BULLETIN

INTERNATIONAL CENTER FOR MATHEMATICS

DECEMBER 2004

17

CONTENTS

COMING EVENTS

THEMATIC TERM ON OPTIMIZATION

COORDINATOR

Luís Nunes Vicente (University of Coimbra)

DATES

July 2005

Optimization (mathematical programming) is a well established discipline of mathematics which has been remarkably capable of finding new applications to science, engineering, and economics.

Two areas where optimization is playing an increasingly important role are finance and medicine. The 2005 CIM Thematic Term includes a workshop and a short-course on Optimization in Finance and a workshop on Optimization in Medicine.

One of the main events of this Thematic Term is the Workshop on PDE Constrained Optimization, where a short course is also planned. Optimization problems governed by PDEs is at the core of simulation-based optimization, an area of high demand and pivotal importance in multi-disciplinary engineering.

A Summer School on Integer Programming is also scheduled, focused on the newest recent developments obtained by geometric and algebraic approaches to combinatorial optimization problems.

The programme of events is the following:

July 5-8: Workshop on Optimization in Finance

Organizers

A. M. Monteiro (University of Coimbra), R. H. Tütüncü (Carnegie Mellon University, Pittsburgh, USA) and L. N. Vicente (University of Coimbra).

Aims

Optimization models and methods play an increasingly important role in financial decision making. Many problems in quantitative finance, originated from asset allocation, risk management, derivative pricing, and model fitting, are now routinely and efficiently solved using modern optimization techniques. This workshop will bring together researchers in the rapidly growing field of financial optimization and intends to provide a forum for innovative models and methods on new topics, novel approaches to well-known problems, success stories, and computational studies in this exciting field. Participants are encouraged to present and discuss their recent work and new, possibly controversial, approaches are particularly welcome.

The targeted audience for this workshop includes graduate students and faculty members working in applied mathematics, operations research, and economics, who have been interested in mathematical finance or plan to do so. The workshop will also be attractive for those doing quantitative modelling in the financial market.

A one-day short-course, intended for optimization researchers interested in quantitative finance as well as finance researchers and practitioners interested in optimization models and methods, will precede the scientific program of the workshop. Invited and contributed presentations will be scheduled during the remaining three days.

The event will be held at the Faculty of Economics - University of Coimbra.

Short Course

It will be delivered by

R. H. Tütüncü (Carnegie Mellon University, USA)

S. Uryasev (University of Florida, USA)

Invited speakers

J. R. Birge (University of Chicago, USA)

T. F. Coleman (Cornell University, USA)

H. Konno (Chuo University, Japan)

J. M. Mulvey (Princeton University, USA)

R. T. Rockafellar (University of Washington, USA)

N. Touzi (CREST, France)

S. A. Zenios (University of Cyprus, Cyprus)

For more information about the event, see

`http://www.mat.uc.pt/tt2005/of/`

July 11-15: Summer School on Geometric and Algebraic Approaches for Integer Programming

Organizers

M. Constantino (University of Lisbon), L. Gouveia (University of Lisbon) and R. Weismantel (Otto-von-Guericke-University of Magdeburg, Germany).

Aims

The School is composed by five set of lectures, designed to introduce young researchers to the more recent advances on geometric and algebraic approaches for integer programming. Each set of lectures will be about six hours long. They will provide the background, introduce the theme, describe the state-of-the-art, and suggest practical exercises. The organizers will try to provide a relaxed atmosphere with enough time for discussion.

Integer programming is a field of optimization with recognized scientific and economical relevance. The usual approach to solve integer programming problems is to use linear programming within a branch-and-bound or branch-and-cut framework, using whenever possible polyhedral results about the set of feasible solutions.

Alternative algebraic and geometric approaches have recently emerged that show great promise. In particular, polynomial algorithms for solving integer programs in fixed dimension have recently been developed. This is a hot topic of international research, and the School will be an opportunity to bring up-to-date knowledge to young researchers.

The school will be held at the Faculty of Sciences, Bloco C6 - located in the main campus of the University of Lisbon.

LECTURES

Generating Functions for Lattice Points
A. Barvinok (University of Michigan, USA)

Geometric Approaches to Cutting Plane Theory
G. Cornuéjols (Carnegie Mellon University, USA)

Fast Algorithms for Integer Programming in Fixed Dimension
F. Eisenbrand (Max-Planck-Institut, Germany)

I. Experimenting and Applying the Rational Function Method: A LattE Tutorial
II. Transportation Polytopes: Structure, Algorithms, and Applications to Optimization and Statistics
J. de Loera (University of California, Davis, USA)

The Integral Basis Method and Extensions
R. Weismantel (Otto-von-Guericke Univ. Magdeburg, Germany)

For more information about the event, see

`http://www.mat.uc.pt/tt2005/ss/`

July 20-22: Workshop on Optimization in Medicine

ORGANIZERS

C. Alves (Technical University of Lisbon), P. M. Pardalos (University of Florida,Gainesville, USA) and L. N. Vicente (University of Coimbra).

AIMS

The study of computing in medical applications has opened many challenging issues and problems for both the medical computing and mathematical communities. This workshop is intended to foster communication and collaboration between researchers in the medical computing community and researchers working in applied mathematics and optimization.

Mathematical techniques (continuous and discrete) are playing a key role with increasingly importance in understanding several fundamental problems in medicine.

For instance, mathematical theory of nonlinear dynamics and discrete optimization has been used to predict epileptic seizures. Next to stroke, epilepsy is among the most common disorders of the nervous system. Measures derived from the theory of nonlinear dynamics and discrete optimization techniques are used for prediction of impending epileptic seizures from analysis of multielectrode electroencephalographic (EEG) data.

Several examples of the use of mathematics in medicine can be found in recent cancer research. Sophisticated mathematical models and algorithms have been used for generating treatment plans for radionuclide implant and external beam radiation therapy. With Gamma Knife treatment, for example, optimization techniques have been used to automate the treatment planning process.

Optimization has been used to address a variety of medical image registration problems. In particular, specialized mathematical programming techniques have been used in a variety of domains including the rigid alignment of primate autoradiographs and the non-rigid registration of cortical anatomical structures as seen in MRI.

The invited presentations will be complemented by sessions of contributed talks.

The event will take place at the Institute of Biomedical Research in Light and Image (IBILI), Faculty of Medicine - University of Coimbra

INVITED PRESENTATIONS

Optimization of Gamma Knife Radiosurgery
M. C. Ferris (University of Wisconsin, USA)

Multicriteria Optimization in Radiation Therapy
H. W. Hamacher (Univ. of Kaiserslautern, Germany)

Optimization in Epilepsy
L. D. Iasemidis (Arizona State University, USA)

Optimal Reconstruction Kernels in Medical Imaging
A. K. Louis (University of Saarbrücken, Germany)

Optimization and Optimal Control in High Intensity Ultrasound Surgery
J. P. Kaipio (University of Kuopio, Finland)

Integer Programming in Radiation Therapy
E. K. Lee (Georgia Institute of Technology, USA)

Optimization in Medical Imaging Registration
A. Rangarajan (University of Florida, USA)

For more information about the event, see

`http://www.mat.uc.pt/tt2005/om/`

July 26-29: Workshop on PDE Constrained Optimization

ORGANIZERS

L. M. Fernandes (Polytechnical Institute of Tomar), M. Heinkenschloss (Rice University, Houston, USA) and L. N. Vicente (University of Coimbra).

AIMS

Optimization problems governed by partial differential equation (PDE) constraints arise in many important applications. Progress in computational and applied mathematics combined with the availability of rapidly increasing computer power steadily enlarges the range of applications that can be simulated numerically and for which optimization tasks, such as optimal design, parameter identification, and control are being considered. For most of these optimization problems, simple approaches combining off-the-shelf PDE solvers and optimization algorithms often lack robustness or can be very inefficient.

Successful solution approaches have to overcome challenges arising from, e.g., the increasing complexity of applications and their mathematical models, the influence of the underlying infinite dimensional problem structure on optimization algorithms, and the interaction of PDE discretization and optimization.

This workshop will combine a wide range of topics important to PDE constrained optimization in an integrated approach, fusing techniques from a number of mathematical disciplines including functional analysis, optimal control theory, numerical optimization, numerical PDEs, and numerical analysis and application specific structures.

A short course will be offered on the first day of the workshop.

Invited and contributed presentations will be scheduled during the remaining three days.

The event will take place at the Escola Superior de Tecnologia de Tomar and Hotel dos Templários, Tomar.

SHORT COURSE

Theoretical background on characterization and properties of solutions to PDE constrained optimization problems
F. Tröltzsch (Technical University of Berlin, Germany)

Numerical solution of PDE constrained optimization problems
M. Heinkenschloss (Rice University, USA)

INVITED PRESENTATIONS

Flow Control
M. D. Gunzburger (Florida State University, USA)

Multiphysics Problems
R. H. W. Hoppe (University of Augsburg, Germany)

State Constraints
K. Kunisch (University of Graz, Austria)

Time Dependent Problems
G. Leugering (Univ. Erlangen-Nürnb., Germany)

Model Reduction
A. T. Patera (MIT, USA)

Adaptive Solution of PDE Constrained Problems
R. Rannacher (University of Heidelberg, Germany)

Preconditioning of KKT Systems
E. W. Sachs (University of Trier, Germany)

For more information about the event, see

`http://www.mat.uc.pt/tt2005/pde/`

Other CIM events in 2005:

International Conference on Semigroups and Languages

University of Lisbon, July 12-15

Organizers:

Jorge M. André, New Univ. of Lisbon

Mário Branco, Univ. of Lisbon

Vitor Hugo Fernandes, New Univ. of Lisbon

John Fountain, Univ. of York, UK

Gracinda M. S. Gomes, Univ. of Lisbon

John Meakin, Univ. of Nebraska, USA

Confirmed Invited Lecturers

J. Almeida, Univ. of Porto, Portugal

R. Gilman, Stevens Institute of Technology, USA

M. Lawson, Heriot-Watt Univ., UK

S. Margolis, Bar-Ilan Univ., Israel

D. McAlister, Northern Illinois Univ., USA

D. Munn, Univ. of Glasgow, UK

F. Otto, Univ. of Kassel, Germany

J.-E. Pin, Univ. Paris 7, France

P. Silva, Univ. of Porto, Portugal

B. Steinberg, Carleton Univ., Canada

M. Szendrei, Univ. of Szeged, Hungary

D. Therien, McGill Univ., Canada

M. Volkov, Ural State Univ., Russia

P. Weil, Univ. of Bordeaux, France

For more information about this event, see

`http://caul.cii.fc.ul.pt/csl2005/`

Workshop on Statistics in Genomics and Proteomics

Hotel Estoril Eden, Monte Estoril, October 6-8

Organizers:

M. Antónia A. Turkman, Univ. of Lisbon

Kamil Feridun Turkman, Univ. of Lisbon

Lisete Sousa, Univ. of Lisbon

Luzia Gonçalves, New Univ. of Lisbon

Aims

The workshop will aim to bring together the leading researchers in the areas of statistics in genomics and proteomics, to describe the state of the art and also to present problems that will change the next generation of Biostatistics and Bioinformatics researchers.

The workshop will have 7 Keynote speakers and 5 Invited speakers (from Portugal) on topics which are at the forefront of research. The main areas of the workshop are:

- Analysis of Gene Expression Data
- Regulatory Networks
- Statistical Proteomics
- Physical Mapping
- Phylogenetics and Evolutionary Genomics

Preliminary List of Keynote Speakers

Terry Speed, Department of Statistics, University of California, USA

Dirk Husmeier, Biomathematics & Statistics Scotland SCRI, UK

Ruedi Aebersold, Institute for Systems Biology, Seatle, USA

Sophie Schbath, Institut National de la Reserche Agronomique, Unité Mathématique, Informatique & Génome, France

Korbinian Strimmer, Department of Statistics, University of Munique, Germany

Chris Cannings, Division of Genomic Medicine, University of Sheffield, UK

Simon Tavaré, Department of Biological Sciences, University of South California, USA

Preliminary List of Invited Speakers

Margarida Amaral, Department of Chemistry and Biochemistry, University of Lisbon and National Institute of Health, Lisbon

Líbia Zé-Zé, Sequencing Unit, ICAT, University of Lisbon

Pedro Fernandes, Gulbenkian Institute of Science, Lisbon

Rogério Tenreiro, Department of Plant Biology , University of Lisbon

Mário Silva, Department of Informatics, University of Lisbon

For more information about this event, see

`http://wsgp.deio.fc.ul.pt/`

For updated information on these events, see

`http://www.cim.pt/new/?q=events`.

CIM News

CIM Events for 2006

The CIM Scientific Council will meet in Coimbra on February 12, to discuss the CIM scientific programme for 2006.

Seminar of the 2005 Annual Scientific Council Meeting

Hotel Quinta das Lágrimas, Coimbra, February 12

Programme:

16:00 - Marcelo Viana (IMPA, Brazil)

Geodesic flows on flat surfaces

17:30 - Marco Avellaneda (Courant Institute, USA)

A market-induced mechanism for stock pinning

19:30 - Dinner

For more information, see

`http://www.cim.pt/`

Research in Pairs at CIM

CIM has facilities for research work in pairs and welcomes applications for their use for limited periods.

These facilities are located at Complexo do Observatório Astronómico in Coimbra and include:

- office space, computing facilities, and some secretarial support;
- access to the library of the Department of Mathematics of the Univ. of Coimbra (30 minutes away by bus);
- lodging: a two room flat.

At least one of the researchers should be affiliated with an associate of CIM, or a participant in a CIM event.

Applicants should fill in the electronic application form

`http://www.cim.pt/new/?q=research`

CIM on the Web

Complete information about CIM and its activities can be found at the site

`http://www.cim.pt`

CIM Thematic Term on Mathematics and the Environment

Scientific Report

1 Introduction

The CIM Thematic Term 2004 was about Mathematics and the Environment. The knowledge about the impact of human activities on our planet's ecosystems is nowadays more vital than ever. Increasing human population to the detriment of others, cutting and burning vast areas of forest, polluting soil, air and water, are just a few examples of how we humans are altering our environment. Within this Thematic Term, some of these issues were addressed from a mathematical and physical modelling point of view.

The first event, the School and Workshop on Dynamical Systems and Applications, was aimed at consolidating the research activities in Portugal in this area of mathematics, fundamental for the understanding of the evolution of ecological environments and the monitoring of global changes. The Workshop on Forest Fires attempted to promote the communication among researchers with an interest in theoretical modelling of forest fires, in particular in fire front propagation. The third and fourth events, the School on Atmospheric Sciences and Climate Dynamics and the School and Workshop on Oceanography, Lakes and Rivers, addressed probably the most important natural processes for the world's ecosystem, and touched on issues such as air quality, weather prediction, ocean waves and currents, estuarine dynamics, and avalanches, among others.

The activities were coordinated by Juha H. Videman (IST) and José Miguel Urbano (Univ. Coimbra).

2 School and Workshop on Dynamical Systems and Applications (Porto, May 3-7, 2004)

The workshop took place in the Departamento de Matemática Pura da Faculdade de Ciências da Universidade do Porto and counted as participants 11 renowned specialists as well as 25 students and junior researchers, of which 10 from Portuguese institutions. The organizers were José Ferreira Alves (Univ. Porto) and Marcelo Viana (IMPA, Brazil).

The workshop had a strong local and national impact, contributing significantly to reinforce the cooperation between the different national groups working on Dynamical Systems and also promoting their international visibility.

The scientific programme faced the challenge of putting together specialists in a wide spectrum of topics in Dynamics, from the more fundamental research to specific applications in experimental areas, and exploring the connections between them. It is our believe, based on the active contributions of the participants, that the result was a success. The programme had two main components:

1. Two short courses (5 hours each) given by renowned specialists, both with a scientific background that combines the interest in very concrete problems with a wide knowledge of fundamental methods and results.

    - Celso Grebogi (Universidade de São Paulo, Brazil): *Chaotic dynamics and applications.*

      Consisted in an introduction to the notions and basic results in Dynamics and a description of several applications to concrete problems in meteorology, electrical circuits, spatial navigation, among others.

    - Carles Simó (Universidad de Barcelona, Spain): *Dynamical systems, numerical experiments and super-computing.*

      A presentation in full depth of the ideas and methods used in the analysis and numerical simulation of dynamical systems, widely illustrated with examples.

2. The second component of the programme consisted of around 20 talks given by specialists from several countries in a wide range of topics, from the interface with arithmetics and number theory

to applications in experimental neurology: problems of numerical computation and rigorous scientific computing (Tucker, Kim, Choe), the behaviour of conservative systems from mechanics and gas dynamics (Le Calvez, Kalisch, Lopes-Dias, Del Magno), recent fundamental progress in non-hyperbolic systems (Díaz, Gelfert, Pacífico), several aspects of the stability of dynamical systems and their models (Araújo, Elia, Viana), synchronization of systems and application in cryptography (Chembo, Ciszak), algebraic and arithmetic dynamical systems (Hric, Marmi, Marzougui), neuronal systems and neuronal coding (Pakdaman).

We believe the overall balance of topics clearly illustrated the diversity of the current research directions in Dynamical Systems, also putting into perspective the relations between the various trends.

3 Workshop on Forest Fires (Coimbra, June 3-5, 2004)

The event consisted of a three-day workshop with talks by invited speakers; an afternoon was reserved for young researchers to present their work in talks of 20min. The organizers were Jorge C. S. André (Univ. Coimbra) and José Miguel Urbano (Univ. Coimbra).

The main goal of the workshop was to promote the communication (i.e., mutual knowledge, criticisms, possible future synergies respecting results and, above all, strategies of research) among researchers with a common interest and competence on theoretical modelling issues of forest fires, with an emphasis on fire front propagation. The invited speakers had different backgrounds (mathematics, physics, mechanical engineering) and came from different parts of the world where wild forest fires are a threat to the environment (Mediterranean Europe, Middle-East, USA, Australia). The scientific level was very good with talks touching different aspects of forest fires research: fire ecology, convection in forest fires and numerical simulation of wild fires, for example.

Unfortunately, the attendance was rather poor.

4 School on Atmospheric Sciences and Climate Dynamics (Lisbon, July 12-16, 2004)

The event consisted of five short courses of five hours each and of an afternoon session with short communications. It took place at the main auditorium of the Complexo Interdisciplinar of the Instituto Superior Técnico in Lisbon. The organizers were Juha Videman (IST, Lisbon), José Miguel Urbano (Univ. Coimbra) and Didier Bresch (CNRS, LMG-Grenoble, France).

The school was intended for graduate and PhD students as well as for researchers pursuing investigation on problems related to atmospheric sciences or climate dynamics. The main goal was to broaden our understanding of the complex processes that control the climate, the chemistry of the coupled atmosphere-ocean system, and the physics of the upper atmosphere.

The courses of the invited lecturers ranged from discussions of the human influence on climate and climate forecasting (Myles Allen) to treatments of transport and mixing phenomena in atmospheric chemistry (Peter Haynes) and in atmospheric-ocean dynamics (Esteban Tabak) and to the analysis of energy balance models (Jesus Ildefonso Díaz). The speakers were excellent and the scientific quality of their lectures reached the highest possible level.

The lectures were attended most keenly by about 50 participants (40 officially registered), with 15 of them coming from 7 different foreign countries.

5 School and Workshop on Oceanography, Lakes and Rivers (Lisbon, July 19-24, 2004)

This event started with a four-day Summer School including five short courses of four hours each and ended with a two-day Workshop consisting of five invited lectures and of an afternoon session with ten short communications. It took place at the main auditorium of the Complexo Interdisciplinar of the Instituto Superior Técnico in Lisbon. The organizers were Juha Videman (IST, Lisbon), José Miguel Urbano (Univ. Coimbra) and Didier Bresch (CNRS, LMG-Grenoble, France).

The main objectives of the event were to initiate and develop the communication and interactions between the specialists working on different frontiers of Oceanography and to introduce to the Portuguese students the fundamentals, as well as some of the most relevant and current problems, of Environmental and Geophysical Fluid Dynamics. The invited lecturers were chosen carefully taking into account the interdisciplinarity of this field: Joseph Pedlosky and Benoît Cushman-Roisin are world-famous physical oceanographers with strong experience in collaborating with applied mathematicians; Peter Constantin is the world's leading expert in modelling turbulent geophysical flows; Benoît Perthame is a top specialist in numerical modelling of shallow-water equations and Emmanuel Grenier is well-known for his studies of rotating fluids. The Workshop speakers discussed a variety of challenging topics such as the mathematical and numerical treatment of the primitive equations (Francisco Guillén); thermohaline circulation (David Marshall); the modelling of avalanches (Reinhard Farwig); roughness-induced effects in large-scale geophysical systems (David Gérard Varet) and turbulence, clouds and climate models (João Teixeira).

The scientific level was superb, in fact it was unseen to see together all these people in the same event. The atmosphere was friendly and cosy which helped the students to approach the speakers in and out of the lecture room. The attendance exceeded all expectations: it rounded to almost 100 participants (80 officially registered; 30 from foreign countries including France (11), Spain (8) Sweden (5), Italy (3), USA (3), Great Britain, Switzerland, Germany, Czech Republic, Lithuania, Croatia, Bosnia-Herzegovina, Turkey, Senegal).

As for the Portuguese participants, they came from different parts of the country (Porto, Vila Real, Aveiro, Coimbra, Évora, Faro and Lisbon) and from different Departments/Institutes (Mathematics, Geophysics, Environmental Engineering, Oceanography, Mechanical Engineering, Civil Engineering).

We think that this last event was a huge success which crowned the entire Thematic Term.

Juha Videman and José Miguel Urbano (with Jorge André, José Ferreira Alves and Marcelo Viana)

Workshop on Nonstandard Mathematics NSM2004

Scientific Report

This conference, aimed at emphasizing the importance of Nonstandard Analysis for Mathematics, was held in honor of its inventor, Abraham Robinson, on the thirtieth anniversary of his death. The Organizing Committee (OC) also intended to provide an opportunity for mathematicians to discuss scientific aspects as well as problems raised by teaching Nonstandard Analysis both at university and pre-university levels. The OC was much gratified by the fact that some of the participants did take the opportunity to proceed with ongoing joint projects. The web page of NSM2004 is still active at `http://www.mat.ua.pt/eventos/nsmath2004/`. A summary of the main activities, between 9:30 a.m. Monday, July the fifth and 8:00 p.m. Friday the ninth, read as follows:

- A 4.5 hours course in Analysis with Infinitesimal
- Nine plenary talks of approximately one hour
- Twenty two talks of about 25 minutes
- A closing debate of roughly one hour

There were seventy participants of varied nationality — Portuguese, English, French, North American (from the USA), Italian, German, Irish, Ukrainian, Algerian, Russian and Hungarian — together with approximately 15 professors and teaching assistants from the Mathematics Department of the University of Aveiro as well as pre-university teachers of Mathematics, who mainly attended the course, which was actually attended by virtually all participants. Among active participants, there were four PhD students, of which two presented short talks and another co-authored; three more graduate students and an undergraduate were present too. The plenary talks ranged through Analysis and Functional Analysis, Control Theory, Differential Equations, Foundations, Perturbation Theory, Number Theory, Quantum Physics, Stochastic Analysis (herein including stochastic solutions to Navier-Stokes equations); there was also a talk of about half an hour on Nonstandard Pre-Calculus; the late José Sousa Pinto was remembered by means of a talk mainly consisting of a survey of his work on Generalized Functions.

The Scientific Board - where Robert Lutz, from the University of Haute-Alsace in France, and Francine Diner from Nice (France), who were not present at the meeting, were substituted by Tewfik Sari, from the University of Haute-Alsace – recognized the high scientific level of the meeting and therefore participants were invited to submit papers to be refereed for inclusion in the proceedings, the editors being Imme van den Berg, professor at the University of Évora and Vítor Neves, professor at the University of Aveiro.

Vítor Neves (on behalf of the Organizing Committee)

Mathematical Finance - a glimpse from the past challenging the future

Maria do Rosário Grossinho
Departamento de Matemática
ISEG, UTL, Portugal

1 Introduction

Mathematical Finance is a flourishing area of modern science that was born in 1900 with Louis Bachelier's Ph.D thesis "Théorie de la speculation" [1]. This work is a result of his attempt to model stock prices on the Paris stock market. Since then, its importance has increased not only as the basis of the hectic financial activity of the modern world but also as a source of many interesting mathematical problems and theories.

Modelling of risky asset prices and modern option pricing techniques are often considered among the most mathematically complex of all applied areas of finance. Their roots rely on stochastic calculus and their development is intimately related to the history of stochastic integration. In fact, financial phenomena and instruments (bank accounts, bonds, stocks, options, rates, currencies, etc.) combine on the one hand a deterministic behaviour and on the other a degree of uncertainty due to time, risks and the (random) environment. That is why the theory of stochastic processes perfectly suits the needs of financial theory and strategy. What is commonly referred to as Mathematical Finance can be considered in a naïf way as the resultant of two vectors, stochastic integration and modelling of asset prices of financial markets operating under uncertainty.

In this text, given that it is impossible to give a panoramic or exhaustive view of the subject, we decided to focus on some key moments that in a pioneering way have determined the development of Mathematical Finance due to contributions either on the analysis and dynamics of financial markets or on the closely related mathematical theory.

2 Starting at the beginning - Bachelier and a work ahead of its time

Bachelier was the first person to model the dynamics of stock prices based on random walks and their limit cases. Combining probability techniques with Markov property, and using the fact that the Gaussian kernel gives the fundamental solution of the heat equation, he was able to model and study the market noise of the Paris Stock Market.

He proposed to regard the stock prices $S = (S_t)_{t\geq 0}$ as a random (stochastic) process

$$S_t = S_0 + \sigma W_t, \qquad t \geq 0,$$

where $W = (W_t)_{t\geq 0}$ is a stochastic term describing the *noise,* that is, the random component of the phenomenon that is called the *Brownian motion* or *Wiener process.*

In the following model, still designated as Bachelier's model, the stock prices $S = (S_t)_{t\leq T}$ follow a Brownian motion with *drift*, that is,

$$S_t = S_0 + \mu t + \sigma W_t, \qquad t \leq T. \tag{2.1}$$

In (2.1), it is considered that there is a (B, S) −market such that the bank account $B = (B_t)_{t\leq T}$ remains fixed, $B_t = 1$. In a differential form, Bachelier's model can be written as

$$dS_t = \mu.dt + \sigma dW_t.$$

In his work, Bachelier gave the price for a *European option.* Recalling the definition, *option* is generally defined as "a contract between two parties in which one party has the right but not the obligation to buy or sell some underlying asset, at an agreed strike price K, at an assigned time T, called maturity, while the second party, the writer, has the obligation to sell or buy it if the first party wants to exercise his right". *Call options* are contracts that give the option holder the right to buy a given asset, while *put options*, conversely, entitle the holder to sell. Having rights without obligations has financial value. So, option holders must purchase these rights, that is, must pay a *premium.* This type of contract derives its value from some underlying asset; so, they are called *derivative assets* or simply *derivatives.*

At expiry or maturity, a call option is worthless if $S_T < K$ but has the value $S_T - K$ if $S_T > K$. This means in financial terms that its *payoff* is

$$(S_T - K)^+ = \max(S_T - K, 0).$$

If, at maturity, $S_T > K$, then the option buyer obtains a profit equal to $(S_T - K)$ since he can buy the stock at the price K and sell it immediately at S_T. On the other hand, if at maturity $S_T < K$, then the buyer simply does not exercise his right and his loss is just the premium paid C_T. Starting from Brownian motion, Bachelier derived a formula for the expectation of the *payoff* $(S_T - K)^+$ of a call option

$$C_T = E(S_T - K)^+$$

which gives us the value of the *reasonable (fair) price* to be paid by the buyer to the writer of a call option at the moment of the contract, that is, as referred above, the *premium.* Considering the density function and the normal distribution, respectively,

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \qquad \text{and} \qquad \Phi(x) = \int_{-\infty}^{x} \varphi(y)\, dy, \tag{2.2}$$

the following formula

$$C_T = (S_0 - K)\,\Phi\left(\frac{S_0 - K}{\sigma\sqrt{T}}\right) + \sigma\sqrt{T}\varphi\left(\frac{S_0 - K}{\sigma\sqrt{T}}\right) \tag{2.3}$$

is called *Bachelier's formula* (which is in fact an updated version of several of Bachelier's results on options). It defines the price C_T of the standard *European call option* with pay-off function $(S_T - K)^+$ for the Bachelier model (2.1). The main interest of this model, besides of course the historical aspect, lies in the fact that it is both *arbitrage free* (does not allow riskless profits) and *complete* (is replicable) [15].

Correlations between price assets and options can be used by the investor to construct a portfolio in such a way that risk can be reduced – *hedging strategy.* So, valuing options becomes of great importance.

We have referred above to *Brownian motion.* Originally named after the biologist Robert Brown, this term has two meanings: the physical phenomenon that describes the random movement of small particles immersed in a fluid and the one of the mathematical models (used, for instance, to describe that movement). In the physical context, it was first modelled by Albert Einstein in 1905 [5]. At that time, the molecular nature of matter was still a controversial idea. Einstein observed that, if the kinetic theory of fluids was right, every small particle of water would receive a random number of impacts of random strength and from random directions in any short period of time. This random bombardment would explain the jittering motion of small particles exactly in the way described by Brown.

However, five years before, in 1900, Louis Bachelier had already given a mathematical theory of Brownian motion in his doctoral thesis, using a stochastic process as a model for the price and relying on his belief in the power of the law of probability to explain the stock market

"*Si, a l'égard de plusieurs questions traitées dans cette étude, j'ai comparé les résultats de l'observation a ceux de la théorie, ce n' eétait pas pour vérifier des formules etablies par les méthodes mathématiques, mais pour montrer seulement que le marché, a son insu, obéit a une loi qui le domine: la loi de la probabilité*".

Bachelier's work had little impact for a long time, in spite of the favourable report of his mentor, Henri Poincaré. His mathematical reasoning was not rigorous; and could not be, since the mathematical techniques used later to make it rigorous, that is, measure theory and axiomatic probability, had not been developed yet. But his results were basically correct. However, the "taste and goals" of the scientific elite of that time were not sensitive to mathematical applications for economics problems. Although there are some references to Bachelier results in later works of Kolmogorov, Doob and Itô, for example, it was only fifty years later that his thesis came to the limelight after having been "discovered" in the MIT library by the economist Paul Samuelson, Nobel Laureate in Economics in 1970. The impact of Bachelier's work in Samuelson's opinion can be clearly seen in his remark "*Bachelier seems to have had something of a one-track mind. But what a track*" [13] (see also [14]).

3 The classical Black-Scholes model

Bachelier's analytical valuation for options exhibited however a weakness as far as financial instruments were concerned, since the prices in the model could take negative values. In the 1960s, in order to overcome that weakness, Samuelson suggested modelling prices using what is now designated as geometric Brownian motion

$$S_t = S_0 e^{\sigma W_t + \left(\mu - \sigma^2/2\right)t} \tag{3.1}$$

and which, from Itô's calculus, can be written in the differential form

$$dS_t = S_t \left(\mu dt + \sigma dW_t\right). \tag{3.2}$$

This suggestion provided a workable model for asset prices and anticipated the central result of modern finance, the Black-Scholes option-pricing formula. It is assumed that the bank account $B = (B_t)_{t \geq 0}$ evolves according to the formula

$$dB_t = rB_t dt,$$

where r is the interest rate, whereas the price of the risky asset evolves according to the stochastic differential equation (3.2) whose solution with initial condition S_0 is (3.1). The coefficients are constant: μ measures the global behaviour of S while the coefficient σ, called the *volatility,* measures the importance of the *noise,* that is, of the influence of the Brownian motion. The larger σ is, the greater the influence of Brownian motion. Expression (3.1) shows that $S(t) > 0$.

Assuming that the function $C = C\left(t, S\right)$ is sufficiently smooth, Fisher Black and Miron Scholes [2] and Robert Merton [12], working independently, obtained as model for the dynamics of a European call option the so-called *fundamental equation*

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0 \tag{3.3}$$

with the final condition

$$C\left(T, S\right) = \max\left(S - K, 0\right). \tag{3.4}$$

An explicit solution can be determined using methods of partial differential equations that involve transforming this problem into the heat equation with an adequate condition. The solution, that is, the price of a call option at time t is given by

$$C\left(t, S\right) = S\Phi\left(d_+\right) - Ke^{-r(T-t)}\Phi\left(d_-\right),$$

where Φ is defined in (2.2) and

$$d_\pm = \frac{\ln\frac{S}{K} + (T - t)\left(r \pm \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T - t}},$$

and Black-Scholes Option Pricing Formula is

$$\begin{aligned} C_T \quad = \quad & S_0\Phi\left(\frac{\ln\frac{S_0}{K} + T\left(r + \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}}\right) \\ & - Ke^{-rT}\Phi\left(\frac{\ln\frac{S_0}{K} + T\left(r - \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}}\right). \end{aligned}$$

This result can be derived by a so-called *martingale* proof but using the solution of the fundamental equation was, in fact, the original proof established by Black, Scholes and Merton. If we look at Bachelier's formula, it can be easily recognized as a forerunner of the Black-Scholes formula.

On account of the above achievement, the Nobel prize in Economics was awarded to R. Merton and M. Scholes in 1997, thus also honoring F. Black (who died in 1995).

The above models concern continuous time. In 1976, three years after the Black-Scholes-Merton model, a model for discrete time was developed: the Cox-Ross-Rubinstein binomial model [3]. It assumed a risk-neutral world, that is, it recognized that investor risk preferences did not interfere in the pricing of derivatives. This model was simple, flexible and suitable for pricing American as well as European options. In fact, Black-Scholes-Merton gave an exact solution for European options, that could be exercised only at maturity, but was not able to provide values for American options that could be exercised before expiry. In general, solutions for American options can only be obtained numerically. This explains the important role played by numerical analysis and computational techniques in option pricing.

Asset and option pricing are fundamental elements in Portfolio Theory. This theory concerns the construction of portfolios, taking into account the benefits of diversification, so that expected returns may be optimized for a given level of market risk. Pioneering work concerning the Modern Portfolio Theory was carried out by Harry Markowitz [11], Nobel Laureate in Economics in 1990.

4 From finance to stochastic analysis

From the above paragraphs it is clear that financial phenomena and risky asset modelling are a wonderful playground for Mathematics.

A rigorous mathematical theory of Brownian motion was developed by Norbet Wiener in 1923 [16] by combining new results on measure theory with Fourier analysis. On account of those studies, Brownian motion is commonly referred to as *Wiener process.*

In the 1930s, a fundamental role was played by Kolmogorov. Among numerous major contributions made in a whole range of different areas of Mathematics, either pure or applied, he built up probability theory in a rigorous way, providing the axiomatic foundation on which the subject has been based ever since, and laid the foundations of the theory of Markov processes [10].

The concept of martingales in probability theory was introduced by Paul Pierre Lévy in the late 1930s. It was developed extensively by Doob who published a fundamental work on the subject [4] in 1953.

Considered as the father of stochastic integration, Kiyosi Itô published his first paper on the subject in 1944 [8]. Reflecting on the studies of Wiener and Kolmogorov and attempting to study the connection between partial differential operators, such as the heat operator, and Markov processes, he constructed a stochastic differential equation of the form

$$dX_t = \sigma(X_t)\, dW_t + \mu(X_t)\, dt$$

where W represents a standard Wiener process. This formula created two problems: the first one was to give sense to $\sigma(X_t)\, dW_t$ and the second one was to relate his work with Kolmogorov's results on Markov processes. He gave a positive answer to those problems. Namely, he developed a new calculus to solve the problem arising from the fact that Riemann-Stieljes integration was no longer valid. This new differential/integral calculus was named after his work as *Itô calculus.*

By 1980, arbitrage pricing theory had become well understood. A close link between nonexistence arbitrage opportunities and martingales was established in the so-called *Fundamental Theorem of Asset Pricing.* This theorem is due to Harrison, Kreps and Pliska [6, 7] and became a pioneering result after which many contributions appeared to improve the understanding of the subject. It points out that stochastic integration is extremely well suited to the study of stochastic processes arising in finance.

5 Conclusion

The financial world is fast-changing and needs constant updating in order to operate financial resources where new financial instruments and strategies are always appearing. Determination of opportunities is becoming more and more reliant on complex mathematics, which drives studies into new areas.

Among all the possible directions, presently most of them are related to incomplete markets, in which Black-Scholes style replication is impossible. Risk neutral world is no longer assumed and any pricing formula must involve some balance of the risks involved. Moving to incomplete markets means that mathematical finance must inevitably demand new approaches and lead to new developments in mathematical research.

In the above paragraphs we have tried to give the reader a combination of a slight flavour of some financial concepts and instruments that concern Financial Theory, together with a quick look at the Mathematics involved. Above all, we aim to provide a stimulating glance at the huge complexity and multidisciplinary features of the subject, as well as at the reciprocal challenges continuously appearing between practical and theoretical aspects.

Bibliography

[1] **Bachelier, L.** Théorie de la Spéculation. *Ann. Sci. Ecole Norm. Sup.* **17** (1900) 21-86 [English translation in P. Cootner (ed.), *The Random Character of Stock Prices*, MIT Press, 1964, reprinted Risk Books, London 2000].

[2] **Black, F.** and**Scholes, M.** The pricing of options and corporate liabilities. *J. Political Econom.* **81** (1973) 637-654.

[3] **Cox, J.; Ross S.** and **Rubinstein, M.** Option pricing, a simplified approach. *J. Financial Economics* **7** (1979) 229-263.

[4] **Doob, J. L.** *Stochastic Processes.* Wiley, New York (1953).

[5] **Einstein, A.** Uber die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen. *Annalen der Physik* **17** (1905) 549-560.

[6] **Harrison, J. M.** and **Kreps, D. M.** Martingales and arbitrage in multiperiod securities markets. *J. Econ. Theory* **20** (1979) 381-408.

[7] **Harrison, J. M.** and **Pliska, S. R.** Martingales and stochastic integrals in the theory of continuous trading. *Stoch. Proc. and Appl.* **110** (1981) 215-260.

[8] **Itô, K.** Stochastic Integral. *Proc. Imp. Acad. Tokyo* **20** (1944) 519-524.

[9] **Itô, K.** On a formula concerning stochastic differentials. *Nagoya Math. J.* **3** (1951) 55-65.

[10] **Kolmogorov, A. N.** On Analytic Methods in Probability Theory, in A. N. Shiryaev (ed.), *Selected Works of A. N. Kolmogorov*, Volume II: Probability Theory and Mathematical Statistics, Kluwer, Dordrecht, 1992, pp. 62-108. [Original: Uber die analytischenMethoden in der Wahrscheinlichkeitsrechnung, *Math. Ann.* **104** (1931) 415-458.]

[11] **Markowitz, H.** Portfolio Selection. *Journal of Finance* **7** (1), (1952) 77-91.

[12] **Merton, R. C.** Theory of rational option pricing. *Bell J. Econom. and Management Sci.* **4** (1973) 141-183.

[13] **Samuelson, P.** Mathematics of Speculative Price. *SIAM Review* **15** (1973) 1-42.

[14] **Samuelson, P**. Modern finance theory within one lifetime, in H. Geman, D. Madan, S.R. Pliska and T. Vorst (eds.), *Mathematical Finance - Bachelier Congress 2000*, Springer-Verlag, Heidelberg, 2002, pp. 41-46.

[15] **Shiryaev, A. N.** *Essentials of Stochastic Finance. Facts, Models, Theory.* World Scientific (1999).

[16] **Wiener, R.** Differential Space. *J. Math. Phys.* **2** (1923) 131-174.

Making wavelets in the art world. "Computers confront the art experts" is a piece by Philip Ball posted on November 22, 2004 at Nature's online site `news@nature.com`. Ball's subheadline reads: "Automated method seems to spot forgeries as well as a connoisseur does." The forgeries are in paintings, and the method in question, devised by Hany Farid, Dan Blackmore and Siwei Lyu of Darmouth University, uses wavelet analysis to characterize the "hand" of an artist. Ball describes it as follows: "They scan a picture at high resolution and then use the wavelet technique to decompose the picture into sets of vertical, horizontal and diagonal lines. [...] From the mathematical distribution of lines, the researchers calculate a set of numbers that characterizes each picture. These numbers can be regarded as coordinates in a multi-dimensional space, like a grid reference. If two images share similarities in their use of line, the points in space defined by their coordinates will lie close together, even if the scenes depicted are totally different." When the Dartmouth team tried their technique on a set of 13 landscapes (genuine and imitation) by Brueghel, "the points corresponding to the eight pictures deemed to be authentic all sat together in a cluster, and the fakes were further away." Then they turned to the Madonna and Child with Saints, attributed to the Italian Renaissance painter Perugino, in the collection of the Hood Museum at Dartmouth. There are four Saints in the picture, so six heads in all. Applying their analysis to the heads, Farid and his collaborators found no fewer than four painters at work. Three of the heads are by three different painters; the three others are by a fourth, "perhaps Perugino himself," as Ball puts it. Article available online.

World Renowned Geometer S.-S. Chern Dies. Shiing-Shen Chern, one of the outstanding mathematicians of the 20th century, passed away in Tianjin, China, on Friday, December 3, 2004, at the age of 93. S.-S. Chern was one of the creators of modern differential geometry as it is known today. Fifty years ago, his global viewpoint, emphasizing relations with topology, was revolutionary. One of his early successes was his elegant proof of the Chern-Gauss-Bonnet theorem, which, together with concepts such as Chern classes and Chern-Simons invariants, made a lasting imprint on the subject. S.-S. Chern was the founding director of the Mathematical Sciences Research Institute in Berkeley. After holding professorial positions at the University of Chicago and the University of California, Berkeley, he returned to China in his retirement and founded the Nankai Institute of Mathematics. He received the first Shaw Prize in 2004. Read more about Chern in an interview that appeared in *Notices of the AMS*.

"Quantum decoys create uncrackable code" by Mark Buchanan, is the title of a short article in the November 13, 2004, *New Scientist*. It describes a recent breakthrough in quantum cryptography by researchers at the University of Toronto. One reason quantum codes were initially thought to be so powerful is that eavesdropping would disturb the photons used to carry the messages and therefore could be detected. Then researchers found a way whereby an eavesdropper could cover up his or her tracks. The new research shows how a message-sender can send out decoy photons that would foil an eavesdropper.

"A fractal life". Interview with Benoit Mandelbrot. Interviewed by Valerie Jamieson. *New Scientist*, 13 November 2004. No mathematical object has become so well known among the general public as has the fractal. Benoit Mandelbrot coined the term and was the first to systematically explore this geometric phenomenon. In the interview, he talks about his views on the popularity of fractals, his background growing up in France, and the influence of his uncle, Szolem Mandelbrojt, who was a mathematician at the Collège de France in Paris. He also talks about his latest research on the concept of "negative dimension" and on the dynamics of financial markets. His book *The (Mis)Behavior of Markets: A fractal view of risk, ruin and reward*, written with Richard Hudson, came out in 2004.

Gödel's Theorem on ABC News. John Allen Paulos, in his "Who's Counting" column on the *ABC News* website, posted "Complexity, Randomness and Impossible Tasks" on November 7, 2004. Paulos starts with algorithmic complexity. He invites us to compare the two sequences

(A) 0010010010010010010010010010010010010010010...

(B) 1000101101101100010101100101111010010111010...

and asks: "Why is it that the first sequence of 0's and 1's ... is termed orderly or patterned and the second sequence random or patternless?" As a quantitative answer, he proposes Greg Chaitin's definition: *The (algorithmic) complexity of a sequence of 0's and 1's is the length of the shortest computer program that will generate the sequence.* The program for (A) could be "print 0,0,1, repeat." But for (B) it is quite possible that the only way to generate the sequence is to spell it out: "print 1,0,0,0,1,0,1,1,..." Paulos continues: "We define a sequence to be random if and only if its complexity is (roughly) equal to its length; that is, if the shortest program capable of generating it has (roughly) the same length as the sequence itself." If the only way to generate (B) is to spell it out, (B) is a random sequence. Next Paulos introduces us to the "Berry sentence," which he attributes to Bertrand Russel, 1908. *Find the smallest whole number that requires, in order to be specified, more words than there are in this sentence.* "The paradoxical nature of the task becomes clear when we realize that the Berry sentence specifies a particular whole number that, by its very definition, the sentence contains too few words to specify." Now the big step: "What yields a paradox about numbers can be modified to yield mathematical statements about sequences that can be neither proved nor disproved." A formal mathematical system can be encoded as a computer program P. As P runs, it generates all the possible theorems which can be proved in that system. "Now we ask whether the system is complete. Is it always the case that for a statement S, the system either proves S or it proves its negation, $\sim S$?" Paulos explains Chaitin's argument, which involves imagining the the following Berry-like task for the program: *Generate a sequence of bits that can be proved to be of complexity greater than the number of bits in this program.* "The program P cannot generate such a sequence, since any sequence that it generates must, by definition of complexity, be of complexity less than P itself." It follows that "statements of complexity greater than P's can be neither proved nor disproved by P." This is Chaitin's new, quantitative twist on Gödel's Theorem. Since any formal mathematical system has a certain finite complexity, it must allow statements which can be neither proved nor disproved, "a limitation affecting human minds as well as computers."

"How Strategists Design the Perfect Candidate" is the title of a piece by Mark Buchanan in the *Science* issue of 29 October 2004. With the United States presidential election on the horizon, writer Mark Buchanan interviewed some political analysts about mathematical models they use to analyze how voters choose candidates. An often-used model depicts voters "as abstract points in a 'policy space'"; a politician can optimize his or her standing with a set of x voters by choosing a point equidistant from all x. (On the other hand, a politician may have a better chance of executing his or her preferred policies – if elected – by moving away from this central location.) A second model, presented by University of California, San Diego, physicist David Meyer, reflects voters inconsistent preferences, implying that a politician's best strategy is "roughly speaking, to be as inconsistent as the voters," according to Buchanan. Given that closer races resulting from candidates' use of the above models – and that unpopular third-party candidates can heavily influence an election – better voting systems were also discussed. These included eliminating the Electoral College in favor of popular voting, an option that seems unlikely to Eric Maskin of the Institute for Advanced Study in Princeton, New Jersey, since it would necessitate a constitutional amendment. What is clear is that there are no simple answers. As political scientist Larry Bartels of Princeton University says, "the surprising reality is that we still understand relatively little about how presidential campaigns affect the vote."

"What Makes an Equation Beautiful", New York Times, 24 October 2004. In a column in *Physics World* magazine, philosopher and historian Robert P. Crease asked readers which equations they considered to be the greatest. He got 120 responses proposing 50 different equations. This article discusses Crease's experiment and also provides readers with a nice context to appreciate the power of mathematical equations. The top vote-getters were Maxwell's equations for electromagnetism and Euler's equation, $e^{i\pi}+1=0$. A list of 18 other winners is given in a sidebar. Most of the equations relate to physics, but the Pythagorean theorem and the Riemann zeta function made it onto the list.

IT and the Riemann Hypothesis. "What is the Riemann Hypothesis and why Should I Care?" is the provocative title of a piece by Robin Bloor posted at `IT-Director.com` on October 5, 2004. The site "provides IT decision makers with a one stop source of all current IT news, information, analysis and advice." (IT = Information Technology). Naturally, there is no attempt at a correct statement of the Riemann Hypothesis ("Without bothering to state the details, it is a proposed formula that calculates the number of primes less than a given number") but the reason why IT decision makers might be concerned is the "worrying predictions that if the Riemann Hypothesis is confirmed mathematically, then most of the encryption schemes we use in commerce and government will suddenly be vulnerable ...” together with news of its possible confirmation by Louis de Branges and perhaps by others. The risk for IT is “if the mathematics surrounding the solution reveals quicker ways to factorize numbers. Actually even then it will only matter if it reveals much quicker ways to factorize numbers.” Because public-key cryptography “is based on the product of prime numbers. The fundamental idea is that it is very difficult to find factors for a number that is created by multiplying two prime numbers together. It’s easy to multiply the numbers together but very difficult to find out what they were if you’re only given the result.” But not to worry: “Strange as it may seem (if you never studied Mathematics) there are mathematical relations that can be used to create encryption that can be proved to be unbreakable. The real problem is that we founded the early encryption on a technique that wasn’t provably unbreakable.” Bloor’s piece is available online.

The Math of Medical Marriages. “Tweaking the Math to Make Happier Medical Marriages” is the title of a piece by Sara Robinson in the August 24 2004 *New York Times*. “Medical marriages” refers to the process by which the National Residency Match Program assigns medical students to residency positions. Residency Match uses an algorithm that turns out to be equivalent to the “marriage algorithm” devised in 1962 by the mathematicians David Gale and Lloyd Shapley, who proved that it converges. Here is how Robinson explains the algorithm:

- “Each boy ranks all the girls in order of his preference, and each girl does the same. Then, each boy asks his first choice for a date. Each girl with one or more offers dates her favorite and says “no” to the rest.

- In the next round, the boys who were rejected move on to their second-choice girl. The girls again date their favorites, possibly throwing over their date from the earlier round for someone better.

- Continuing in this way, the mathematicians showed, the dating frenzy eventually subsides into a stable situation where each girl has only one boy, and there is no boy and girl who prefer each other to the people they are dating. That is, every time a boy does not get his first choice, he has no hope of getting anything better. Each of the girls he prefers is paired with someone she prefers to him. The same is true for a girl.”

The *Times* diagrams a 3×3 example in which Adam and Eve, Romeo and Juliet, Tristan and Isolde end up paired even though Isolde was only Tristan’s second choice to start with, and he was her third. [An interesting point about this algorithm, unfortunately obscured by the *Times* presentation (boys ask girls in the text, girls ask boys in the diagram) is that it *favors the askers*. The simplest example is with two boys and two girls.

- Suppose Romeo ranks Juliet #1 and Isolde #2, while Tristan ranks Isolde #1 and Juliet #2. And suppose Juliet ranks Tristan #1 and Romeo #2, while Isolde ranks Romeo #1 and Tristan #2.

- Following the *Times* text, Romeo invites Juliet and Tristan invites Isolde. Each girl has only one offer, and has to take it, so Romeo and Tristan get their wish, but Juliet and Isolde do not.

- Following the *Times* diagram, Juliet invites Tristan and Isolde invites Romeo; now the girls get their heart’s desire, and the boys do not.]

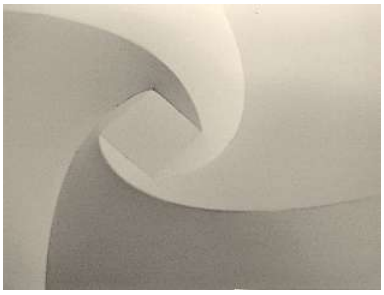
Hospitals used to do the asking. Even though cases like this are very rare (a 1996 analysis, available online, by August Colenbrander, MD, from which the example above was taken, estimates that in the residency match the chance of a discrepancy is less than 0.1%) the algorithm has been reversed since 1996 to make the students the askers. [See also Mathias Lindemann’s The Stable Marriage Problem, available online.] The algorithm is in the news because it is suspected of allowing hospitals to underpay residents.

Bird Logic. “Bigger than” is a transitive relation: if X is bigger than Y, and Y is bigger than Z, then we can infer, without comparing X to Z, that X is bigger than Z. “Pinion jays use transitive interference to predict social dominance,” by Guillermo Paz-y-Miño and three collaborators (*Nature*, August 12, 2004) shows how some birds apply this principle to their pecking order. The experimental setup involved three initially isolated groups of jays; in each group a dominance hierarchy had established itself: $A > B > C > D > E > F$, $1 > 2 > 3 > 4 > 5 > 6$ and $P > Q > R > S$. In a typical run, bird-3 (the “observer”) would watch, on three consecutive days, bird-2 defer to bird-B. Then bird-3 and bird-B were placed in a competitive encounter. If birds can use transitive inference then bird-3, having seen its dominator bird-2 defer to bird-B, should infer that B is its superior. And in fact: “During the first minute of the first encounter, observers displayed subordinance levels that were nearly four times as high as those of controls.” Controls would have watched similar displays involving two birds with which they were not acquainted. Pinyon jays are "among the most social of North American corvids." They also are better than their less gregarious cousins the scrub jays, at applying transitive inference in experiments involving colored markers. The authors' closing remark: "This work ... supports the hypothesis that social complexity provided a crucial context for the evolution of cognitive abilities."

Calculus, the play (New End Theatre, London, until August 24) is reviewed in the August 12 2004 *Nature* by Philip Ball. The play, written by Carl Djerassi, "centres on the deliberations of a Royal Society committee appointed in 1712 to pronounce on the priority issue." The issue being whether or not Leibnitz had plagiarized Newton's discovery of calculus. Newton appears in a play within a play which allows his interlocutor "to anticipate the audience's dismay (and indeed I sensed such a response) at having to hear about the calculus." Ultimately, Ball finds that "there is just not quite enough at stake here to sustain the drama." He adds parenthetically: "I did, however, enjoy the portrayal of the eminent French mathematician Abraham de Moivre as a gluttonous reprobate." *Calculus* was also reviewed online by Rachel Thomas in *+plus* magazine.

Happy 100th Birthday, Henri Cartan! Legendary French mathematician Henri Cartan turned 100 on July 8th. The son of Élie Cartan and a major figure in 20th century mathematics, Henri Cartan made outstanding contributions to several fields of mathematics. He led the famed "Cartan Seminar" in Paris and is also well known for his 1956 book Homological Algebra, written jointly with Sammy Eilenberg. On June 28, his birthday was celebrated in the Journée Cartan, held at his home institution, the École Normale Supérieure in Paris. In addition, the International Mathematical Union has issued a resolution congratulating Cartan. Read more about Henri Cartan in the *Notices of the AMS*: "Happy 100th Henri Cartan!" and "Interview with Henri Cartan".

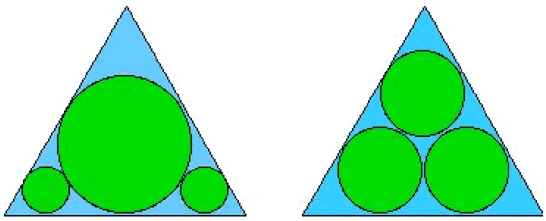
Mathematical Origami. "Cones, Curves, Shells, Towers: He Made Paper Jump to Life" is a piece by Margaret Wertheim in the June 22 2004 *New York Times*. She writes about David Huffman, a computer scientist who died in 1999, and his work on mathematically informed origami.



As the image above exemplifies, Huffman's specialty was folds along curves. He wanted to be able "to calculate precisely what structures could be folded to avoid putting strain on the paper." Huffman, who is best known for the "Huffman codes" he discovered as an MIT graduate student, is also "a legend in the tiny world of origami sekkei," or computational origami. He published only one paper on the subject but his models and his notes are being carefully studied by today's mathematical paper-folders. Wertheim quotes Robert Lang: "he anticipated a great deal of what other people have since rediscovered or are only now discovering. At least half of what he did is unlike anything I've seen." And Michael Tanner, who says that what fascinated Huffman above all else "was how the mathematics could become manifest in the paper."

Local Boy Makes Good?

- The *Zaman Daily Newspaper* (Istanbul) online edition ran a dispatch dated May 18, 2004 from Omer Oruc in Izmir. "200 Year Old Math Problem Solved" is the headline; the story tells how Mustafa Tongemen, a retired mathematics teacher, has solved "a math problem brought forward by the Italian mathematician, Malfatti, in 1803," after working on the problem two hours a day for seven years. "Malfatti's problem aims to extract three circular vertical cylinders from a triangular vertical prism made of marble, with the least material loss." In fact, there are two non-equivalent problems, initially confused by Malfatti. The problem just posed, which Conway calls problem (1), and the problem of inscribing three mutually tangent circles in a triangle, problem (2).



In an equilateral triangle, in fact, the incircle and two smaller inscribed circles give a larger area than the three mutually tangent circles. Malfatti solved problem (2), which, as is clear from the image on the Zaman website, is the problem Mustafa Tongeman actually addressed. The much more difficult problem (1) was only solved in 1992. See the *Historia Mathematica Mailing List Archive* for a summary of the history and the source of the equilateral counterexample.

- The Purdue University *Purdue News* ran "Purdue mathematician claims proof for Riemann Hypothesis" on June 8, 2004. "Louis de Branges ... has posted a 124-page paper detailing his attempt at a proof on his university Web page. While mathematicians ordinarily announce their work at formal conferences or in scientific journals, the spirited competition to prove the hypothesis – which carries a $1 million prize for whoever accomplishes it first – has encouraged de Branges to announce his work as soon as it was completed." The jury is out on this one.

- On June 22, 2004, the *Daily Star* (Beirut) ran May Habib's online story: Has local mathematician proven the '5th Postulate?' "Rachid Matta, a Lebanese mathematician and engineer, claims to have proven Euclid's parallel theorem – a theorem that has remained unproved since Euclid wrote it in 300 BC and one that many of the world's greatest minds have deemed improvable. If verified, Matta's work could have an enormous impact on mathematics because both elliptical and hyperbolic geometry – branches of geometry that violate the parallel theorem – would be discredited." We are told that Matta has spent 10 years working on this problem.

Mind, Music and Math. From the desk of *New York Times* cultural critic Edward Rothstein comes "Deciphering the Grammar of Mind, Music and Math" (June 19, 2004). The piece, under the Connections rubric, is a meditation on the nature of musical intelligence, in the light of recent work on the neural concomitants of musical perception and on the differences between the way the brain processes speech sounds and music. Rothstein emphasizes the "unique" power of music: how, even if completely unfamiliar music is heard in a locked room, where there is no reference to the outside world, "it can teach itself. Gradually, over repeated hearings [...] music shows how it is to be understood. [...] Sounds create their own context. They begin to make sense. [...] Music may be the ultimate self-revealing code." He goes on: "This is one reason that connections with mathematics are so profound. Though math requires reference to the outside world, it too proceeds by noting similarities and variations in patterns, in contemplating the structure of abstract systems, in finding the ways its elements are manipulated, connected and transformed. Mathematics is done the way music is understood." The moral of the story: "This means that music can be fully understood only by maintaining access between the room and the world: neither can be closed off."

The Pythagorean Theorem of Baseball has just been simplified. This news from the web-based *Science Daily* for March 30, 2004. The original PTOB is due to the baseball statistician and connoisseur Bill James. It estimates a team's winning chances in terms of two numbers: R_s, the number of runs scored, and R_a, the number of runs allowed. The formula is

$$P = \frac{R_s^2}{R_s^2 + R_a^2}.$$

Suppose that in 12 games your team scored 72 hits and allowed 64 hits. The PTOB gives $P = 0.56$ so it should have won 7 games and lost 5. If it won more than 7, it is "overperforming," if less, then "underperforming." This is supposed to help in predicting future performance. The simplification is due to Michael Jones and Linda Tappin (Montclair State University). Their formula runs $P = 0.5 + \beta(R_s - R_a)$, where β is a constant, "chosen to give best results" for each season, ranging between 0.0053 and 0.0078 and averaging 0.0065. For your hypothetical team the streamlined formula with $\beta = 0.0065$ gives $P = 0.55$ and leads, after round-off, to the same prediction as the PTOB. (*Science Daily*'s βs were off by a factor of 10). [Linearizing the PTOB about the equilibrium $R_s = R_a$ gives $P \sim 0.5 + (R_s - R_a)/(R_s + R_a)$.]

Originally published by the American Mathematical Society in What's New in Mathematics, a section of AMS Website, `http://www.ams.org/new-in-math`. Reprinted with permission.

An Interview with John Horton Conway

Curia, 11th of September of 2004. Breakfast with John Horton Conway[1]

Professor Conway, can we start this interview with the genesis of On Numbers and Games *(ONAG)*[2] *and the Theory of Surreal Numbers?*

Of course. I was in Cambridge at the time, and used to play Go with a colleague, who was the English champion. I'm not good at Go, but became very curious about the game. Simultaneously, I'd been thinking about sums of partizan games for a long time. I already knew that such games formed a group. I've investigated the structure of that group. I've found a sequence of games of type A, $B = 2A$, $C = 3A$, etc., it was natural to associate them with the natural numbers; another sequence satisfied $2B = A$, $2C = B$, etc., it was natural to associate it with the dyadic rationals. I've realised in this way that the group of the games had interesting subgroups, isomorphic to other well-known ones. Later on I've convinced myself that I had obtained more than this, games were indeed numbers, they were not bound to contain subgroups isomorphic to the integers and the fractions alone, there were more general ones, like the irrationals and the infinite ordinals. It took me more than one year to obtain the definition in the final form. In 1970 I successfully presented my construction at the California Institute of Technology, suddenly we realised the stuff was important, a generalization of Dedekind's construction of the real numbers, and produced many other numbers. In the following year I went to Calgary's University in order to work with Richard Guy on this matter, and ended up writing down a paper, "All games bright and beautiful", where I presented this theory. Somewhat later, in a break during a conference, I mentioned this work of mine to Donald Knuth. Shortly after, on the pretext of having discussed with his wife, the latter spent one week in an hotel, in Norway, writing *Surreal Numbers*[3], which is the first book mentioning my construction. Actually, the term "surreal numbers" was coined by Knuth. Other authors have been writing some books and papers about the subject.

How was it possible to write ONAG in one week?

Well, I was involved, with Elwin Berlekamp and Richard Guy, in the project of writing a book about impartial games, I mean, games of Nim type. Nevertheless, I was uncovering so many things about the other class of games, that I preferred to take it out of my attention, by writing a book. I locked myself in the office and only stopped for eating and sleeping. In one week the book was finished. It remained the final chapter, which I have completed two years later, and some tables, but essentially the book was finished after one week.

John H. Conway, talking about tangles and knots in Curia, September 11, 2004

This book almost gave rise to a quarrel between friends ...

It's true. I sent a letter to my colleagues Berlekamp and Guy, which started "Dear colleagues, last week I wrote a book, which you will get by mail in a couple of days ...". Even before they have got the book, I already had a letter from Berlekamp, threatening me with a lawsuit. He thought I was stealing stuff from our joint project to publish under my name alone.

That lawsuit didn't go ahead, did it?

I was so distressed with Berlekamp's reaction, that I answered him by explaining what my really intention was, and offering to withdraw my name from the joint work we were doing. In the end he set down and we have

[1] with Nuno Crato (ISEG, Lisbon) at the same table.

[2] Shortly to be published in Portuguese by Gradiva house editor.

[3] Also published in Portuguese by Gradiva, with the title *Números Surreais*.

been three good friends until today. The collaboration continued and has culminated with the publication of *Winning Ways* (WW). Our typical method of work consisted in having me going to the blackboard and explain the theory, while Guy was taking notes. Later, Richard Guy would expose on the blackboard the result of his work on the notes he had taken. That was the point where the battle would begin, I would say "Richard, that was not what I have written!" and Richard Guy would answer "Of course it isn't. This is better!". The sessions were always very vibrant. The strong point of Berlekamp consisted in the analysis of some games, the most interesting of them is Dots and Boxes, which I know makes part of your National Championship of Mathematical Games[4].

But Berlekamp's style is different from yours, how does that not show up in "WW"?

The text suffered many stylistic changes, mainly done by Richard Guy. You may note, in certain chapters, mainly in the annexes, that the language of Berlekamp shows up more.

How did the possibility of working with Richard Guy came up, as he is much older than the two other co-authors?

My friendship started with his son, Michael Guy, who had been my colleague in Cambridge and is an excellent mathematician. It was through him that I got to know Richard, who was very interested in games, and that's it. Michael was my best friend for many years, but today it is his father which I know better ... In the beginning I had a strange feeling, I was in my twenties and he was almost fifty! But everything worked out well, we have had great fun together. Once we rented a house in New Jersey, where we were both working at Bell. Richard came in the house with me and said: the largest room is for me, as I'm the oldest. There were two books in that house: a novel and a children's book with magic tricks. Richard kept the novel with him and gave me the children's book (it was there that I learned the trick I have done some days ago, at Gulbenkian[5], though I have modified it a bit). The weeks we spent together in New Jersey were very amusing, and we worked a lot on mathematical games.

We know you were born in Liverpool. How were you as a student?

I was a good student at high school, which got me into Cambridge. Here I did my first degree and the PhD.

How influent was Cambridge in your career?

The system there, with lectures and tutorials worked out very well with my fellows and me. Some teachers were really good and the ones who dedicated themselves to the tutorial classes did that with high competence, the students could learn a lot.

You did your PhD under Harold Davenport, an expert in Number Theory, but your thesis was in Logic. How did that happen?

Well, I have always been very fond of Number Theory. Davenport's lectures were excellent, I even liked his accent from the North! It was natural to choose him for supervisor. Davenport gave me a problem to think about (Waring's problem). We met every Thursday, so that I could show him my progress. There was none along the first year, and I started feeling guilty. At the end of the academic year I spent some weeks thinking on the problem and could solve it. When the classes resumed I showed him my work. He took it for one week. In the following meeting he told me: Conway, what we have here is a poor PhD thesis ... Davenport never congratulated anybody, so this was the best we could expect to hear from him. In this way the message he wanted to get through was the following: if you don't do anything more, this work will give you the PhD, but you should work more. Actually, the average time for a PhD is three years, and this happened right after the first year.

After this our Thursday's meetings were always open to discussions on nearly any subject in mathematics, philosophy, etc.

Why not presenting the very same work after the three years?

A Chinese mathematician solved the same problem and published his work meanwhile. Therefore, after this, my work was no longer worth a PhD. Another reason is that I also got interested on some problems of Logic and Set Theory which, fortunately, gave me enough material to finish my degree. Davenport also had interest in these subjects, so I could keep the same supervisor.

Your interest in Logic didn't last for long ...

In the conferences I used to go I could see many important problems in the theory being solved, but all proofs extended over hundreds of pages ... it was difficult to work. At the same time, some colleagues introduced me to some hot questions in Group Theory, and my dormant old interest for this theory waked up again. In the following fifteen years I dedicated myself professionally to this field. After this period I published, together with

[4] Pavilhão do Conhecimento, Lisbon, 26th of November of 2004. `http://ludicum.org`

[5] John Conway visited Portugal last September to present a series of lectures, at *Fundação Gulbenkian* in Lisbon, in a Summer School of the "New Talents in Mathematics" programme.

some colleagues, the Atlas of Finite Groups, in 1984. I simultaneously moved to Princeton.

This is when the packing of spheres enters the scene, right?

Right, with Sloane, I devoted myself to the sphere-packing problem and Kepler's conjecture. So, I became a geometer! All my work after this point has to do with geometry.

What is the mathematical discovery you are most proud of?

Well, the answer must be the surreal numbers. This is, nevertheless, a surprising answer, even for me, due to the short mathematical content involved. In this theory, after introducing the definitions, everything is constructed in a few pages. The simplicity of the process is amazing. The amount of work I invested in the Atlas was enormous, along many years. The reconstruction of the Monster was also a nontrivial task. If I had to prove someone I am a competent mathematician, I'd show him my production in Group Theory. The book *Packing of Spheres*, which I have written with Sloane, got very positive criticisms. One of them in particular, from Gian-Carlo Rota, was so enthusiastic, that I copied it and hanged it on the wall. It helped me during times of depression. The *Book of Numbers*[6] was also very well accepted, being translated in nine languages, I believe.

And what do you think was the most important result of last century?

Well, we have Gödel's theorems, for example, which are extremely important. We have also the recently announced proof of Poincaré's conjecture, if it proves correct. But maybe the work of Wyles, in the proof of Fermat's last theorem, is the most remarkable result. It is hard to say, because, as we all know, the relevance of a result depends more on its future than on its past ...

Interview by Jorge Nuno Silva (University of Lisbon)

John H. Conway was born in 1937 in Liverpool, and received both his BA (1959) and his PhD (1964) from Cambridge. He is one of the preeminent theorists in the study of finite groups and the mathematical study of knots, and has written over 10 books and more than 140 journal articles on a wide variety of mathematical subjects. He has also done pathbreaking work in number theory, combinatorial game theory, coding theory, the sphere-packing problem, tiling and quadratic forms.

Before joining Princeton University in 1986 as the John von Neumann Distinguished Professor of Mathematics, Conway served as professor of mathematics at Cambridge University. There, from 1962 until 1986 he was Lecturer, Reader, and Professor in Mathematics. He remains an honorary fellow of Caius College.

Among the general public he is best known for his work on combinatorial game theory, including the classic game of Nim and many others, and for the invention of the Game of Life, popularized by Martin Gardner's columns in *Scientific American* in the early 1970s. He is also one of the inventors of sprouts, as well as philosopher's football, and he developed detailed analyses of many other games and puzzles, such as the Soma cube. He also created a new system of numbers, the surreal numbers, the subject of a mathematical novel by Donald Knuth. Conway may well have the distinction of having more books, articles and Web pages devoted to his creations than any other living mathematician.

He was elected a Fellow of the Royal Society in 1981, is a Member of the American Association for the Advancement of Science, and recipient of the Berwick Prize of the London Mathematical Society (1971), Pólya Prize of the London Mathematical Society (1987), Frederic Esser Nemmers Prize (1999), Leroy P. Steele Prize of the American Mathematical Society (2000), and Joseph Priestley Award (2001). He was also awarded an Honorary DSc by the University of Liverpool in 2001.

---

[6] In Portuguese it was published by Gradiva under the title *Livro dos Números*.

José Morgado*

José Morgado was born in Pegarinhos, a small village near Favaios, in the *Douro* region, north of Portugal, on February 17, 1921.

He enrolled Elementary School at Pegarinhos and later Secondary School in Favaios. Since his family could not afford his studies at the *Liceu* of Vila Real, he did not register for the third year. Some of his teachers knowing that he was "*an outstanding student, not only in one or two disciplines, but in all of them*", personally guaranteed the pursuit of his studies in Vila Real.

José Morgado made his university studies at the Faculty of Sciences of the University of Porto, where he completed his degree in Mathematical Sciences in 1944. Although it was wartime in Europe, the period he was a student at Porto was a singular moment in the history of Science and Mathematics in Portugal. By that time, a scientific revolution in Portugal was in process under the leadership of António Monteiro, who had recently returned to Portugal after his doctoral studies in Paris. Ruy Luís Gomes, Bento de Jesus Caraça, Aureliano Mira Fernandes, Manuel Zaluar Nunes, José da Silva Paulo, Manuel Valadares, Hugo Ribeiro and A. Pereira Gomes were among the participants of this scientific breakthrough. It was in the context of this truly outstanding generation in the history of Mathematics in Portugal that José Morgado studied and lived in Porto. Those were years full of activity and enthusiasm brought to an end by the repression of the *Estado Novo*. As José Morgado puts it [13]:

"*These activities were undertaken at the Physics Laboratory of the Faculty of Sciences of Lisbon, but in 1947 [...] fascism started a major offensive against the scientific workers and the universities.*"

Meanwhile, a long interim occurred in the life of José Morgado. According to his Curriculum Vitæ, "*between his removal from the official teaching in 1947 and his departure to Brasil in 1960, he survived by giving private lessons. First in Lisbon, he taught Calculus, Infinitesimal Analysis, Algebra, Descriptive Geometry, Projective Geometry and other mathematical disciplines, to students of the* Instituto Superior de Agronomia*,* Instituto Superior Técnico *and* Faculdade de Ciências de Lisboa*. After October 1958 he moved to Porto, and gave private lessons to students of the Faculty of Economics and the Faculty of Sciences of Porto.*" It was perhaps during this long experience of proximity with students that he developed his remarkable pedagogical qualities that made him a very popular tutor, and, later on, an exceptional teacher. Those 13 years of private teaching were interrupted by some periods in prison, due to his political activities. It was in prison that he wrote the second work in his list of publications [6], the first volume of a book about lattices, published by the *Junta de Investigação Matemática* in 1956.

José Morgado

Six years later, already in Recife, Brasil, he published the notes [11], from a series of lectures in Lattice Theory. These were presented at a seminar at the University of Ceará, and contain some interesting and original results.

As a teacher, José Morgado distinguished himself through his dedication, clarity and constant dialogue with his students. His high motivation and dynamism characterized his lectures.

*This is an adaptation of the article [J. Almeida, A. Machiavelo, *José Morgado: in memoriam*, Boletim da SPM **50** (2004), 1-18]. The editors thank the authors for this adaptation and the *Boletim da SPM* for permission to include it here.

In Brasil, he played a preeminent role, together with Ruy Luís Gomes. They reinforced mathematical activities at the Federal University of Pernambuco, Recife, where he worked for 14 years. After the Revolution of April 25, 1974, he returned to Portugal. On October 4, 1974, he was reintegrated as an Assistant Professor at the *Instituto Superior de Agronomia*. On November 7, he was nominated for 2 years, as an invited full professor at the Pure Mathematics Group of the Faculty of Sciences of the University of Porto. On July 24, 1979, he got a permanent position there as a full professor. On February 1991, he retired and became a "Professor Jubilado" at the same university, where, in the following seven years, he continued to teach several optional courses.

José Morgado died in Porto on October 8, 2003.

José Morgado

José Morgado published at least 124 papers on research, history and popularization of Mathematics; 89 have been referred in the *Zentralblatt für Mathematik*.

The scientific work of José Morgado falls, essentially, in 3 areas: Lattice Theory, Group Theory and Number Theory. Next, we briefly mention his earliest work in Lattice Theory, and his later research in Number Theory.

The work of José Morgado in Lattice Theory includes 2 books, published in 1956 and 1962, and 25 papers from 1960 to 1966. We decided to choose his first articles because of the deep and ingenious way which he used to attack the problems. The first paper of José Morgado on Lattice Theory [7] gives a characterization of partially ordered sets P whose set $\Phi(P)$ of closure operators, ordered by $\varphi \leq \psi$ if $\varphi(x) \leq \psi(x)$ for all $x \in P$, is a complete lattice.[1] Ward [16] had proved that $\Phi(P)$ is a complete lattice whenever the same holds for P. Morgado showed that $\Phi(P)$ is complete if and only if, for every element $x \in P$, and every subset Σ of $\Phi(P)$, the set of the elements of P above x which are closed for some element of Σ admits a "quasi-infimum"[2] with respect to x and, moreover, there is some element above x which is essentially closed. Morgado was led to this study following a paper by Monteiro and Ribeiro [5] in which the authors aim to extend the notion of topological space to partially ordered sets, which play the role of the power set of a given set, using closure operators.

In the papers [9, 8], Morgado introduced the notion of *quasi-isomorphism* between complete lattices: this is a bijection $q : L \to M$ that takes the maximum of L to the maximum of M and such that, for all non-empty subsets $S \subseteq L$ and $T \subseteq M$, there exist nonempty sets $S' \subseteq S$ and $T' \subseteq T$ such that $q(\bigwedge S) = \bigwedge q(S')$ and $q^{-1}(\bigwedge T) = \bigwedge q^{-1}(T')$. In [8], he established that the group of automorphisms of $\Phi(L)$ is isomorphic to the group of quasi-automorphisms of L. In [9], he proved that $\Phi(L)$ and $\Phi(M)$ are isomorphic if and only if L and M are quasi-isomorphic. In [10], he showed, among other results, that, if L is a finite lattice, then a permutation q of L is a quasi-isomorphism if and only if q fixes the maximum of L and

$$q(x \wedge y) = q(x) \wedge q(y) \tag{1}$$

whenever x and y are not comparable;

he also observed that, in the finite case, a bijection $q : L \to M$ taking the maximum of L to the maximum of M and satisfying condition (1) is not necessarily a quasi-isomorphism.

The work of José Morgado in Number Theory can be naturally divided in 3 periods:

- a first period, from 1962 to 1964, when he published papers about arithmetic functions, where he studies unitary analogs of several arithmetic functions, these analogs being defined on the basis of the unitary divisors of a number;[3]

[1] Given a partially ordered set P, a function $\varphi : P \to P$ is said to be a *closure operator* if, for all $x, y \in P$, $x \leq \varphi(x)$, $\varphi(x) \leq \varphi(y)$ whenever $x \leq y$, and $\varphi(\varphi(x)) = \varphi(x)$. An element x is said to be *closed for* φ if $\varphi(x) = x$ and it is said to be *essentially closed* if it is closed for all $\varphi \in \Phi(P)$.

[2] Given a non-empty set X of elements of P above x, a *quasi-infimum of* X *with respect to* x is the largest element y such that $x \leq y \leq X$ and, for every non-empty subset S of X and every s such that $x \leq s \leq S$, there exists z such that $\{y, s\} \leq z \leq S$. Note that, given subsets $T, U \subseteq P$, we write $T \leq U$, if $t \leq u$ for all $t \in T$ and $u \in U$.

[3] We recall that d is said to be an unitary divisor of n if $gcd(d, n/d) = 1$.

- a second period, from 1972 to 1977, when he introduced some results about generalizations of Euler's theorem[4], starting from the study of the regular integers (in the sense of von Neumann) mod n;
- a third period, from 1982 to 1995, when he published a series of articles about Fibonacci numbers and its generalizations, as well as about finite sets of integers in arithmetic progression and satisfying certain properties; all of this motivated by a problem going back to an observation of Fermat, suggested by a problem from the *Arithmetica* of Diophantus, and subsequently studied by L. Euler.

The subject of the papers about Fibonacci numbers and its generalizations, related to the third period, can be described in a somewhat more concise way if we introduce the following notation (see [1, 2]): a finite set $S \subseteq \mathbb{N}$ is said to be a *P_t-set with m elements*, or, when $t = 1$, a *Diophantine m-tuple*, if for every pair x, y of distinct elements of S, $xy + t$ is a perfect square. In the question of Diophantus studied by Fermat, one is asked to construct what is nowadays known as a *Diophantine quadruple*. Starting from the numbers 1, 3 and 8, Fermat shows how to search for a fourth element, finding the number 120. In 1969, A. Baker[5] and H. Davenport proved that $\{1, 3, 8, 120\}$ is the unique Diophantine quadruple containing the set $\{1, 3, 8\}$. In 1977, Hoggatt and Bergum showed that, for all $n \in \mathbb{N}$, the set

$$\{F_{2n}, F_{2n+2}, F_{2n+4}, 4F_{2n+1}F_{2n+2}F_{2n+3}\},$$

where $(F_n)_n$ denotes the Fibonacci sequence, is a Diophantine quadruple. Morgado generalized this result, showing in [12] that the set

$$\{F_n, F_{n+2r}, F_{n+4r}, 4F_{n+r}F_{n+2r}F_{n+3r}\},$$

is a P_t-set for some t, which can itself be expressed by means of certain Fibonacci numbers. On the basis of this work of Morgado, A. Dujella gives in [3] more examples of P_t-sets with 4 elements consisting of Fibonacci and Lucas numbers. In [15], G. Udrea generalized the results of Morgado to sets formed by Chebyshev polynomials of the second kind, and in [14] Morgado generalized these results to polynomials including as particular cases Chebyshev polynomials of the second kind and of the first kind, thus proving a theorem that includes all previous results of the same type. Morgado also found some impressive identities between Fibonacci numbers, as for example:

$$F_nF_{n+1}F_{n+2}F_{n+4}F_{n+5}F_{n+6} + (F_{n+2} + F_{n+4})^2 = \left(\frac{F_{n+1}F_{n+2}F_{n+6} + F_nF_{n+4}F_{n+5}}{2}\right)^2,$$

which holds for all $n \in \mathbb{N}_0$.

It is worth noticing that the question of the existence of a Diophantine quintuple is still an open problem, although it was recently proved by A. Dujella, in [4], that there cannot be more than a finite number of such quintuples and that there are no Diophantine sextuples.

[4] namely that $a^{\varphi(n)} \equiv 1 \pmod n$ holds whenever $gcd(a, n) = 1$.

[5] who was awarded the Fields medal in 1970 for his work on Diophantine equations and transcendental numbers.

Bibliography

[1] George Berzsenyi, *Adventures among P_t-sets*, Quantum Magazine (1991), Mar/Apr issue.

[2] Andrej Dujella, *Diophantine m-tuples*, web page: `http://www.math.hr/~duje/dtuples.html`.

[3] Andrej Dujella, *Diophantine quadruples for squares of Fibonacci and Lucas numbers*, Portugal. Math. **52** (1995), 305–318.

[4] Andrej Dujella, *There are only finitely many Diophantine quintuples*, J. Reine Angew. Math. **566** (2004), 183–214.

[5] António Monteiro and Hugo Ribeiro, *L'opération de fermeture et ses invariants dans les systèmes partiellement ordonnés*, Portugal. Math. **3** (1942), 171–184.

[6] José Morgado, *Reticulados (Cap. I - sistemas parcialmente ordenados)*, 1956, Junta de Investigação Matemática, Porto.

[7] José Morgado, *Some results on closure operators of partially ordered sets*, Portugal. Math. **19** (1960), 101–139.

[8] José Morgado, *Note on the automorphisms of the lattice of closure operators of a complete lattice*, Nederl. Akad. Wetensch. Proc. Ser. A 64 = Indag. Math. **23** (1961), 211–218.

[9] José Morgado, *Quasi-isomorphisms between complete lattices*, Portugal. Math. **20** (1961), 17–31.

[10] José Morgado, *Some remarks on quasi-isomorphisms between finite lattices*, Portugal. Math. **20** (1961), 137–145.

[11] José Morgado, *Introdução à Teoria dos Reticulados*, Textos de Matemática, no. 10 e 11, Instituto de Matemática, Recife, 1962.

[12] José Morgado, *Generalization of a result of Hoggatt and Bergum on Fibonacci numbers*, Portugal. Math. **42** (1983/84), 441–445.

[13] José Morgado, *Hugo Baptista Ribeiro, matemático português que só pôde ensinar numa Universidade portuguesa depois do 25 de Abril*, Bol. Soc. Port. Mat. **12** (1989), 31–42.

[14] José Morgado, *Note on the Chebyshev polynomials and applications to the Fibonacci numbers*, Portugal. Math. **52** (1995), 363–378.

[15] Gheorghe Udrea, *A problem of Diophantos-Fermat and Chebyshev polynomials of the second kind*, Portugal. Math. **52** (1995), 301–304.

[16] Morgan Ward, *The closure operators of a lattice*, Ann. of Math. (2) **43** (1942), 191–196.

Jorge Almeida and António Machiavelo

Centro de Matemática da Universidade do Porto

Editors: António Caetano (acaetano@mat.ua.pt)
Jorge Picado (picado@mat.uc.pt).

Address: Departamento de Matemática, Universidade de Coimbra, 3001-454 Coimbra, Portugal.

The CIM Bulletin is published twice a year. Material intended for publication should be sent to one of the editors. The bulletin is available at http://www.cim.pt.

The CIM acknowledges the support of: Fundação para a Ciência e a Tecnologia, Apoio do Programa Operacional Ciência, Tecnologia, Inovação do Quadro Comunitário de Apoio III; Departamento de Matemática da Universidade de Coimbra; Departamento de Matemática da Universidade de Aveiro; Centro de Matemática da Universidade de Coimbra.