Leonori, Pedro J. Martínez-Aparicio, Luigi Orsina and Francesco Petitta.

Reaction-diffusion equations and perturbation of the domain

José M. Arrieta
Univ Complutense, Madrid

In this talk we analyze the behavior of the dynamics of a reaction-diffusion equations with homogeneous Neumann boundary condition when the domain undergoes a perturbation. We will be mainly interested in the so-called “dumbbell-type” perturbation of the domain, which consists of two fixed smooth and disjoint domains joined by a thin channel whose width tends to zero. The limit “domain” consists of the two fixed disjoint domains joined by a line. We will be able to develop an appropriate functional setting where we can analyze this perturbation problem proving among other things, the continuity of the set of stationary solutions.

Large time asymptotics for partially dissipative hyperbolic systems

Karine Beauchard
ENS Cachan

This work is concerned with (n-component) hyperbolic systems of balance laws in m space dimensions. First we consider linear systems with constant coefficients and analyze the possible behavior of solutions as $t \rightarrow \infty$. Using Fourier transform we exhibit the role that control theoretical tools, such as the classical Kalman condition, play. We build Lyapunov functionals allowing to establish explicit decay rates depending on the frequency variable. In this way we extend the previous analysis by Shizuta and Kawashima under the so-called algebraic condition (SK). In particular we show the existence of systems exhibiting a more complex behavior than the one that the (SK) condition allows. We also discuss the link of this analysis with previous literature in the context of damped wave equations, hypoellipticity and hypocoercivity. To conclude we analyze the existence of global solutions around constant equilibria for nonlinear systems of balance laws. Our analysis of the linear case allows proving existence results in situations that the previous existing theory does not cover.

Hardy's uncertainty principle and Schrödinger evolutions

Luis Escauriaza
UPV/EHU, Bilbao

I will describe an extension of Hardy's uncertainty principle (f must be identically zero, when $e^{|x|^2/\beta^2} f$, $e^{4|\xi|^2/\alpha^2} \widehat{f}$ are in $L^2(\mathbb{R})$ and $\alpha\beta \leq 4$) to solutions in $\mathbb{R}^n \times [0, 1]$ of the Schrödinger equation

$$\partial_t + i(\Delta + V(x, t)) = 0.$$

Applications of PDE's in piezoelectricity and in endoscopic image processing

Isabel Figueiredo
Univ Coimbra

We describe several PDE's applications and their numerical simulations. The first PDE model deals with the optimal placement of actuators in piezoelectric plates and the second one concerns the detection of an unknown obstacle in contact with a piezoelectric plate. The third application focus on a PDE image segmentation model which is used to segment aberrant crypt foci captured in vivo by endoscopy in the human colon.

Eigenvalues of the Laplacian: geometric aspects, explicit expressions and numerics

Pedro Freitas
FMH-UTL and GFM, Lisboa

The purpose of this talk is to report on recent progress on the following question: how far can we go in describing eigenvalues of the Dirichlet Laplacian explicitly? It is well known that apart from a handful of exceptional cases it is not possible to obtain explicit expressions for, say, the first eigenvalue of a planar domain. In spite of this, we will show that there are several directions that may still be explored at different levels. This is based on collaborations with D. Krejcirik (geometric aspects), D. Borisov (explicit expressions) and P. Antunes (numerics).

Error estimates for the approximation of the effective Hamiltonian

Diogo Gomes
Instituto Superior Técnico, Lisboa

We study approximation schemes for the cell problem arising in homogenization of Hamilton-Jacobi equations. We prove several error estimates concerning the rate of convergence of the approximation scheme to the effective Hamiltonian, both in the optimal control setting and as well as in the calculus of variations setting. This is a joint work with F. Camilli and I. C. Dolcetta.

A nonlocal convection-diffusion equation

Liviu Ignat

Univ Autónoma, Madrid

In this talk we study a nonlocal equation that takes into account convective and diffusive effects, $u_t = J * u - u + G * (f(u)) - f(u)$ in \mathbb{R}^d , with J radially symmetric and G not necessarily symmetric. First, we prove existence, uniqueness and continuous dependence with respect to the initial condition of solutions. This problem is the nonlocal analogous to the usual local convection-diffusion equation $u_t = \Delta u + b \cdot \nabla(f(u))$. In fact, we prove that solutions of the nonlocal equation converge to the solution of the usual convection-diffusion equation when we rescale the convolution kernels J and G appropriately. Finally we study the asymptotic behaviour of solutions as $t \rightarrow \infty$ when $f(u) = |u|^{q-1}u$ with $q > 1$. We find the decay rate and the first order term in the asymptotic regime. Joint work with Julio Rossi.

On the definition and properties of superparabolic functions

Juha Kinnunen

Helsinki Univ Technology

We discuss potential theoretic aspects of degenerate parabolic partial differential equations of p-Laplacian type. Solutions form a similar basis for a nonlinear parabolic potential theory as the solutions of the heat equation do in the classical theory. In the parabolic potential theory, the so-called superparabolic functions are essential. For the ordinary heat equation we have supercaloric functions. They are defined as lower semicontinuous functions obeying the comparison principle. The superparabolic functions are of actual interest also because they are viscosity supersolutions of the equation. We discuss their structural, convergence and Sobolev space properties.

Optimal internal stabilization of the linear system of elasticity

Pablo Pedregal

Univ Castilla-la-Mancha

The optimal design problem consisting in finding the best distribution and shape of the internal damping set for the stabilization of the linear system of elasticity will be addressed. Non-existence of optimal solutions is, in general, well-established, so that a relaxation capable of describing the intricate, never-ending microgeometry of optimal shapes is sought. This relaxation turns out to be extremely simple, and it consists in the most straightforward extension of the original problem one could think of. It only involves the distribution of the damping set by means of a density. The microshape of such optimal damping set does not influence or improve the stabilization. We will try to explain in a non-technical way these two facts. We will also show some numerical simulations in dimension 2, and explore through them the dependence on the Lamé coefficients.

The nonlinear n-membranes evolution problem with time-dependent Dirichlet data

José Francisco Rodrigues
Univ Lisboa / CMAF

We consider the evolution N-membranes problem for the p-Laplace operator with time-dependent Dirichlet data. We characterize the system satisfied by the solution, obtaining continuous dependence results and the asymptotic behaviour in time, as well as the convergence of the coincidence sets under natural non degeneracy conditions on the external forces.

3 Titles and Abstracts — Short Communications

Characterization of two-scale Young-measures and applications to homogenization

Margarida Baía
Inst. Superior Técnico, Lisboa

This work is devoted to the study of two-scale gradient Young measures naturally arising in non-linear elasticity homogenization problems. Precisely, a characterization of this class of measures is derived and an integral representation formula for homogenized energies, whose integrands satisfy very weak regularity assumptions, is obtained in terms of two-scale gradient Young measures.

(Joint work with Jean-Francois Babadjian and Pedro Santos)

Observability of time discrete schemes

Sylvain Ervedoza
Univ Versailles

We consider various time discretization schemes of abstract conservative evolution equations of the form $z' = Az$, where A is a skew-adjoint operator. We analyze the problem of observability through an operator B . More precisely, we assume that the pair (A, B) is exactly observable for the continuous model, and we derive uniform observability inequalities for suitable time-discretization schemes within the class of conveniently filtered initial data. The method we use is mainly based on the resolvent estimate given in Burq-Zworski, Geometric Control in the presence of a black box. We present some applications of our results to time-discrete schemes for wave, Schrödinger and KdV equations and fully discrete approximations schemes for wave equations. Joint work with Chuang Zheng and Enrique Zuazua.

The boundary behaviour of blow-up solutions related to a stochastic control problem with state constraint

Tommaso Leonori
CMUC, Coimbra

We consider solutions of the equation $-\Delta u + u + |\nabla u|^q = f$ which blow-up uniformly at the boundary of a smooth domain, that can be interpreted as the value function of a state constraint control problem for a Brownian motion. We prove a complete asymptotic expansion of the gradient at the boundary, giving precise behaviour of normal and tangent components.

Nonlinear Maxwell systems in mathematical problems of electromagnetism

Fernando Miranda
Univ Minho, Braga

We study an evolution system coupling the electromagnetic and thermal fields considering first the elliptic and the parabolic quasi-stationary electromagnetic problems in a general abstract framework. In order to prove the existence of solutions of those problems we present some properties of the functional spaces $\nabla p\Omega$ and $\Delta p\Omega$, with particular emphasis to the null normal trace boundary condition. Next we consider an extension of Bean's critical-state model for type-II superconductors, imposing that the current density does not exceed some critical value J_c , that may depend on the magnetic field. We obtain a quasi-variational inequality for which we are able to prove the existence of solution. (Joint work with José Francisco Rodrigues and Lisa Santos).

A note on a Benney-type system

Filipe Oliveira
Univ Nova de Lisboa

In this talk we will present recent results concerning some Benney-type systems. This set of equations was introduced by David Benney in the sixties in order to study the interactions between long and short waves. We will emphasize on the adiabatic limit to the Cubic Nonlinear Schrodinger equation and on the existence of local strong solutions, obtained in a joint work with João Paulo Dias and Mário Figueira.

The influence of boundary conditions on the numerical solution of convection-diffusion problems

Ercília Sousa
Univ Coimbra

The mechanism of convection diffusion appears in many physical applications and accurate modeling of the interaction between convective and diffusive processes can be a difficult task. Although the majority of physical experiments are performed in the presence of boundaries if we consider the approximation of the unsteady convection-diffusion problem, we can observe that much of the literature is concerned with choices for the whole real line. It is very common that the approximate solutions we derive for the whole line present some difficulties when we need to deal with the presence of a physical boundary. This difficulty is more obvious if we are interested in simulations next to the boundary and at short times. Even if they perform efficiently far away from the physical boundary, next to it they can have a poor performance. In this talk, we present new numerical methods derived by using an evolution operator which takes into account the presence of an inflow physical boundary. We also show some numerical simulations to observe their advantage near the boundary.

4 Organizers

- José Miguel Urbano (Univ Coimbra)
- Enrique Zuazua (IMDEA-Matemáticas & Univ Autónoma, Madrid)

5 Support

- BPI – Banco Português de Investimentos;
- FCT – Fundação para a Ciência e a Tecnologia.

6 List of Participants

Name	Institution	E-mail
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