Mathematical Needs for Behavioral Modeling of Telecommunication Circuits and Systems

(First Part – Model Formulation)

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Presentation Outline

1. Introduction to Behavioral Modeling

2. General Nonlinear Behavior of Wireless Systems

3. Basics on System Identification Theory

4. Nonlinear Behavioral Modeling of Wireless Systems

5. Conclusions

1. Introduction to Behavioral Modeling

Nonlinear Modeling of RF Nonlinear Devices

- Models are necessary for CAD of microwave circuits and systems

- Nonlinear CAD of Circuits is already in a mature state;
  However, it can not support large and heterogeneous circuits
  such as complete telecommunication systems

⬇

Reduced Complexity ➔ Higher Levels of Hierarchical Description

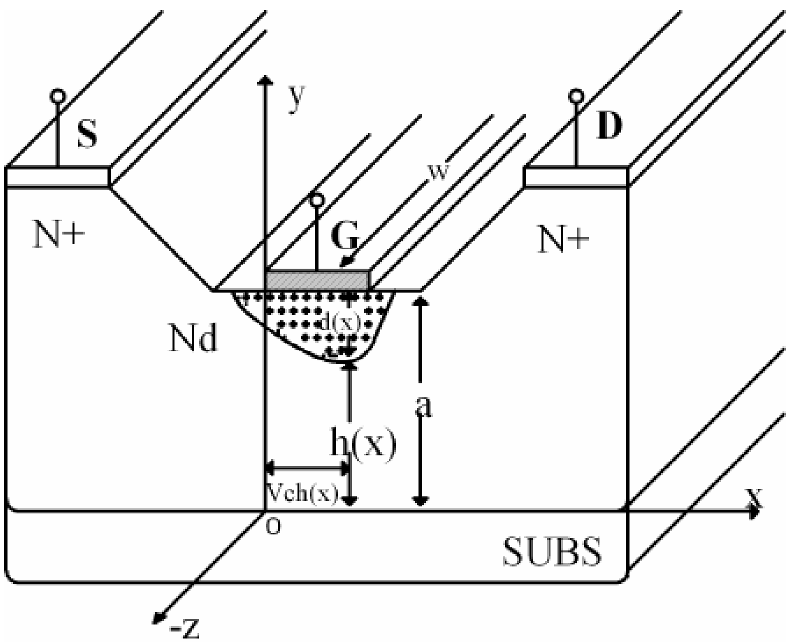
⬇

Circuit Level CAD/Modeling ➔ System Level CAD/Modeling

1. Introduction to Behavioral Modeling

Physical Modeling vs. Behavioral Modeling

Physics Based Modeling:



$$\nabla^2 \psi = -\frac{q}{\varepsilon}\left[p + N_d^+ - n - N_a^-\right]$$

$$\frac{\partial n}{\partial t} = -\mu_n \nabla^2 \psi n - \mu_n \nabla \psi \nabla n + D_n \nabla^2 n - G_n + R_n$$

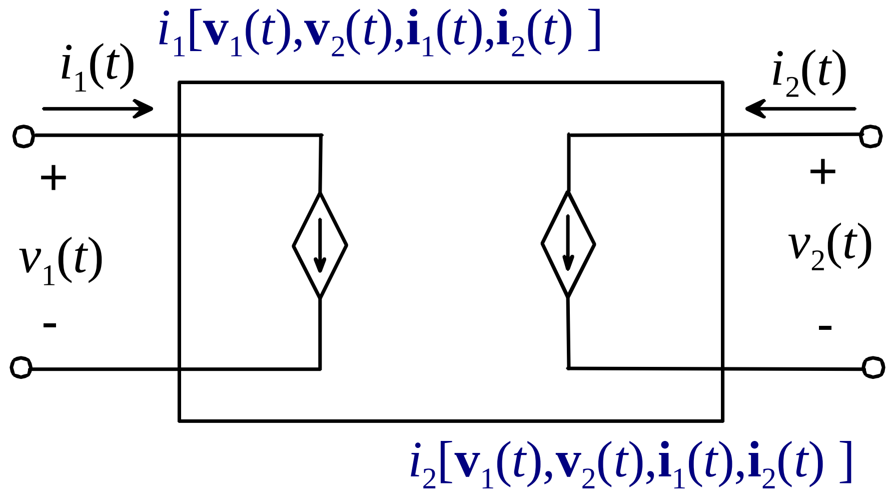
$$\frac{\partial p}{\partial t} = -\mu_p \nabla^2 \psi p - \mu_p \nabla \psi \nabla p + D_p \nabla^2 p - G_p + R_p$$

$$\psi(x,y)\big|_S = 0, \quad \psi(x,y)\big|_G = V_{GS}, \quad \psi(x,y)\big|_D = V_{DS}$$

1. Introduction to Behavioral Modeling

Physical Modeling vs. Behavioral Modeling

Empirical, Behavioral or Black Box Modeling:



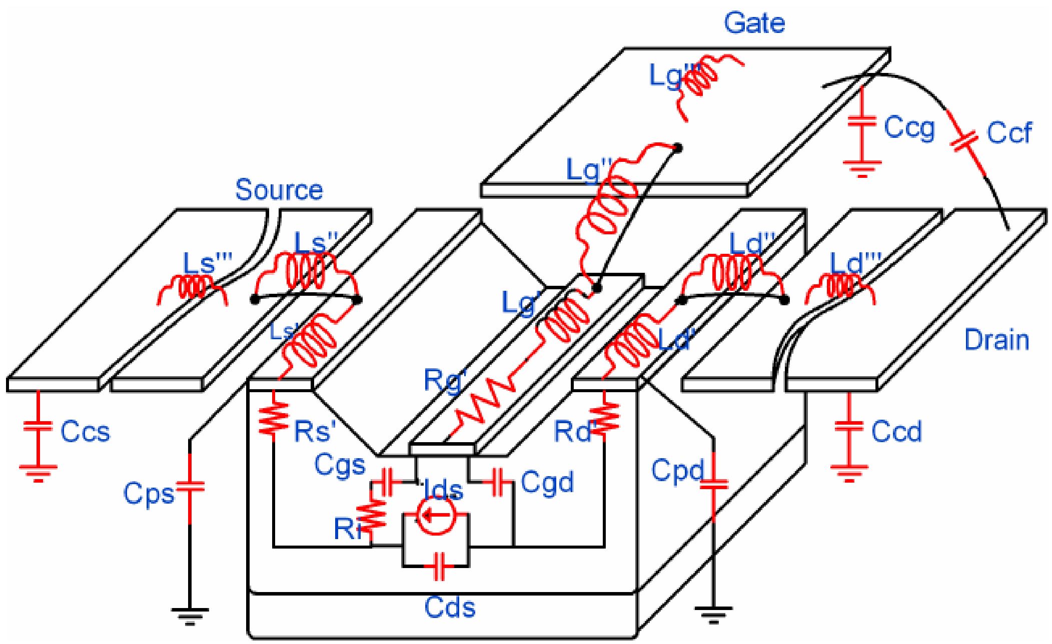
where $$\mathbf{v}_{1,2}(t) = \left[v_{1,2}(t),\ \dot{v}_{1,2}(t),\ \ddot{v}_{1,2}(t),\ \ldots\right]$$

$$\mathbf{i}_{1,2}(t) = \left[i_{1,2}(t),\ \dot{i}_{1,2}(t),\ \ddot{i}_{1,2}(t),\ \ldots\right]$$

1. Introduction to Behavioral Modeling

Physical Modeling vs. Behavioral Modeling

Equivalent Circuit Modeling can be seen as Behavioral Modeling using a-priori physics knowledge of the topology:



1. Introduction to Behavioral Modeling

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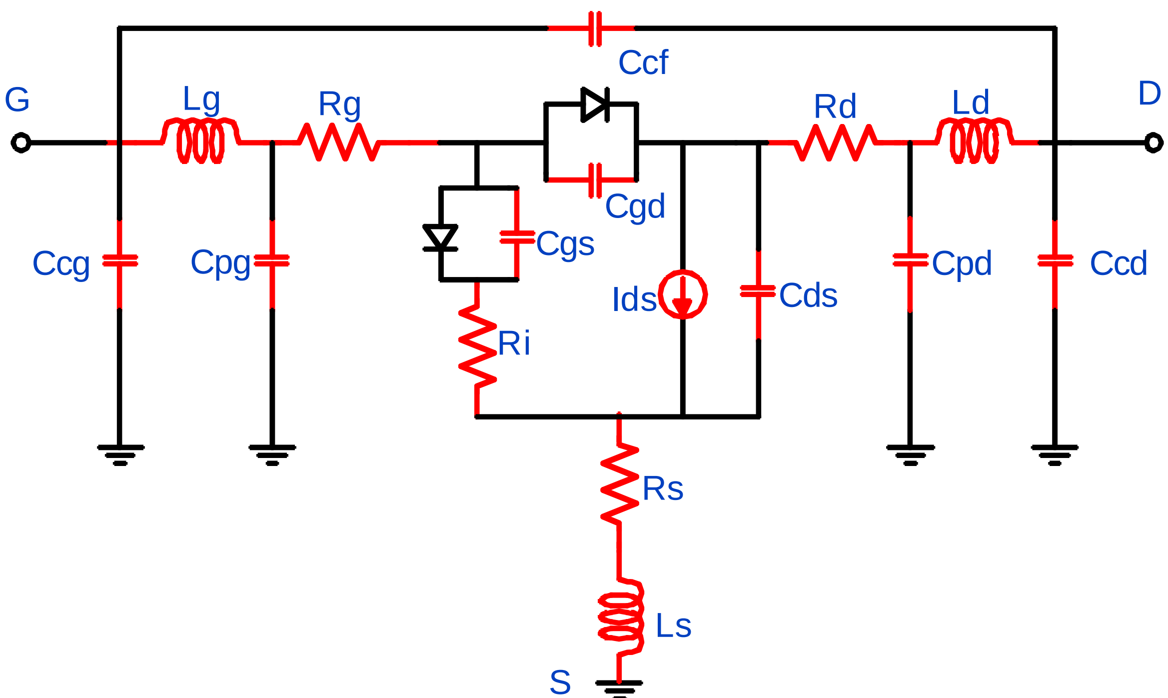
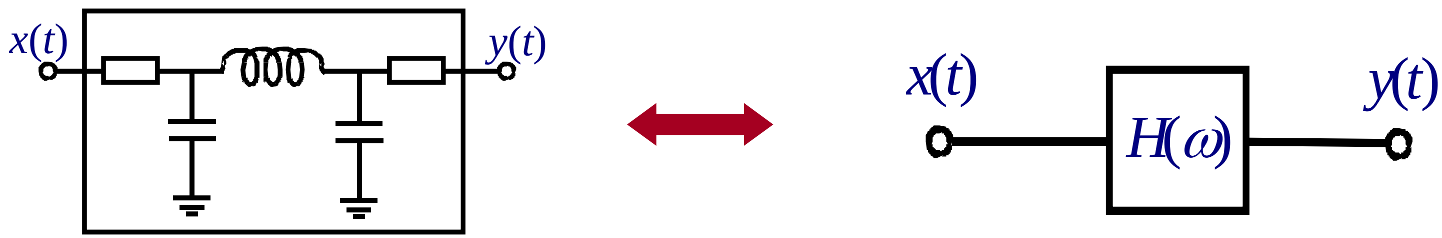


1. Introduction to Behavioral Modeling

Physical Modeling vs. Behavioral Modeling



Physical Models can be deduced from the physics of the device

Behavioral Models are ***Empirical*** in Nature:

➔ They rely on input-output (***Behavioral***) observations

➔ They need to compensate the lack of knowledge of device constitution (***Black-Box Models***) with observation data

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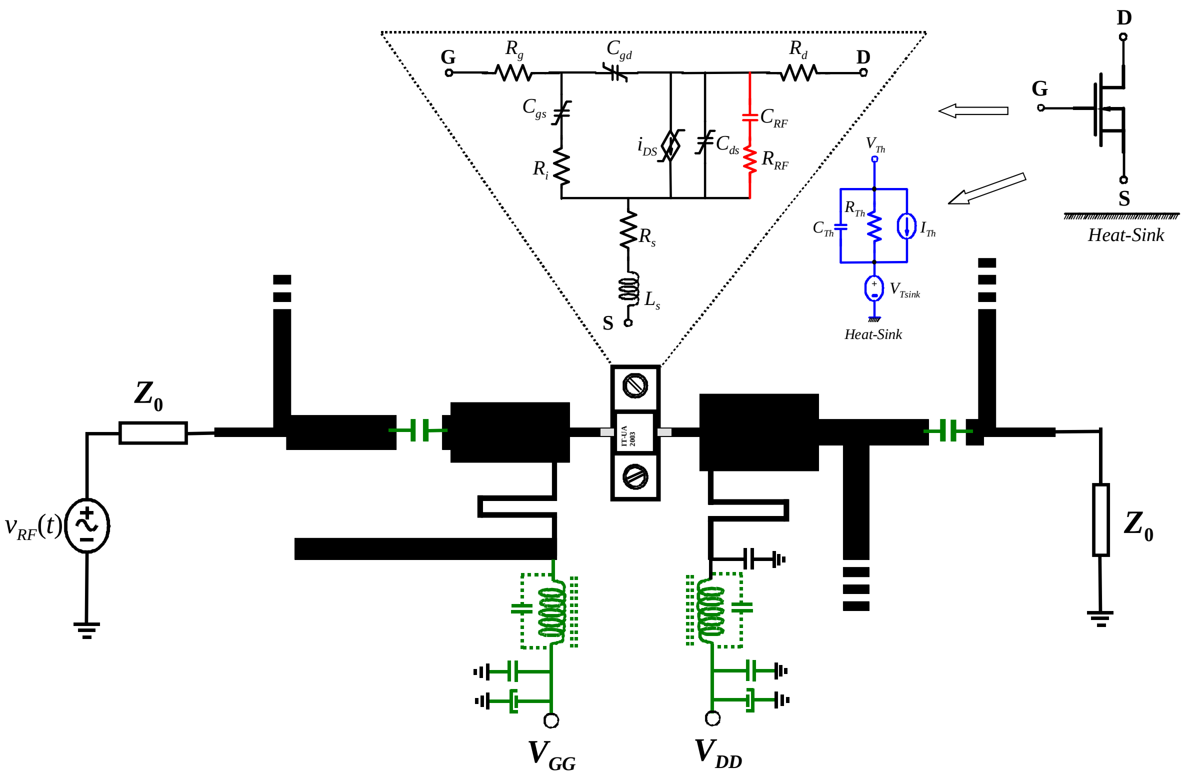
2. General Nonlinear Behavior of Wireless Systems

General Distortion Behavior of a Microwave PA

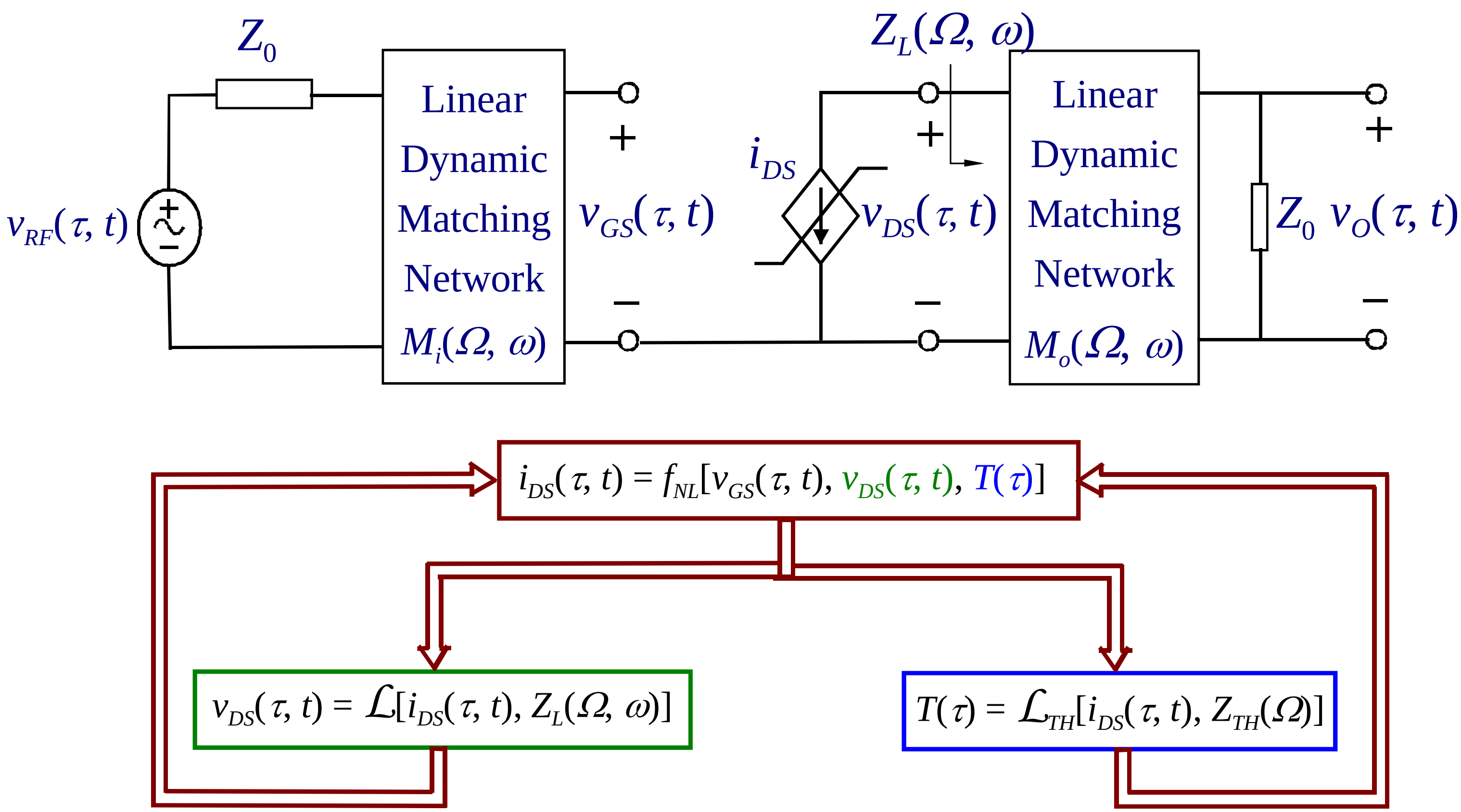
A wireless power amplifier includes, not only short-term (RF) memory effects, as long-term (envelope) memory caused by *bias circuitry*, *charge-carrier traps* and *self-heating*.



2. General Nonlinear Behavior of Wireless Systems

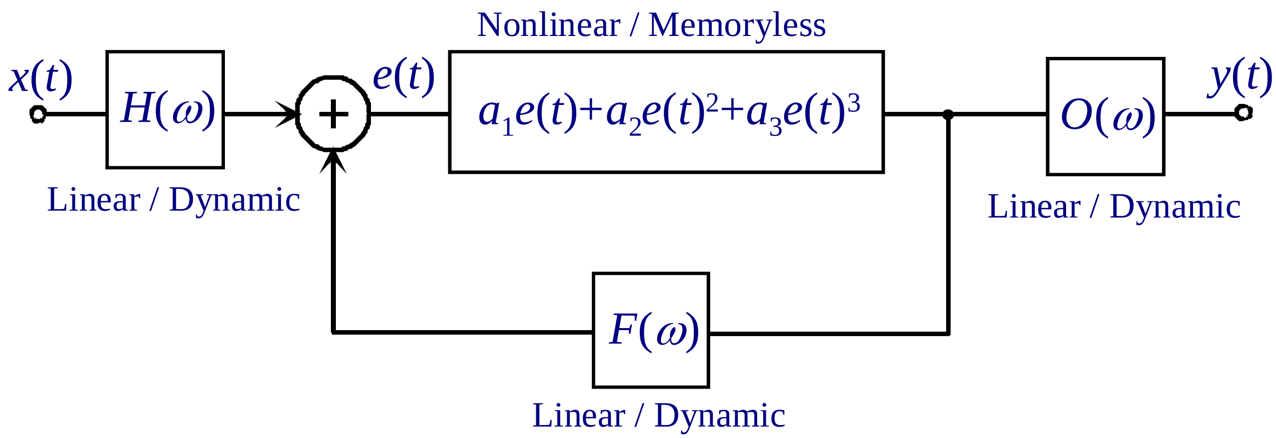
General Distortion Behavior of a Microwave PA

These disparate origins of memory effects interact to create a very complex dynamic feedback behavior.



2. General Nonlinear Behavior of Wireless Systems

Nonlinear Dynamic Model of a Feedback Wireless System



$$S_1(\omega)=H(\omega)\frac{a_1}{D(\omega)}O(\omega)$$

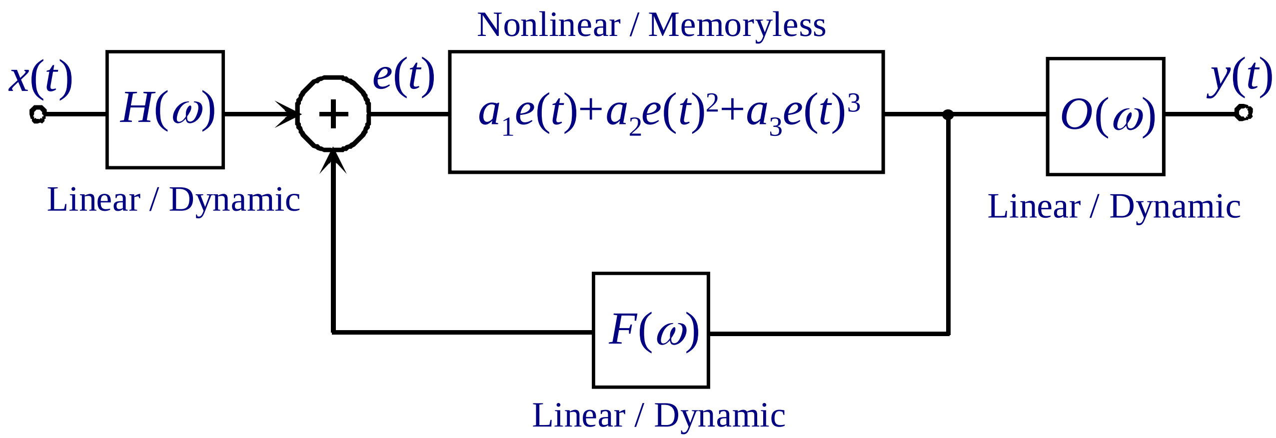
$$D(\omega)=1-a_1F(\omega)$$

$$S_3(\omega_1,\omega_2,-\omega_3)=\frac{H(\omega_1)\,H(\omega_2)\,H^*(\omega_3)}{D(\omega_1)\,D(\omega_2)\,D^*(\omega_3)}\,\frac{O(\omega_1+\omega_2-\omega_3)}{D(\omega_1+\omega_2-\omega_3)}\left\{a_3+\frac{2}{3}a_2^{\,2}\left[\frac{F(\omega_1+\omega_2)}{D(\omega_1+\omega_2)}+\frac{F(\omega_1-\omega_3)}{D(\omega_1-\omega_3)}+\frac{F(\omega_2-\omega_3)}{D(\omega_2-\omega_3)}\right]\right\}$$

A wireless system will then present *Linear* and *Nonlinear Dynamic Effects*.

2. General Nonlinear Behavior of Wireless Systems

Nonlinear Dynamic Model of a Feedback Wireless System



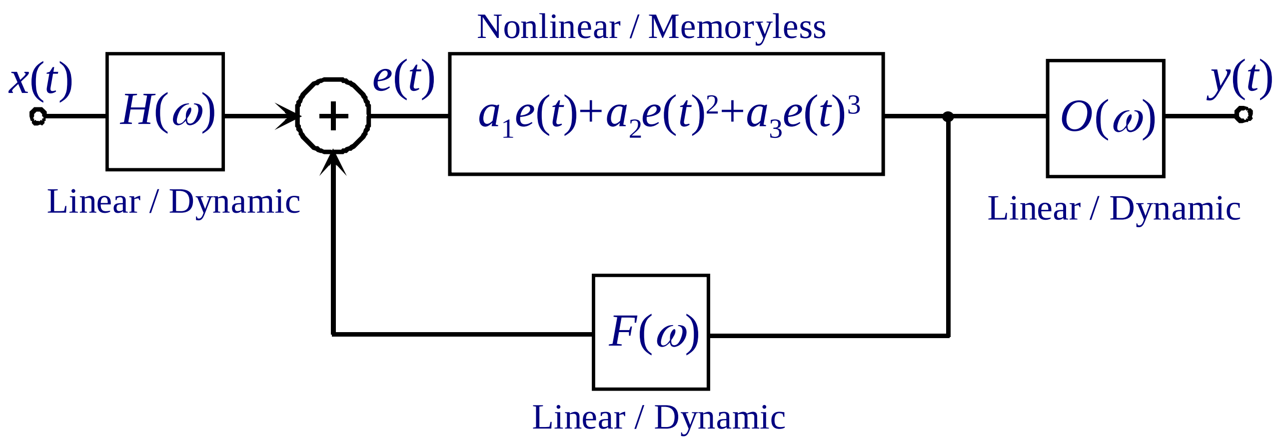
So, nonlinear dynamics cannot be represented by any Filter-Nonlinearity (Wiener model) …

$$S_3(\omega_1,\omega_2,-\omega_3)=\frac{H(\omega_1)H(\omega_2)H^*(\omega_3)}{D(\omega_1)D(\omega_2)D^*(\omega_3)}\frac{O(\omega_1+\omega_2-\omega_3)}{D(\omega_1+\omega_2-\omega_3)}$$

$$\left\{a_3+\frac{2}{3}a_2^{\,2}\left[\frac{F(\omega_1+\omega_2)}{D(\omega_1+\omega_2)}+\frac{F(\omega_1-\omega_3)}{D(\omega_1-\omega_3)}+\frac{F(\omega_2-\omega_3)}{D(\omega_2-\omega_3)}\right]\right\}$$

$$S_3(\omega)=\Gamma(\omega)\left|\Gamma(\omega)\right|^2\ a_3$$

Nonlinear Dynamic Model of a Feedback Wireless System



… nor any Nonlinearity-Filter (Hammerstein model) cascade !

$$S_3(\omega_1,\omega_2,-\omega_3)=\frac{H(\omega_1)H(\omega_2)H^*(\omega_3)}{D(\omega_1)D(\omega_2)D^*(\omega_3)}\frac{O(\omega_1+\omega_2-\omega_3)}{D(\omega_1+\omega_2-\omega_3)}$$

$$\left\{a_3+\frac{2}{3}a_2{}^2\left[\frac{F(\omega_1+\omega_2)}{D(\omega_1+\omega_2)}+\frac{F(\omega_1-\omega_3)}{D(\omega_1-\omega_3)}+\frac{F(\omega_2-\omega_3)}{D(\omega_2-\omega_3)}\right]\right\}$$

$$S_3(\omega)=a_3\Gamma(\omega)$$

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3. Basics on System Identification Theory

The Behavioral Modeling Problem

Is it possible to produce a behavioral model with predictive capabilities (i.e. describing the whole system), from a finite set of observations ?

Yes, it is !!

Contrary to some a-priori intuition and many heuristic behavioral models, system identification theory shows that our wireless system can be described by a mathematical operator (a function of functions) that maps a function of time $x(t)$ (the input signal) onto another function of time $y(t)$ (the output):

$$x(t) \xrightarrow{S[f(t)]} y(t) = S[x(t)]$$

The Behavioral Modeling Problem

This input-output map can be represented by the following forced nonlinear differential equation:

$$f\left[\frac{d^{p}\,y(t)}{d\,t^{p}},\ldots,\frac{d\,y(t)}{d\,t},y(t),\frac{d^{r}\,x(t)}{d\,t^{r}},\ldots,\frac{d\,x(t)}{d\,t},x(t)\right]=0$$

3. Basics on System Identification Theory

Recursive and Non-Recursive Models

In a digital computer, time is a succession of uniform time samples:

$$x(t) \rightarrow x(s) \quad \longrightarrow \quad y(t) \rightarrow y(s)$$

so, our nonlinear differential equation becomes a difference equation.

The solution of this nonlinear difference equation can be expressed in the following *Recursive Form* (*Nonlinear IIR Filter*):

$$y(s) = f_R\left[y(s-1),\ldots,y(s-p),\ldots,x(s),x(s-1),\ldots,x(s-r),\ldots\right]$$

Recursive and Non-Recursive Models

If the system is causal, stable and of fading memory it can also be represented by a *Non-Recursive*, or *Direct, Form*, where the relevant input past is restricted to $q \in \{0,1,2,\ldots,Q\}$*, the system's Memory Span (Nonlinear FIR Filter):*

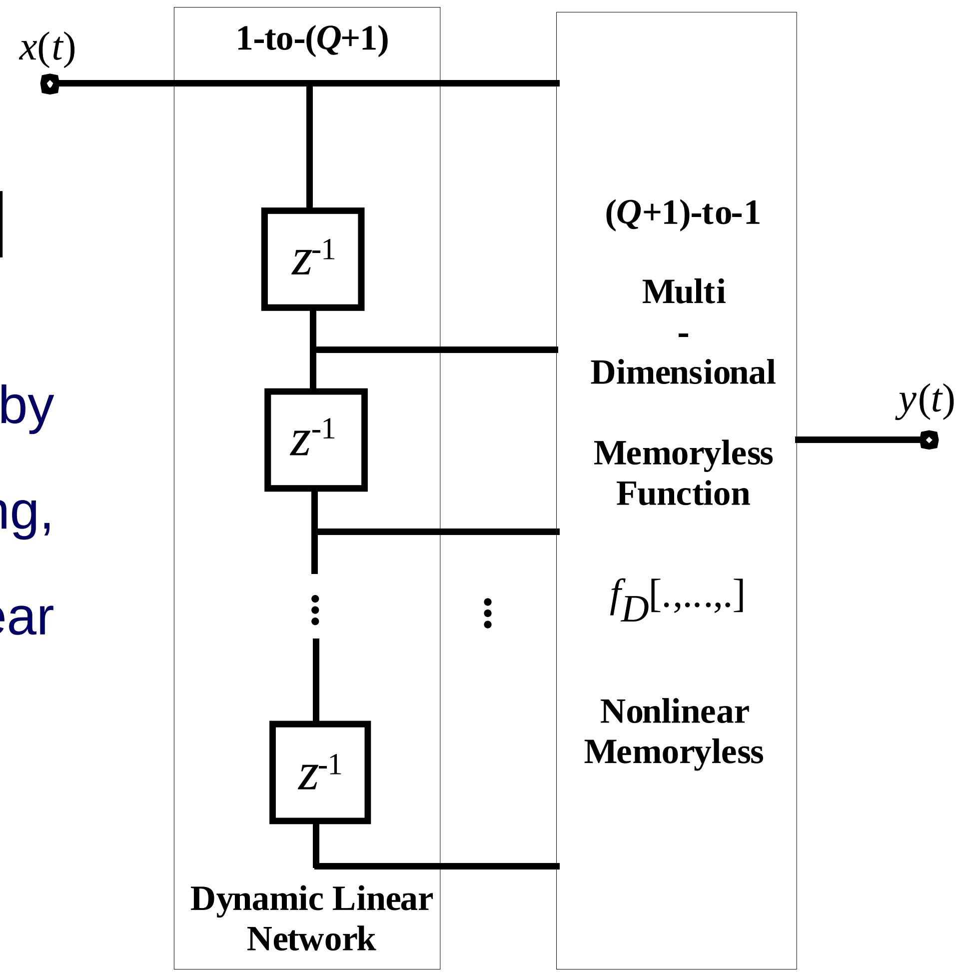
$$y(s) = f_D\left[x(s), x(s-1), \ldots, x(s-Q)\right]$$

3. Basics on System Identification Theory

Recursive and Non-Recursive Models

$$y(s) = f_D\left[x(s), x(s-1), \ldots, x(s-Q)\right]$$

The $S[x(t)]$ operator is replaced by a 1-to-$(Q+1)$ linear mapping, followed by a $(Q+1)$-to-1 nonlinear (static) function.



3. Basics on System Identification Theory

Polynomial Filters and Artificial Neural Networks

The multi-dimensional functions $f_R(.)$ and $f_D(.)$, have been expressed in two different forms, leading to:

➔ $f_R(.)$ and $f_D(.)$ are approximated by Polynomials ➔ *Polynomial Filters*

➔ $f_R(.)$ and $f_D(.)$ are approximated by *Artificial Neural Networks*

Nonlinear IIR Filters

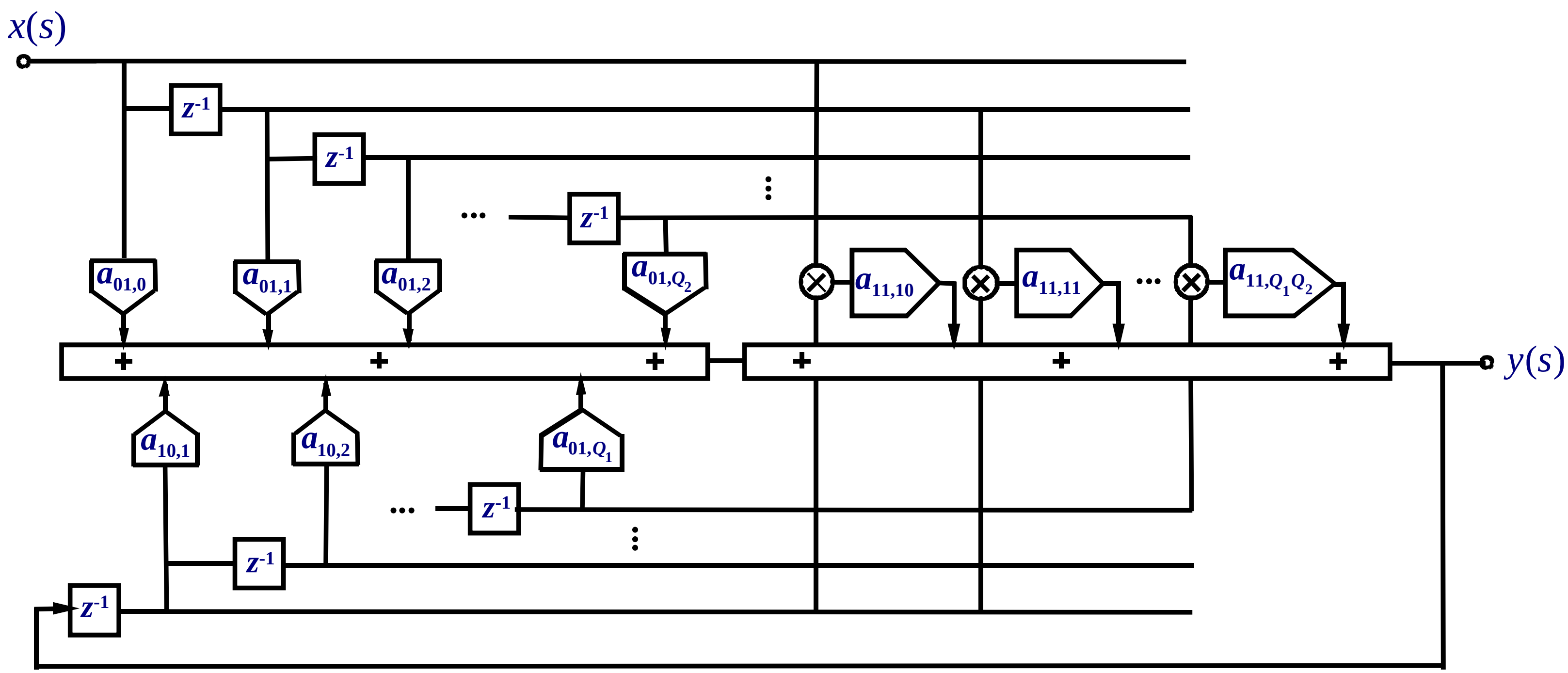
As a *Recursive Polynomial Filter*, $f_R(.)$ is replaced by a multi-dimensional polynomial approximation:

$$
\begin{aligned}
y(s) &= P_R\left[y(s-Q_1),\dots,y(s-1),x(s-Q_2),\dots,x(s-1),x(s)\right] \\
&= \sum_{q=1}^{Q_1} a_{10}(q)y(s-q) + \sum_{q=0}^{Q_2} a_{01}(q)x(s-q) + \sum_{q_1=1}^{Q_1}\sum_{q_2=1}^{Q_1} a_{20}(q_1,q_2)y(s-q_1)y(s-q_2) \\
&\quad + \sum_{q_1=1}^{Q_1}\sum_{q_2=0}^{Q_2} a_{11}(q_1,q_2)y(s-q_1)x(s-q_2) + \sum_{q_1=0}^{Q_2}\sum_{q_2=0}^{Q_2} a_{02}(q_1,q_2)x(s-q_1)x(s-q_2) \\
&\quad + \dots \\
&\quad + \sum_{q_1=1}^{Q_1}\cdots\sum_{q_{N_1}=1}^{Q_1}\sum_{q_{N_1+1}=0}^{Q_2}\cdots\sum_{q_{N_1+N_2}=0}^{Q_2} a_{N_1N_2}(q_1,\dots,q_{N_1},q_{N_1+1},\dots,q_{N_1+N_2}) \\
&\qquad\qquad .y(s-q_1)\dots y(s-q_{N_1})\,x(s-q_{N_1+1})\dots x(s-q_{N_1+N_2})
\end{aligned}
$$

Nonlinear IIR Filters

E.g., a *Recursive Bilinear Filter* (*2nd order IIR*) is implemented as:

$$y(s)=\sum_{q=1}^{Q_2} a_{01}(q)x(s-q)+\sum_{q=0}^{Q_1} a_{10}(q)y(s-q)+\sum_{q_1=1}^{Q_1}\sum_{q_2=0}^{Q_2} a_{11}(q_1,q_2)y(s-q_1)x(s-q_2)$$



3. Basics on System Identification Theory

Nonlinear FIR Filters

As a *Direct Polynomial Filter,* $f_D(.)$ is replaced by a multi-dimensional polynomial approximation:

$$
\begin{aligned}
y(s) &= P_D\left[x(s), x(s-1), \ldots, x(s-Q_2)\right] \\
&= \sum_{q=0}^{Q_2} a_1(q)x(s-q) + \sum_{q_1=0}^{Q_2} \sum_{q_2=0}^{Q_2} a_2(q_1,q_2)x(s-q_1)x(s-q_2) \\
&\quad + \ldots \\
&\quad + \sum_{q_1=0}^{Q_2} \cdots \sum_{q_N=0}^{Q_2} a_N(q_1,\ldots,q_N).x(s-q_1)\ldots x(s-q_N)
\end{aligned}
$$

3. Basics on System Identification Theory

Nonlinear FIR Filters

If $f_D(.)$ is approximated by a Taylor series, then this nonlinear FIR filter is known as the *Volterra Series* or *Volterra Filter*

Optimal approximation (in uniform error sense) near the expansion point, provides:

- ➔ Good modeling properties of small-signal (or mildly nonlinear) regimes
- ➔ Catastrophic degradation under strong nonlinear operation

3. Basics on System Identification Theory

Nonlinear FIR Filters

But $f_D(.)$ can also be approximated by any other *Multi-Dimensional* (*Orthogonal*) *Polynomial*, generating a *General Nonlinear FIR Filter*

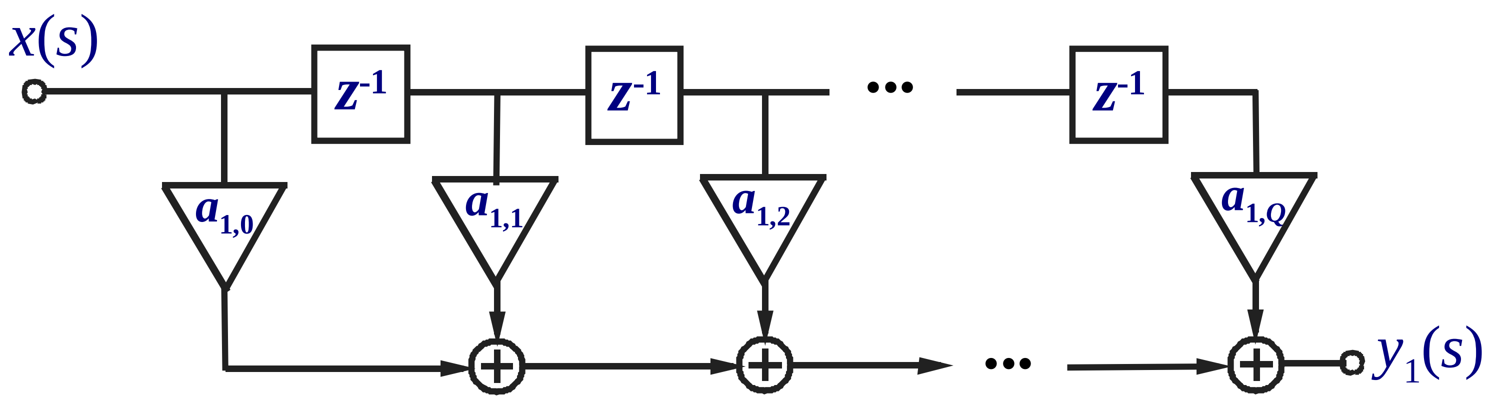
Optimal approximation (in mean square error sense) in the vicinity of a certain operating power level, and for a particular type of input, provides:

- ➔ Good modeling properties of strong nonlinear regimes
- ➔ As optimum as the input signal statistics are close to those of the stimulus for which the polynomial is orthogonal

Nonlinear FIR Filters

E.g., a *Direct 1st Order Polynomial Filter* (*Linear FIR*) is implemented as:

$$y_1(s) = \sum_{q=0}^{Q_2} a_1(q) x(s-q)$$

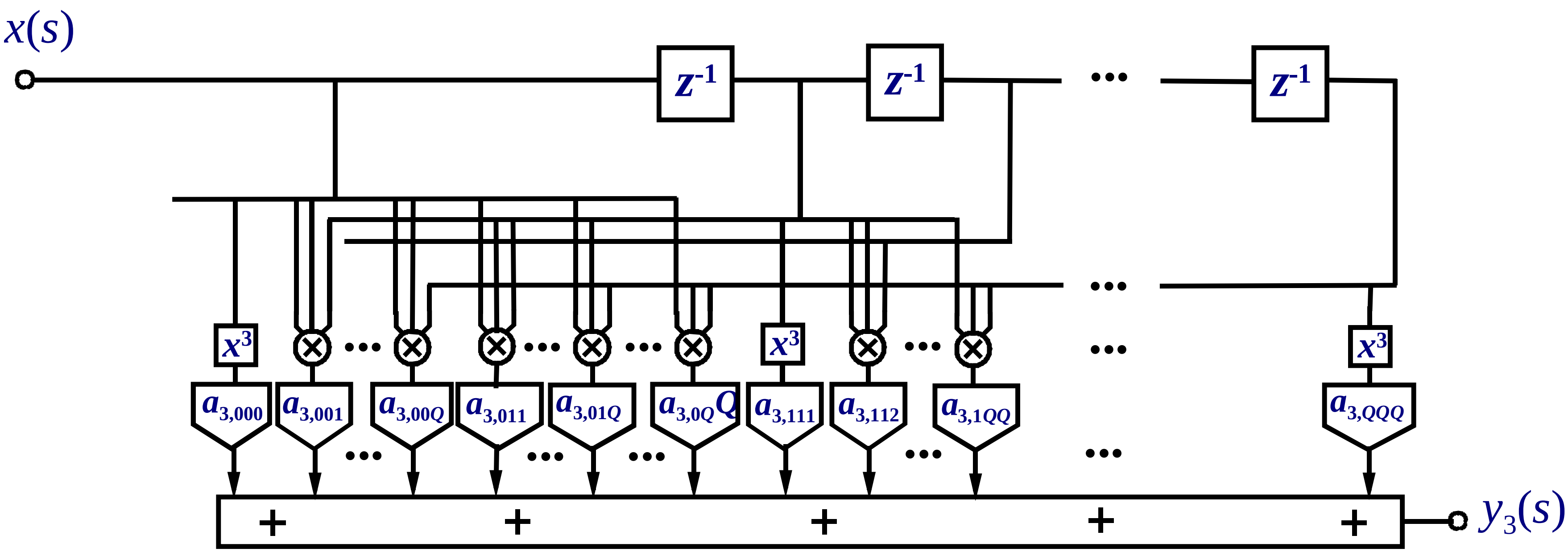


3. Basics on System Identification Theory

Nonlinear FIR Filters

while a *Direct 3rd Order Polynomial Filter* (*3rd Order FIR*) would be:

$$y_3(s) = \sum_{q_1=0}^{Q_2} \sum_{q_2=0}^{Q_2} \sum_{q_3=0}^{Q_2} a_2(q_1, q_2, q_3) x(s-q_1) x(s-q_2) x(s-q_3)$$



3. Basics on System Identification Theory

Dynamic Feedforward Artificial Neural Networks

$$y(s) = f_D\left[x(s), x(s-1), \ldots, x(s-Q_2)\right]$$

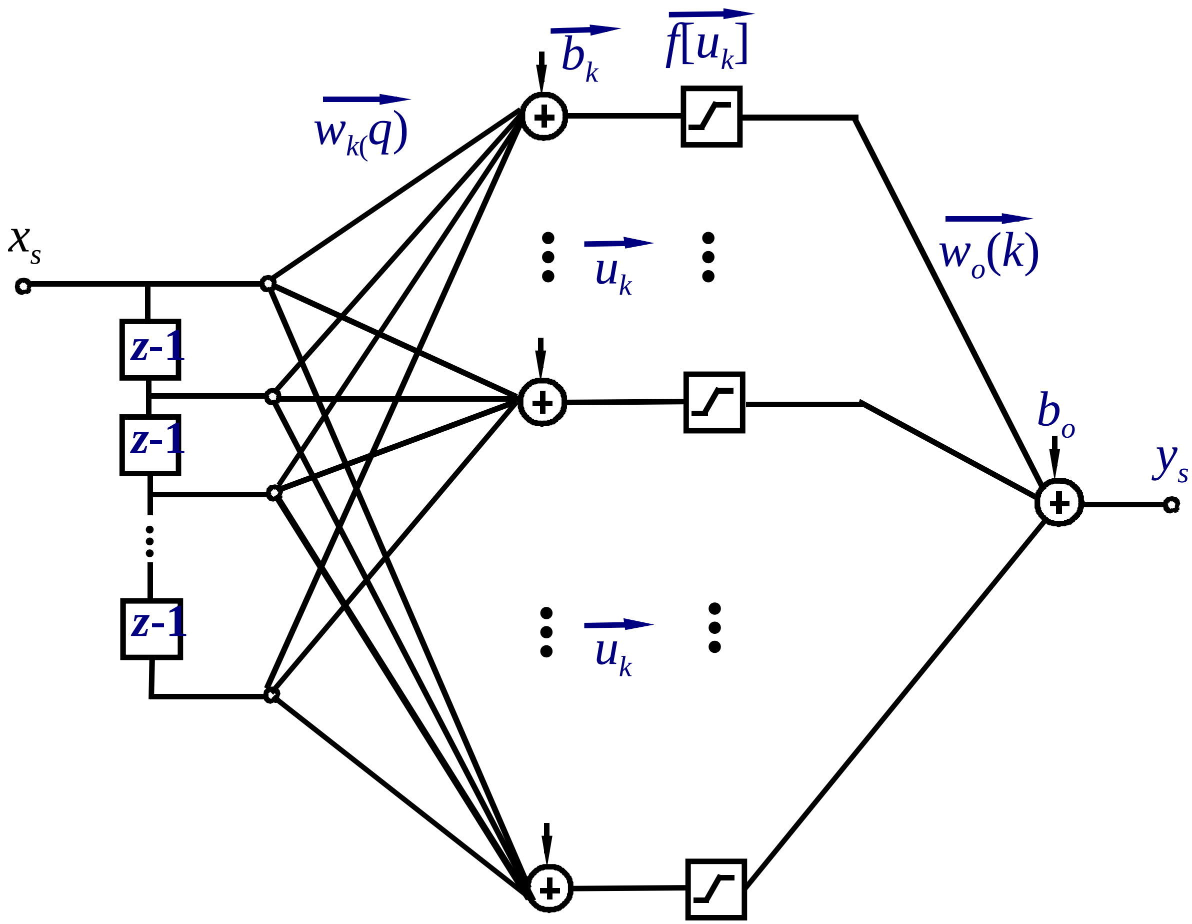
$$= b_o + \sum_{k=1}^{K} w_o(k) f_a\left[b_k + \sum_{q=0}^{Q_2} w_k(q) x(s-q)\right]$$

in which b_o, b_k, $w_o(k)$ and $w_k(q)$ are the model parameters, and $f_a[.]$ is a one-dimensional nonlinearity (typically a sigmoid) known as the activation function.

3. Basics on System Identification Theory

Dynamic Feedforward Artificial Neural Networks

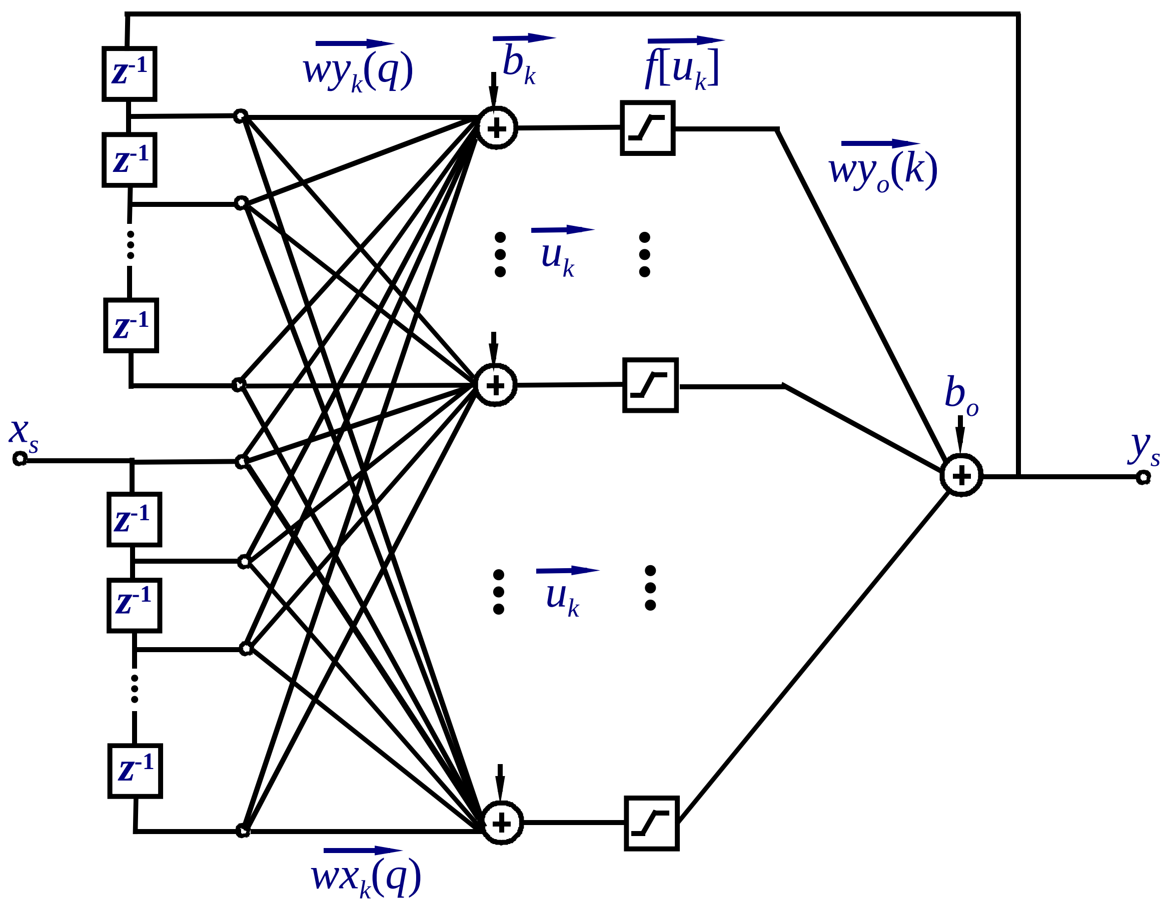
$$y(s) = f_D\left[x(s), x(s-1), \ldots, x(s-Q_2)\right]$$



3. Basics on System Identification Theory

Dynamic Recursive Artificial Neural Networks

$$y(s) = f_R\left[y(s-Q_1),\ldots,y(s-1),x(s-Q_2),\ldots,x(s-1),x(s)\right]$$



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Band-Pass and Low-Pass Equivalent Behavioral Models

[*Pedro and Maas, IEEE T-MTT 2005*]

Considering that a general wireless system processes a modulated RF carrier, its band-pass input and output signals can be expressed as:

$$s(t) = \mathrm{Re}\left\{ r(t)\, e^{j[\omega_0 t + \phi(t)]} \right\} = r(t) \cos[\omega_0 t + \phi(t)]$$

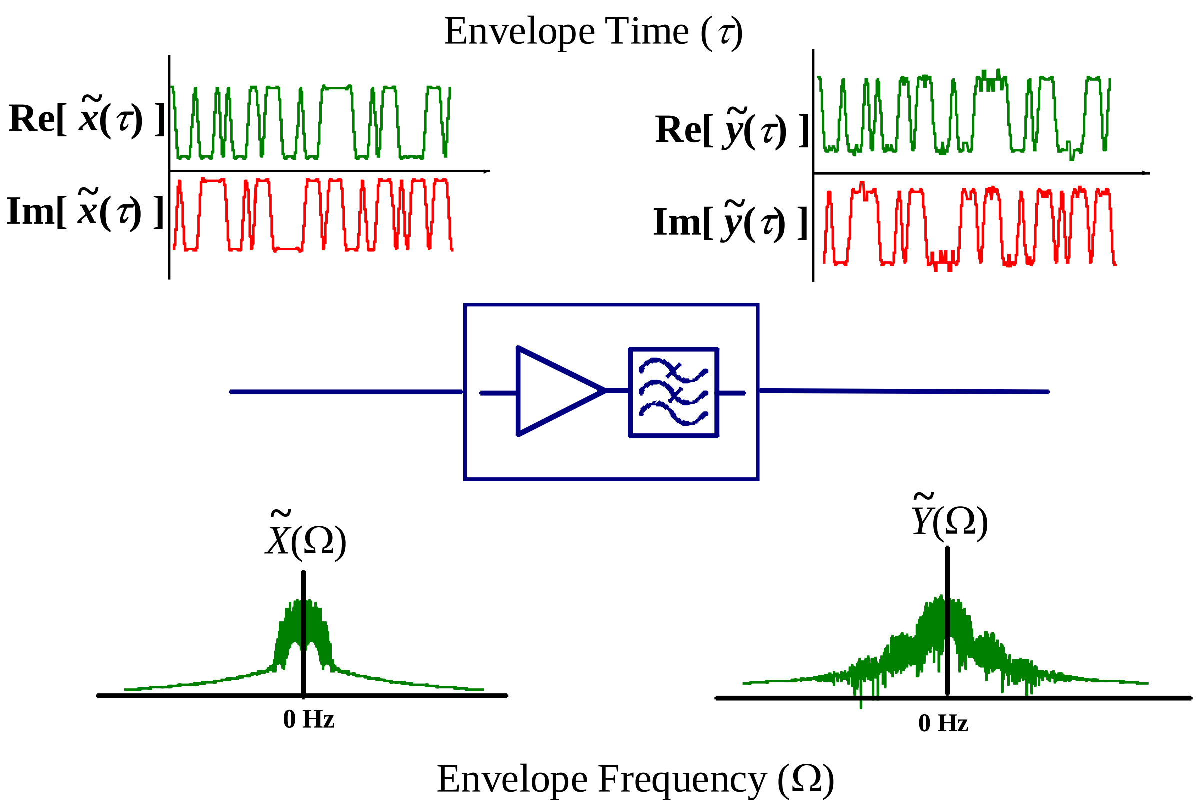
whose RF carrier is: $s_c(t) = \cos(\omega_0 t)$

and low-pass equivalent complex envelope is:

$$\tilde{s}(t) = r(t)\, e^{j\phi(t)}$$

Band-Pass and Low-Pass Equivalent Behavioral Models

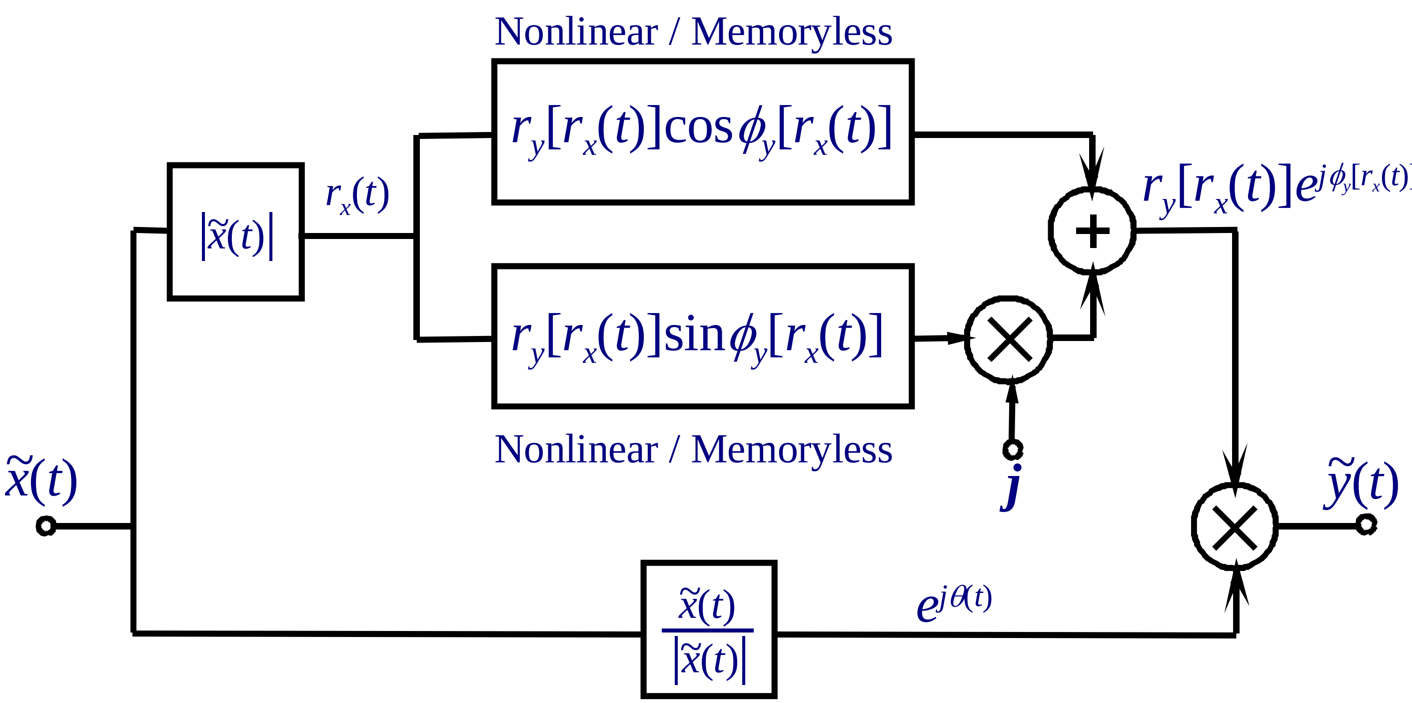
Therefore, we may conceive a *Low-pass Equivalent Behavioral Model* to handle only the complex base-band envelope, $\tilde{s}(t)$:



Low-Pass Equivalent Behavioral Models

[*Pedro and Maas, IEEE T-MTT 2005; Isaksson et al., IEEE T-MTT 2006*]

Simple *AM-AM / AM-PM Model*

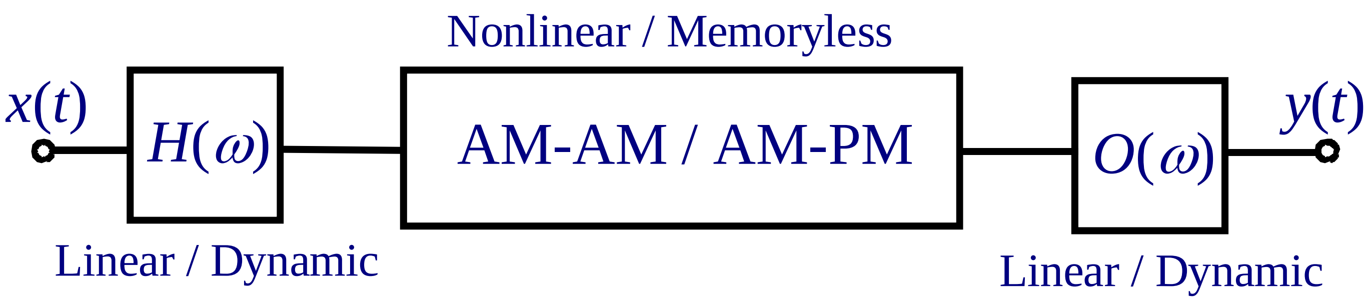


Memoryless AM-AM/AM-PM Saleh Model [*Saleh., IEEE T-COM 1981*]

Low-Pass Equivalent Behavioral Models

Two-Box and Three-Box Models are simplified Nonlinear FIR Filters where memory effects are separated from Nonlinearity.

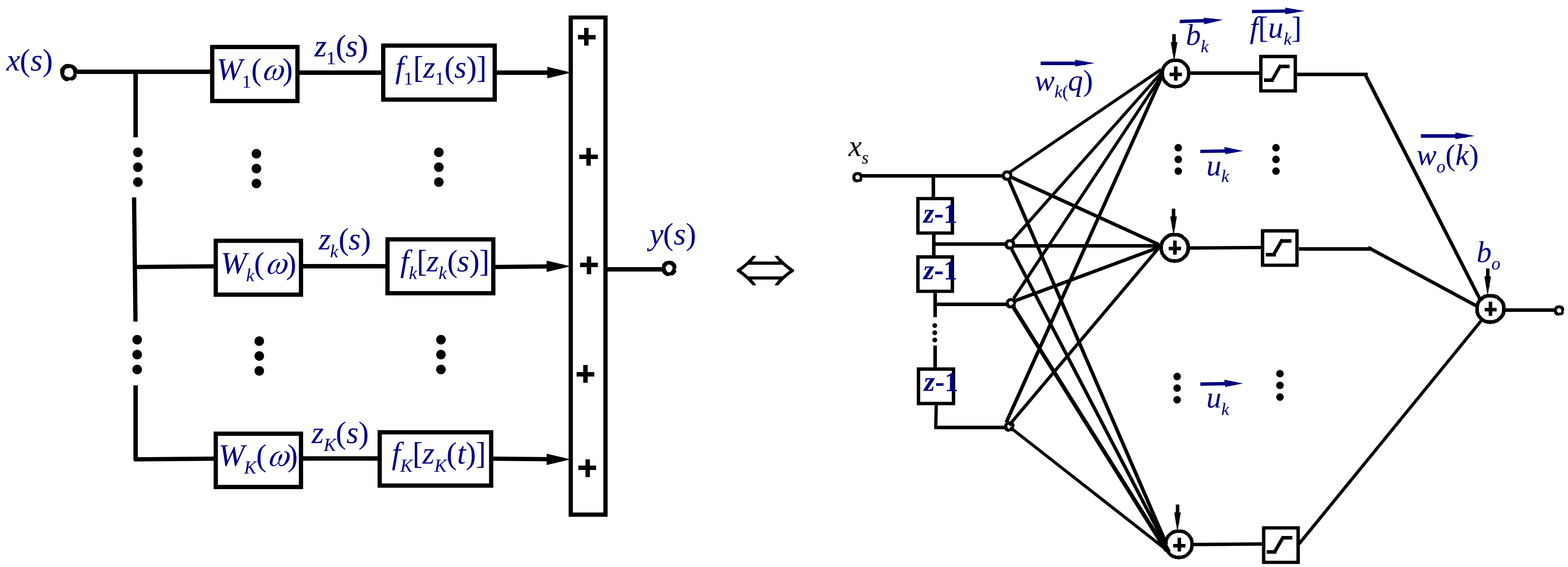
They assume a Filter-Nonlinearity (*Wiener Model*),
a Nonlinearity-Filter (*Hammerstein Model*)
or Filter-Nonlinearity-Filter (*Wiener-Hammerstein Model*) structures:



4. Nonlinear Behavioral Modeling of Microwave PAs

Low-Pass Equivalent Behavioral Models

Modifications of these Two or Three-Box models have also been used…



Parallel Wiener Model [*Ku, Mckinley and Kenney, IEEE T-MTT 2002*]
or Non-Recurrent ANN

4. Nonlinear Behavioral Modeling of Microwave PAs

Low-Pass Equivalent Behavioral Models

In an effort to reach a more systematic structure for the model a recursive ANN [*O´Brien et al., IMS'2006*], but mostly several Polynomial FIR Filters have been tried.

The complete Volterra Model has been tried by Zhu, Wren and Brazil.

[*Zhu, Wren and Brazil, IEEE IMS'2003*]

$$y(s)=\sum_{q=0}^{M}h_1(q)x(s-q)+\sum_{q_1=0}^{M}\sum_{q_2=0}^{M}h_2(q_1,q_2)x(s-q_1)x(s-q_2)$$
$$+\ldots$$
$$+\sum_{q_1=0}^{M}\cdots\sum_{q_N=0}^{M}h_N(q_1,\ldots,q_N).x(s-q_1)\ldots x(s-q_N)$$

4. Nonlinear Behavioral Modeling of Microwave PAs

Low-Pass Equivalent Behavioral Models

Unfortunately, the complexity of the parameter extraction poses severe restrictions on both the order N (degree of nonlinearity) and the number of delays $M+1$ (dynamic behavior).

This demanded the use of several alternatives for pruning the Volterra coefficients …

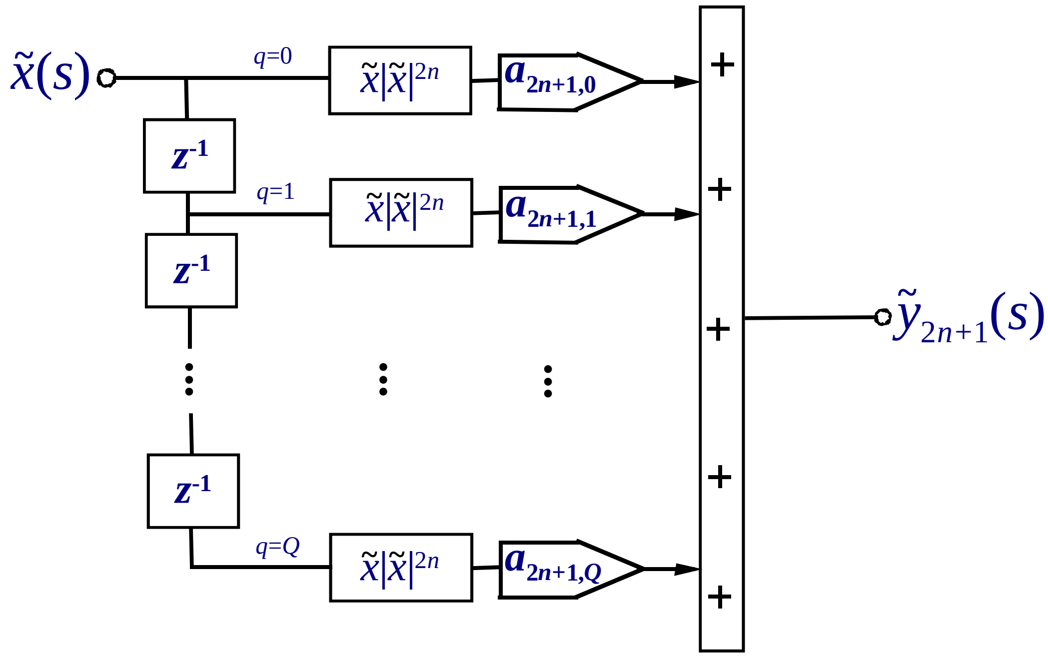
[*Zhu & Brazil, IEEE MWCL-2004; Zhu & Brazil, IEEE IMS'2005, Zhu et al., IEEE IMS'2006; Dooley et al., IEEE IMS'2006; Isaksson et al., IMS'2006*]

… or to extract them, one by one, in an orthogonal (separable) way

[*Lavrador et al., APMC'2005; Pedro et al., IEEE IMS'2006*]

Low-Pass Equivalent Behavioral Models

A one-dimensional alternative was recently proposed:



One-Dimensional Volterra Model [*Ku, Mckinley and Kenney, IEEE IMS'2003*]

… which can be shown to be similar to the *Nonlinear Integral Model*.

4. Nonlinear Behavioral Modeling of Microwave PAs

Low-Pass Equivalent Behavioral Models

Supposing the system may be strongly nonlinear, but memoryless, while showing approximately linear dynamic effects, allows the application of the *Nonlinear Integral Model* [*Filicori et al., IEEE T-MTT 1992*]:

$$\tilde{y}(s) = \sum_{q=0}^{Q} \tilde{f}_q\left[\tilde{x}(s), q\right]\tilde{x}(s-q)$$

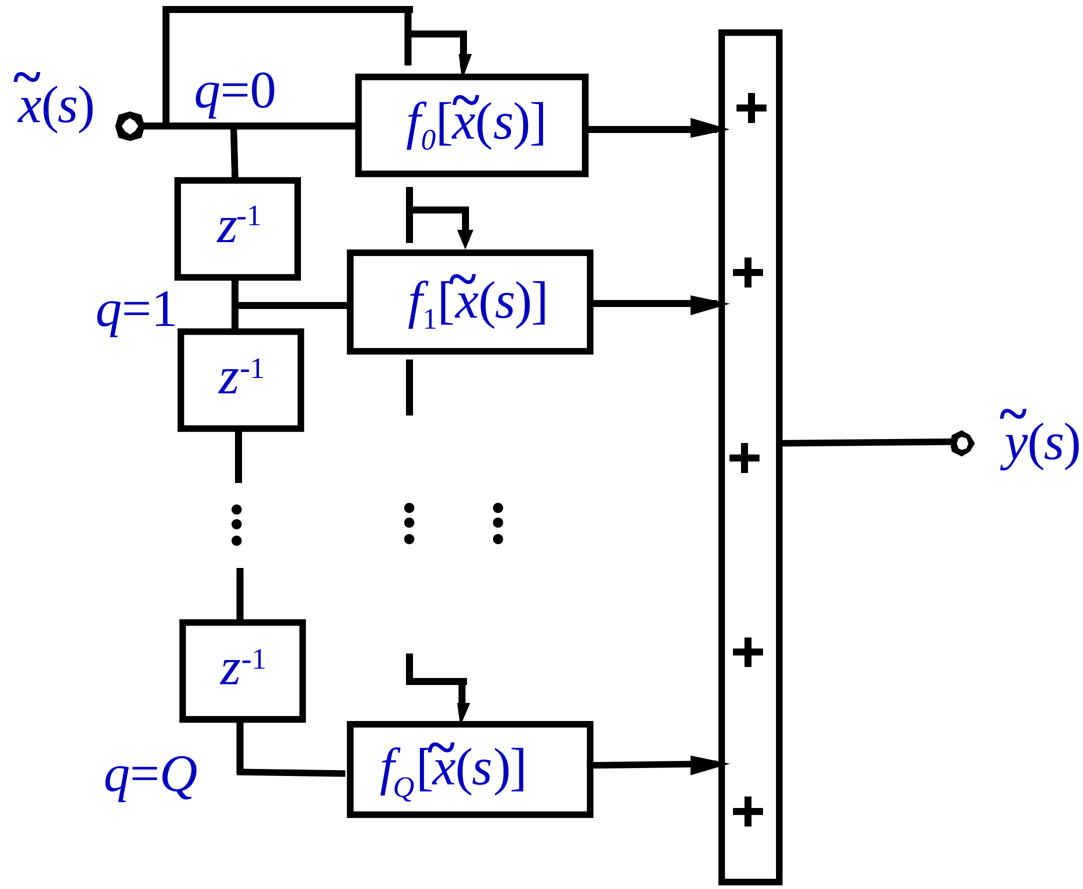
Complex Envelope Nonlinear Integral Model [*Mirri et al., IMTC/99*],
[*Soury et al., IEEE IMS'2003, Zhu et al., IMS'2006*]

in which $f_q[.]$ is a nonlinear impulse response, defined at a certain instantaneous excitation level.

Low-Pass Equivalent Behavioral Models

Accordingly, the NIM could be implemented as:

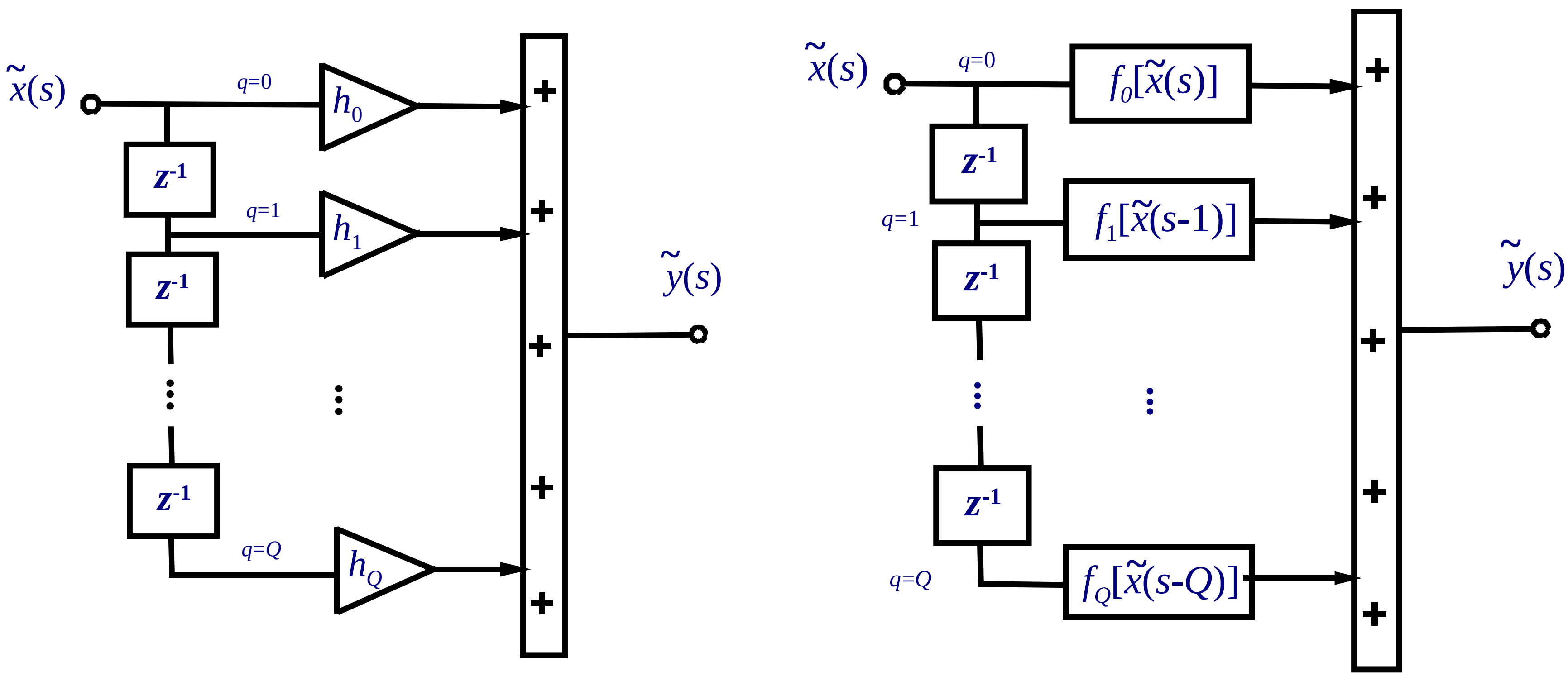
$$\tilde{y}(s) = \sum_{q=0}^{Q} \tilde{f}_q[\tilde{x}(s), q]\tilde{x}(s-q)$$



Low-Pass Equivalent Behavioral Models

… or, in a slightly different form, as a generalization of a liner FIR filter:

$$\tilde{y}(s) = \sum_{q=0}^{Q} h_q \, \tilde{x}(s-q) \quad \longrightarrow \quad \tilde{y}(s) = \sum_{q=0}^{Q} \tilde{f}_q[\tilde{x}(s-q), q]\tilde{x}(s-q)$$



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5. Conclusions

1. Nonlinear behavior of wireless systems shows significant dynamic effects caused by the interactions between the tuning networks, bias circuits and active device low-frequency dispersion.

2. System identification shows that many modeling activities can be framed into a small set of canonic behavioral model structures.

3. Behavioral modeling of wireless systems has been directed to complex envelope low-pass equivalents that process the amplitude and phase data.

4. Although formal structures as the Polynomial FIR Filters, or Feedforward ANNs, provide guaranteed predictive capabilities, they involve a large number of parameters and are very difficult to extract.

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Mathematical Needs for Behavioral Modeling of Telecommunication Circuits and Systems

(Second Part – Excitation Design for Model Extraction and Validation)

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Mathematical Techniques and Problems in Telecommunications – Sept. 2006

Presentation Outline

1. Excitation Design for Behavioral Model Extraction

2. The Role of Excitation's Statistics
on Nonlinear Dynamic Systems' Response

3. Multisine Design for Behavioral Model Validation

4. Conclusions

1. Excitation Design for Behavioral Model Extraction

Fundamentals of Behavioral Model Extraction

Contrary to other modeling techniques, such as the Artificial Neural Networks,

$$y(s) = b_o + \sum_{k=1}^{K} w_o(k)\, f_a\left[b_k + \sum_{q=0}^{Q} w_k(q) x(s-q) \right]$$

… which are nonlinear in their parameters, [b_k and $w_k(q)$], and thus require nonlinear optimization techniques for parameter extraction, …

1. Excitation Design for Behavioral Model Extraction

Fundamentals of Behavioral Model Extraction

… Polynomial Filters are linear in the parameters (the Volterra Kernels),

$$y(s)=\sum_{q=0}^{Q_2} h_1(q)x(s-q)+\sum_{q_1=0}^{Q_2}\sum_{q_2=0}^{Q_2} h_2(q_1,q_2)x(s-q_1)x(s-q_2)$$

$$+\ldots$$

$$+\sum_{q_1=0}^{Q_2}\cdots\sum_{q_n=0}^{Q_2} h_n(q_1,\ldots,q_n).x(s-q_1)\ldots x(s-q_n)+\ldots$$

… therefore enabling the use of standard linear regression methods. This also easies the gathering of knowledge for excitation design.

1. Excitation Design for Behavioral Model Extraction

Fundamentals of Behavioral Model Extraction

In fact, in the same way the Impulse Response Function, $h_1(q)$, of a Linear Dynamic System can be estimated solving the following linear regression system :

$$y(s) = \sum_{q=0}^{Q} h_1(q)x(s-q) \quad \Rightarrow \quad \begin{bmatrix} y(0) \\ \vdots \\ y(s) \\ \vdots \\ y(Q) \end{bmatrix} = \begin{bmatrix} x(0) & \cdots & x(-q) & \cdots & x(-Q) \\ \vdots & & \vdots & & \vdots \\ x(s) & \cdots & x(s-q) & \cdots & x(s-Q) \\ \vdots & & \vdots & & \vdots \\ x(Q) & \cdots & x(Q-q) & \cdots & x(0) \end{bmatrix} . \begin{bmatrix} h_1(0) \\ \vdots \\ h_1(q) \\ \vdots \\ h_1(Q) \end{bmatrix}$$

… provided the input $x(s)$ is sufficiently rich in content …

1. Excitation Design for Behavioral Model Extraction

Fundamentals of Behavioral Model Extraction

… the n´th order Nonlinear Impulse Response Function, $h_n(q_1,\ldots,q_n)$, of the *Volterra-Wiener Model*

$$y(s)=\sum_{q=0}^{Q} h_1(q)x(s-q)+\cdots+\sum_{q_1=0}^{Q}\cdots\sum_{q_n=0}^{Q} h_n(q_1,\ldots,q_n)\prod_{i=1}^{n} x(s-q_i)+\cdots$$

Fundamentals of Behavioral Model Extraction

… can be estimated solving the following linear regression system :

$$\begin{bmatrix} y_n(0) \\ \vdots \\ y_n(s) \\ \vdots \\ y_n(Q^n) \end{bmatrix} = \begin{bmatrix} x(0)^n & \cdots & \prod_i x(-q_i) & \cdots & x(-Q)^n \\ \vdots & & \vdots & & \vdots \\ x(s)^n & \cdots & \prod_i x(s-q_i) & \cdots & x(s-Q)^n \\ \vdots & & \vdots & & \vdots \\ x(Q)^n & \cdots & \prod_i x(Q-q_i) & \cdots & x(0)^n \end{bmatrix} . \begin{bmatrix} h_n(0,\ldots,0) \\ \vdots \\ h_n(q_1,\ldots,q_n) \\ \vdots \\ h_n(Q,\ldots,Q) \end{bmatrix}$$

… provided the multi-input $\Pi x(s\text{-}q_i)$ is again sufficiently rich in content.

1. Excitation Design for Behavioral Model Extraction

Fundamentals of Behavioral Model Extraction

Or, in the same way the Transfer Function, $H(\omega)$, of a Linear Dynamic System can be estimated from the input-output cross-correlation, $S_{yx}(\omega)$, and in input auto-correlation $S_{xx}(\omega)$:

$$\overline{H}(\omega) = \frac{S_{yx}(\omega)}{S_{xx}(\omega)} = \frac{\left\langle Y(\omega).X(\omega)^* \right\rangle}{\left\langle X(\omega)X(\omega)^* \right\rangle}$$

… provided the input $X(\omega)$ is sufficiently rich in content …

Fundamentals of Behavioral Model Extraction

… the n'th order Nonlinear Transfer Function of a Nonlinear Dynamic System, $H_n(\omega_1,\ldots,\omega_n)$,

$$y(s)=\sum_{k=1}^{K}H_1(\omega_k)X(\omega_k)e^{j\omega_k s}+\cdots$$

$$+\sum_{k_1=1}^{K}\cdots\sum_{k_n=1}^{K}H_n(\omega_{k_1},\ldots,\omega_{k_n})\prod_{i=1}^{n}X(\omega_{k_i})e^{j\left(\omega_{k_1}+\ldots+\omega_{k_n}\right)s}+\cdots$$

1. Excitation Design for Behavioral Model Extraction

Fundamentals of Behavioral Model Extraction

… can be estimated from the higher-order input-output cross-correlations, $S_{yx\ldots x}(\omega_1,\ldots,\ \omega_n)$, and in higher-order input auto-correlations $S_{xx}(\omega_1,\ldots,\ \omega_n)$:

$$\overline{H}_n(\omega_1,\ldots,\omega_n)=\frac{S_{yx\ldots x}(\omega_1,\ldots,\omega_n)}{S_{x\ldots x}(\omega_1,\ldots,\omega_n)}=\frac{\left\langle Y(\omega_1+\ldots+\omega_n).X(\omega_1)^*\ldots X(\omega_n)^*\right\rangle}{\left\langle X(\omega_1)\ldots X(\omega_n).X(\omega_1)^*\ldots X(\omega_n)^*\right\rangle}$$

… provided the multi-input $\Pi X(\omega_i)$ is again sufficiently rich in content.

1. Excitation Design for Behavioral Model Extraction

Fundamentals of Behavioral Model Extraction

This shows that, no matter the domain (time or frequency), or the type of stimulus used for the tests, the model extraction procedure will be successful as long as the stimulus excites all wireless system's states:

- $x(s), x(s\text{-}1), \ldots , x(s\text{-}Q)$

- $x(s)^2, x(s).x(s\text{-}1), \ldots, x(s).x(s\text{-}Q), x(s\text{-}1)^2, \ldots, x(s\text{-}1).x(s\text{-}Q), \ldots, x(s\text{-}Q)^2$

- $x(s)^3, x(s)^2.x(s\text{-}1), \ldots, x(s).x(s\text{-}Q)^2, x(s\text{-}1)^3, \ldots, x(s\text{-}1).x(s\text{-}Q)^2, \ldots, x(s\text{-}Q)^3$

⋮

- $x(s)^n, x(s)...x(s\text{-}q_{n\text{-}1}), \ldots, x(s\text{-}Q)^n$

1. Excitation Design for Behavioral Model Extraction

Fundamentals of Behavioral Model Extraction

… or:

- $X(\omega_1), X(\omega_2), \ldots, X(\omega_K)$
- $X(\omega_1)^2, X(\omega_1).X(\omega_2), \ldots, X(\omega_1).X(\omega_K), \ldots, X(\omega_K).X(\omega_1), \ldots, X(\omega_K)^2$
- $X(\omega_1)^3, X(\omega_1)^2.X(\omega_2), \ldots, X(\omega_1).X(\omega_K)^2, \ldots, X(\omega_K)^2.X(\omega_1), \ldots, X(\omega_K)^3$

$\vdots$

- $X(\omega_1)^n, X(\omega_1)\ldots X(\omega_{n-1}), \ldots, X(\omega_K)^n$

1. Excitation Design for Behavioral Model Extraction

Some Historical Steps Towards Behavioral Model Extraction

In the 40's Wiener proved that *White Gaussian Noise* was rich enough to excite a Volterra system. Then, in the 60's, Schezten and Lee used that excitation to extract the Wiener Model – a polynomial filter orthogonal to this input.

The n'th order Wiener functional was obtained by time-domain correlation between the output, $y(t)$, and a n'th order delayed version of the input, $x(t-\tau_1).x(t-\tau_2)\ldots x(t-\tau_n)$.

In the 80's, Boyd, Tang and Chua and then Chua and Liao proposed methods for extracting Volterra kernels in the frequency-domain, using *Sparsely Distributed Harmonically Related Sinusoids*.

1. Excitation Design for Behavioral Model Extraction

Some Historical Steps Towards Behavioral Model Extraction

More recently, much of the effort has been directed to obtain useful data from *Multisines* …

… or even time-domain *Real Modulated Wireless* excitations.

However, the traditional frequency-domain *Sinusoidal Excitation* and the time-domain *Step Stimulus* have also been extensively tried.

1. Excitation Design for Behavioral Model Extraction

Nonlinear System Identification Theory May Help Again ...

Recognizing that both digitally synthesized *Multisines*, *Pseudo-Random Noise Sequences* or *Finite Modulation Sequences* are discrete periodic functions in time and frequency domains, they can be related by the Discrete Fourier Series:

$$X(k\omega_0) = \frac{1}{N} \sum_{n=-N}^{N} x(nT_s)\,\mathrm{e}^{-jk\omega_0 nT_s} \quad \Longleftrightarrow \quad x(nT_s) = \sum_{k=-K}^{K} X(k\omega_0)\,\mathrm{e}^{jk\omega_0 nT_s}$$

… which shows that they are simply two distinct ways of extracting same type of information.

1. Excitation Design for Behavioral Model Extraction

Nonlinear System Identification Theory May Help Again ...

Furthermore, since the response of a system excited with various realizations of:

- *Random Multisine* - Multisine with randomized phases
- *Periodic Noise* - Multisine with randomized amplitudes and phases

⬇

the response of that same system when excited with

- *Band-Limited White Gaussian Noise*

Volterra-Wiener theories prove that both of these stimuli could be used to extract a behavioral model of, at least, a Nonlinear System of Fading Memory !

Presentation Outline

1. Excitation Design for Behavioral Model Extraction

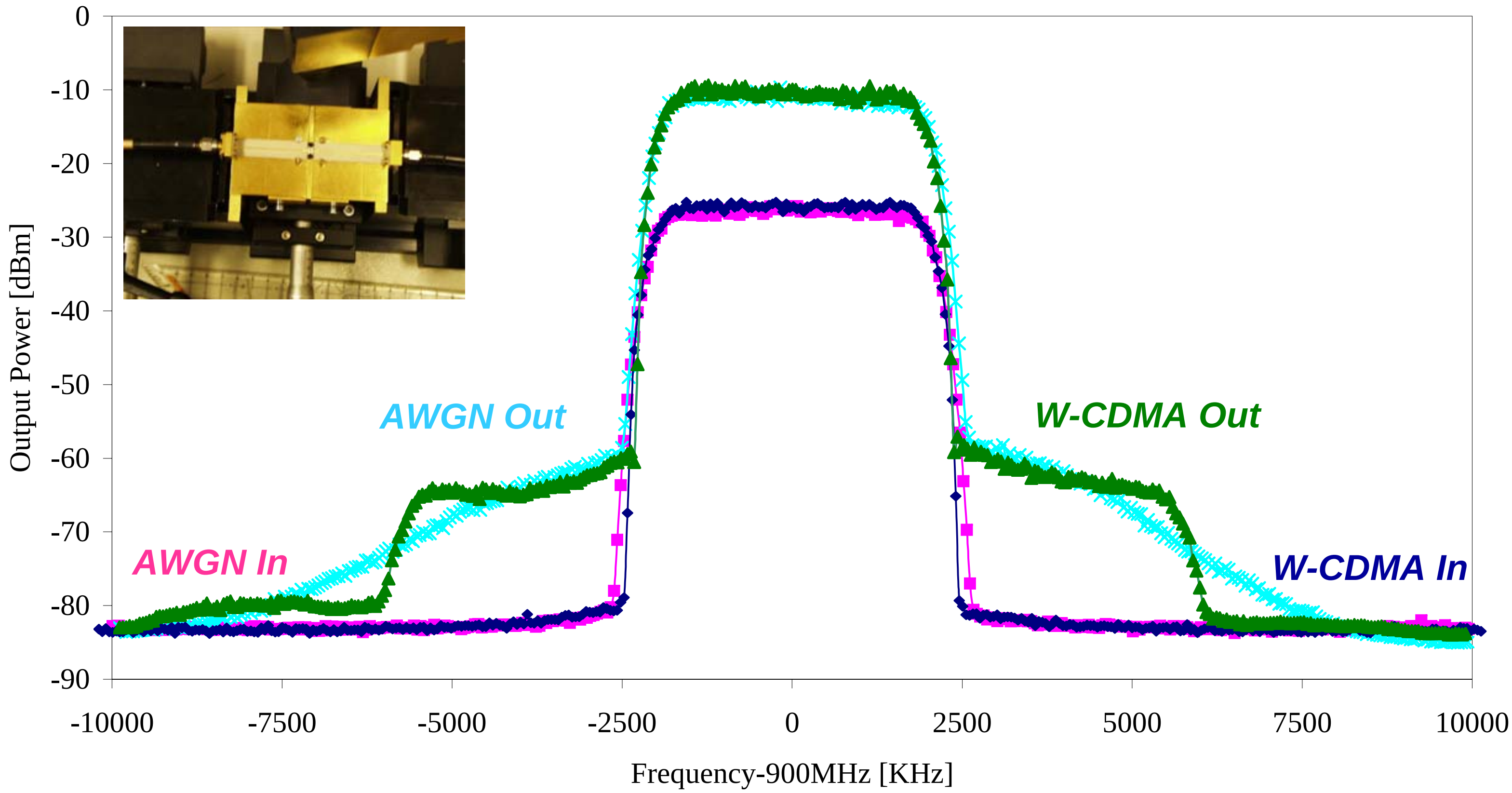
2. The Role of Excitation's Statistics
on Nonlinear Dynamic Systems' Response

3. Multisine Design for Behavioral Model Validation

4. Conclusions

2. The Role of Excitation's Statistics on Nonlinear Dynamic Systems' Response

Why do excitations with similar PSD and integrated power produced so distinct nonliner responses ?



2. The Role of Excitation's Statistics on Nonlinear Dynamic Systems' Response

Probability Density, $pdf_x(x)$, as a Weighting Function

Since typical laboratory data like Output Power, Power Spectrum, etc., is averaged in nature:

$$P_{in} = E\left\{x(s)^2\right\} = \int_{-\infty}^{\infty} x^2 \, pdf_X(x)\, dx$$

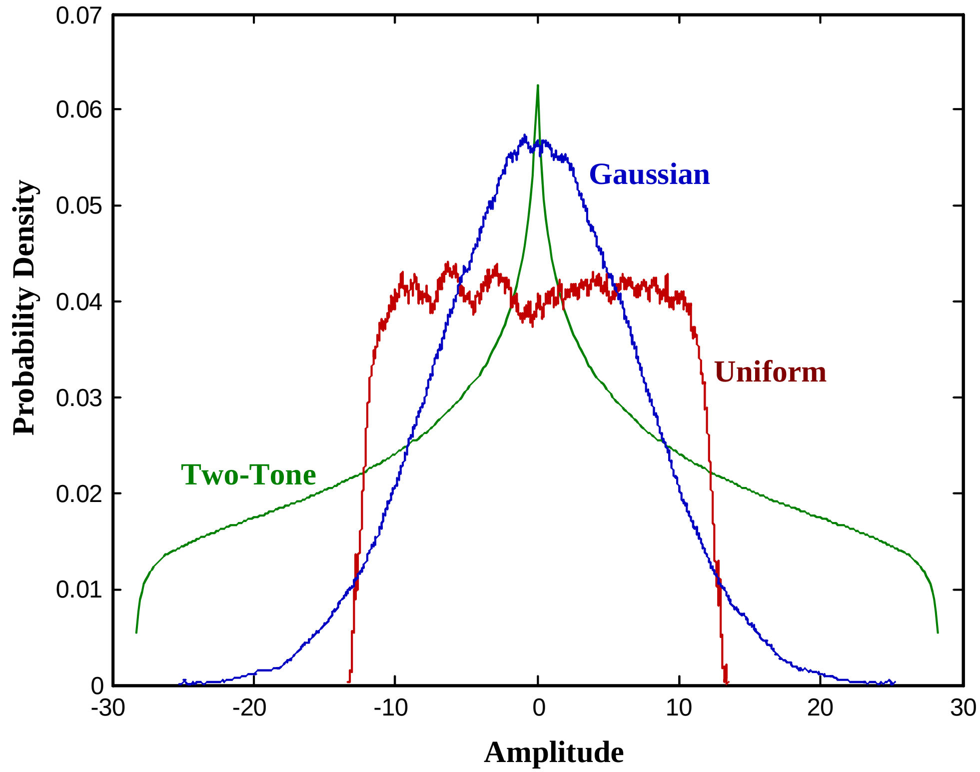
$$P_{out} = E\left\{y(s)^2\right\} = \int_{-\infty}^{\infty} y^2 \, pdf_Y(y)\, dy = \int_{-\infty}^{\infty} f_{NL}(x)^2 \, pdf_X(x)\, dx$$

… it is intuitive to expect that, more important than the trajectory of amplitude values assumed by the excitation, $x(t)$, should be the *Probability* with which each value is reached, i.e., *the Excitation's* $pdf_x(x)$.

2. The Role of Excitation's Statistics on Nonlinear Dynamic Systems' Response

Probability Density, $pdf_x(x)$, as a Weighting Function

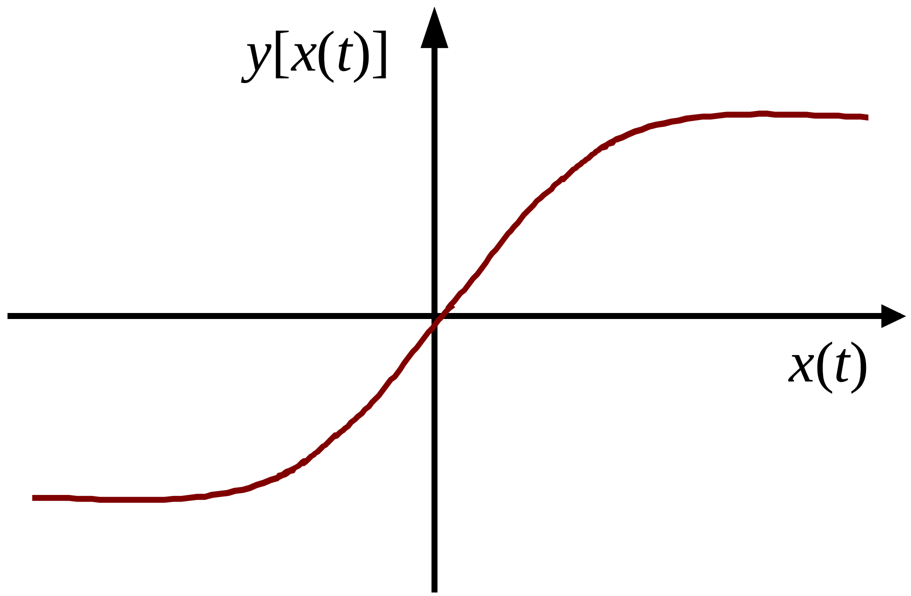
To expose the role of the $pdf_x(x)$ we tested a static nonlinear system with three signals of equal integrated power but distinct amplitude distributions:



Memoryless Nonlinear System Model

The selected nonlinear system was a simple sigmoid function:

$$y(t) = f_{NL}\left[x(t)\right] = \tan\left[\frac{x(t)}{20}\right] \approx \sum_{n=1}^{N} k_n \, x^n$$



since its linear region, followed by a smoothly saturating behavior, is many times used to represent practical memoryless nonlinearities.

2. The Role of Excitation's Statistics on Nonlinear Dynamic Systems' Response

Multisines for Memoryless Nonlinear Systems

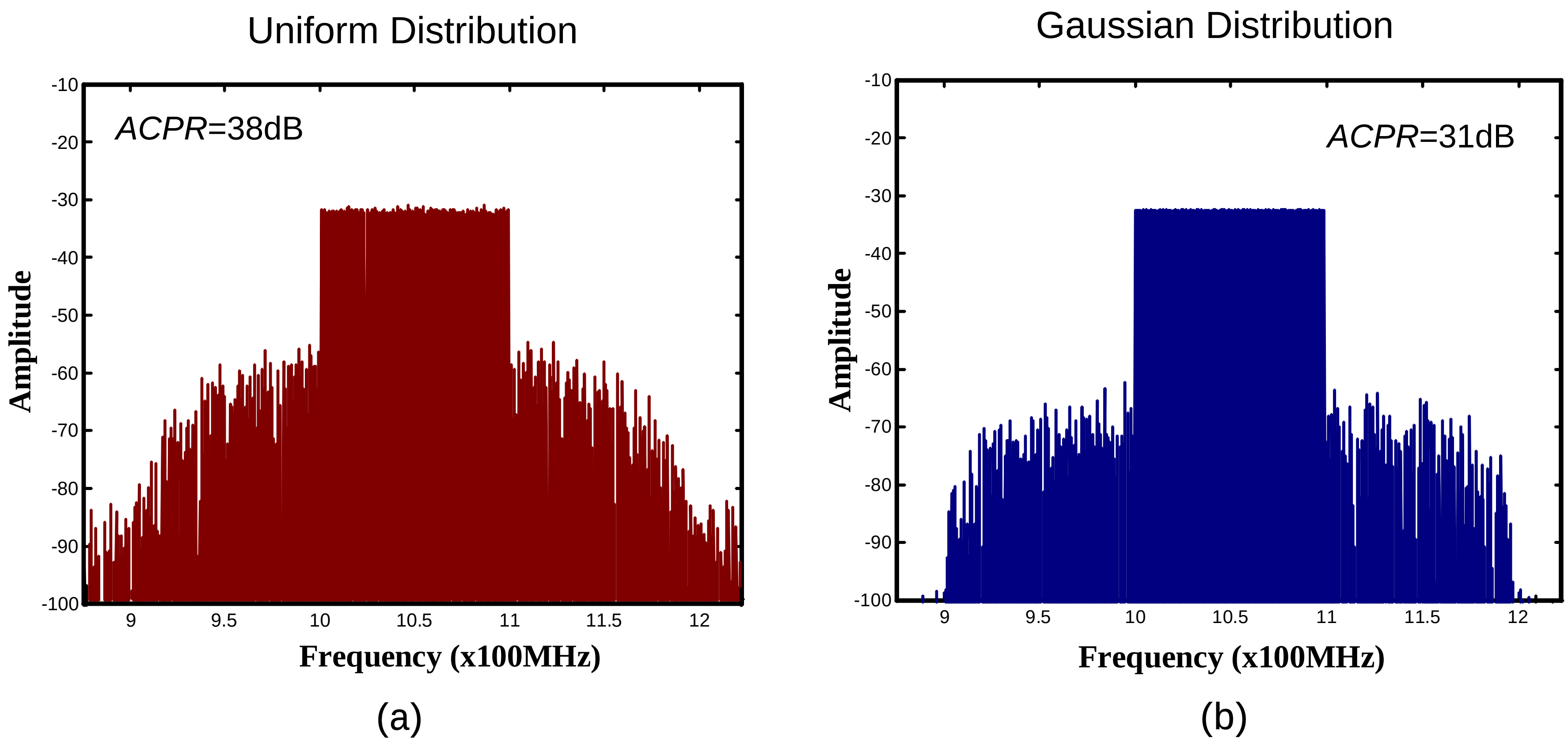
The excitations used were two evenly spaced constant amplitude *Band-Pass Multisines*:

$$x(t) = \sum_{k=1}^{K} A_k \cos\left(\omega_k t + \phi_k\right); \qquad A_k = A : \forall\, k$$

whose phases, ϕ_k, were designed for Gaussian and Uniform $pdf_x(x)$ using a specially conceived algorithm.

(J. Pedro and N. Carvalho, IEEE IMS'2004)

Multisines for Memoryless Nonlinear Systems



Power spectrum of the response of a sigmoid memoryless system excited by the uniformly distributed - (a) and Gaussian distributed multisines - (b).

Nonlinear Dynamic System Model

A dynamic system is one whose output is dependent on the present input and on its past:

$$y(s) = f_D\left[x(s), x(s-1), \ldots, x(s-Q)\right]$$

A nonlinear FIR filter approximation would be:

$$y(s) \approx P\left[x(s), x(s-1), \ldots, x(s-Q)\right]$$

$$= \sum_{q=0}^{Q} h_1(q)\, x(s-q) + \cdots$$

$$+ \sum_{q_1=0}^{Q} \cdots \sum_{q_n=0}^{Q} h_n(q_1, \ldots, q_n)\, x(s-q_1) \ldots x(s-q_n) + \cdots$$

2. The Role of Excitation's Statistics on Nonlinear Dynamic Systems' Response

Nonlinear Dynamic System Model

If $y(s) = f_D\left[x(s)\right]$ ⟶ $y(s) = f_D\left[x(s), x(s-1), \ldots, x(s-Q)\right]$

the system's output should no longer be only dependent on the input statistics, $pdf_x(x)$, but on the *Joint Statistics* of the input and its past samples, $pdf_{x \ldots x\text{-}Q}[x(s), \ldots, x(s\text{-}Q)]$.

So, the n'th order response of a nonlinear dynamic system to a certain stimulus now depends on the Memory Span, Q, of $h_n(q_1, \ldots, q_n)$ and on the correlation between $x(s)$ and all other $x(s\text{-}1), \ldots, x(s\text{-}Q)$.

Not only the amplitude distribution is important as is the signal evolution with time, i.e., its *Time-Domain Waveform* or *Frequency-Domain Spectrum*.

2. The Role of Excitation's Statistics on Nonlinear Dynamic Systems' Response

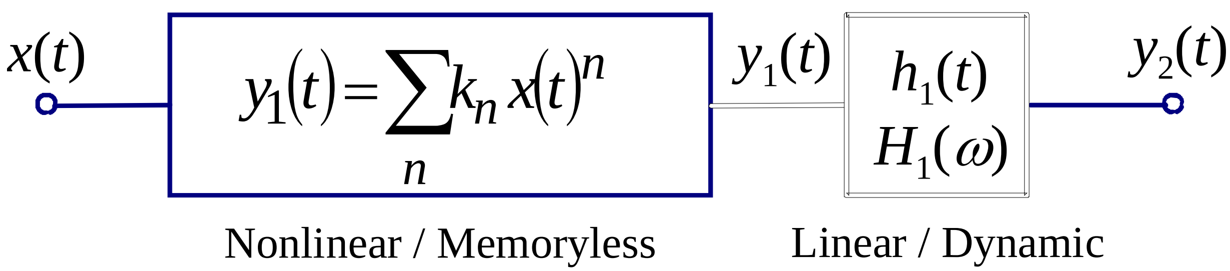
Nonlinear Dynamic System Model

As an example, consider a Hammerstein Model composed of a memoryless nonlinearity,

$$y_1(s) = \sum_{n=1}^{N} k_n x(s)^n$$

followed by a linear filter:

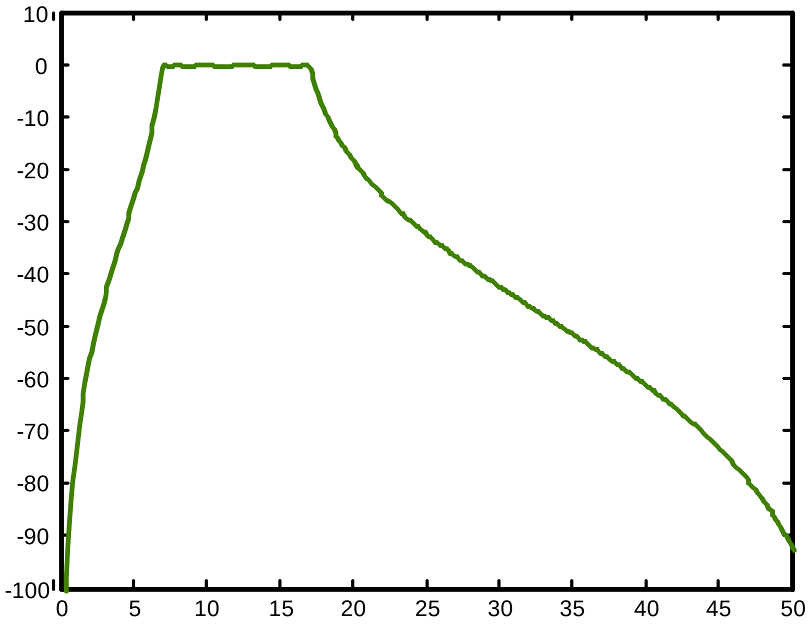
$$y_2(s) = \sum_{q=0}^{Q-1} h_1(q) y_1(s-q)$$



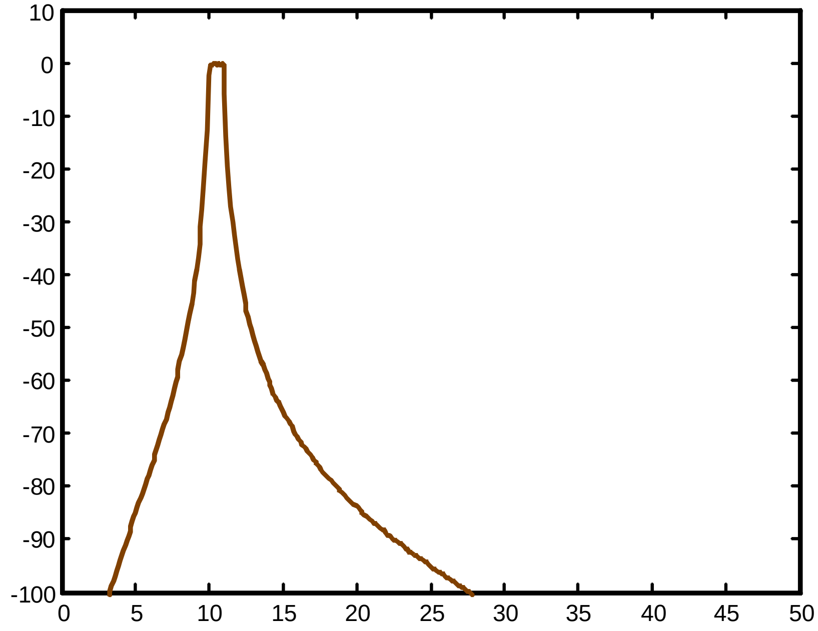
The system's response now depends on the memory span of $h_1(q)$, Q.

Nonlinear Dynamic System Model

$$y_1(s) = \sum_{q=0}^{Q-1} h_1(q)x(s-q)$$



(a)



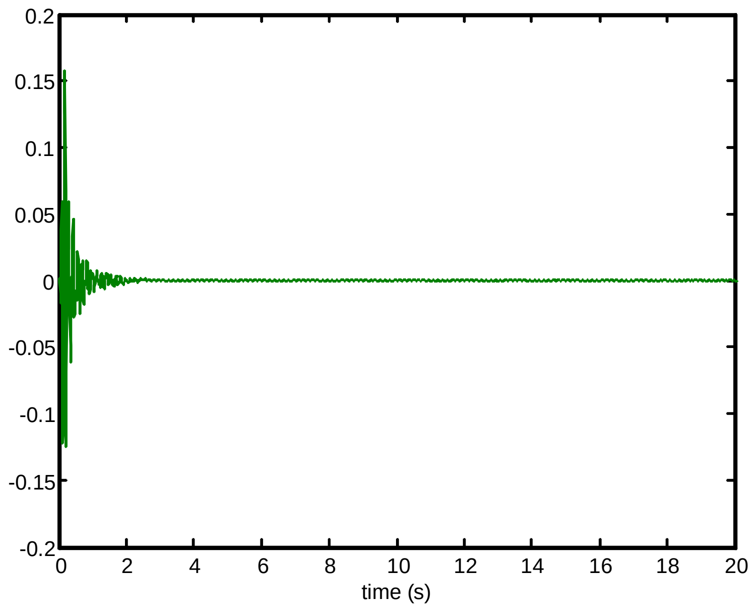
(b)

Linear pos-filters used in the distortion simulations. (a) – Filter with short memory span. (b) – Filter with long memory span.

Nonlinear Dynamic System Model

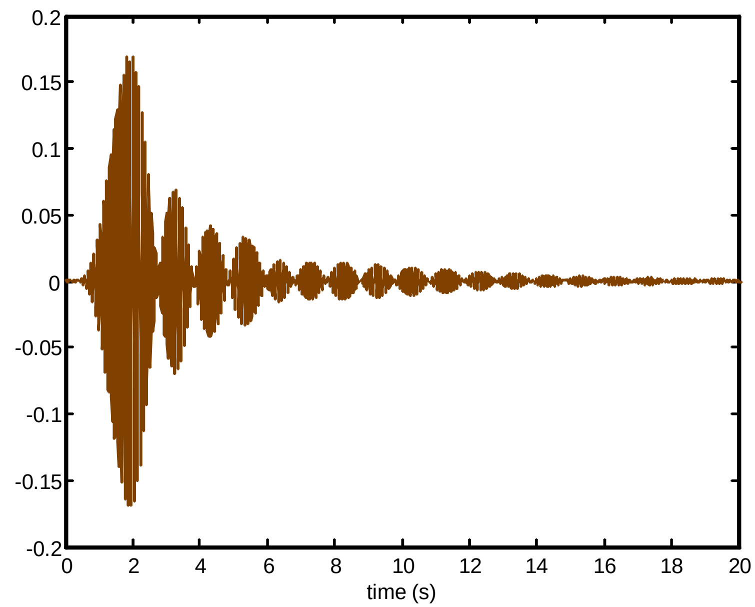
$$y_1(s) = \sum_{q=0}^{Q-1} h_1(q)x(s-q)$$

Short Memory Span, *Q*



(a)

Long Memory Span, *Q*

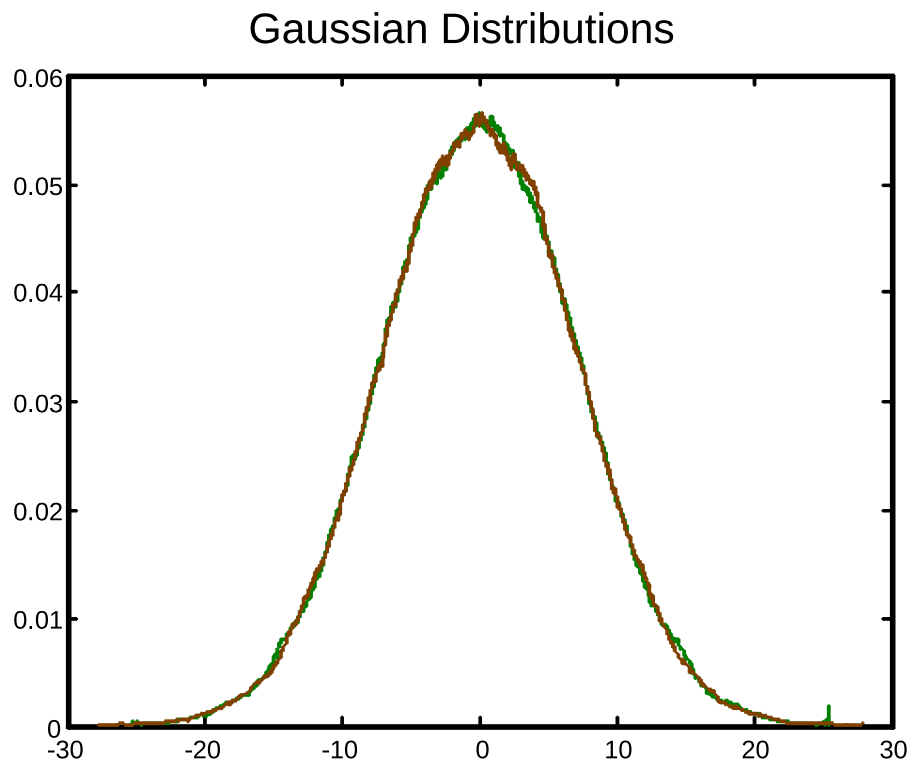


(b)

Linear pos-filters used in the distortion simulations. (a) – Filter with short memory span. (b) – Filter with long memory span.

Response of the Hammerstein Model to Multisines of Equal *pdf*

$$x(t)=\sum_{k=1}^{K} A_k \cos(\omega_k t+\phi_k)\,; \qquad A_k = A : \forall k$$

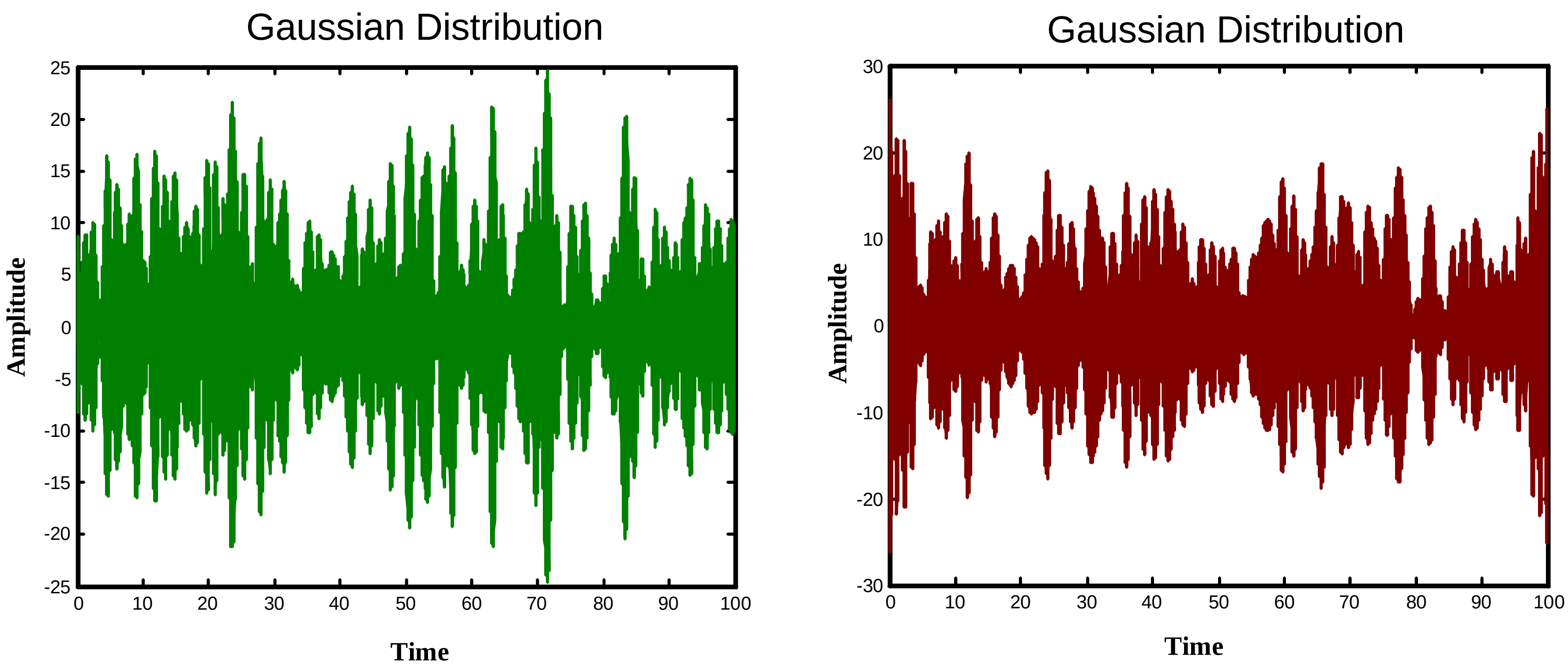


Gaussian *pdf* of the two multisines of equal power spectrum.

2. The Role of Excitation's Statistics on Nonlinear Dynamic Systems' Response

Response of the Hammerstein Model to Multisines of Equal *pdf*

$$x(t)=\sum_{k=1}^{K} A_k \cos(\omega_k t+\phi_k)\,; \qquad A_k = A : \forall k$$



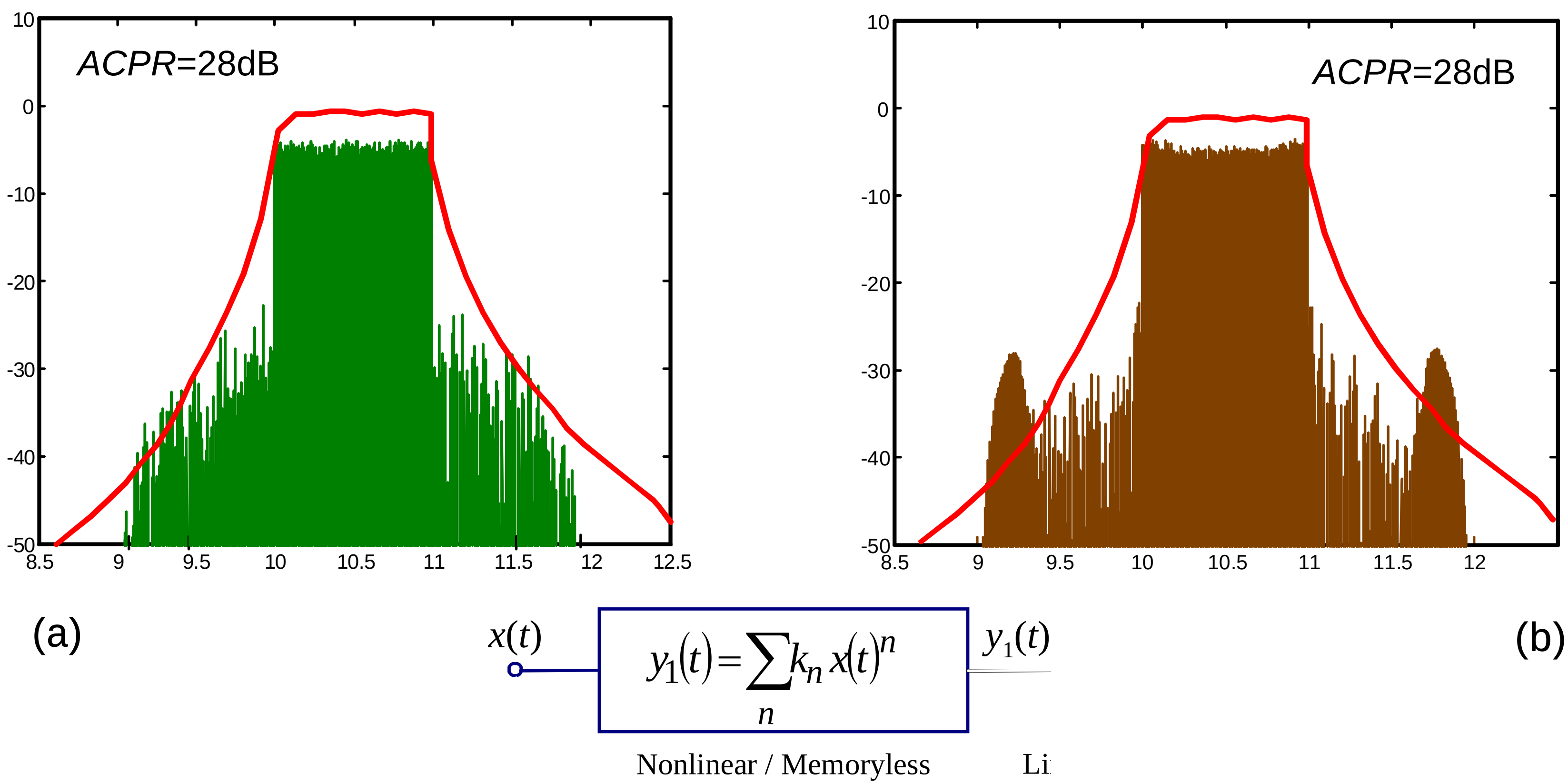
(a) (b)

Time-domain waveforms of the two multisines of equal *pdf* and power spectrum.

2. The Role of Excitation's Statistics on Nonlinear Dynamic Systems' Response

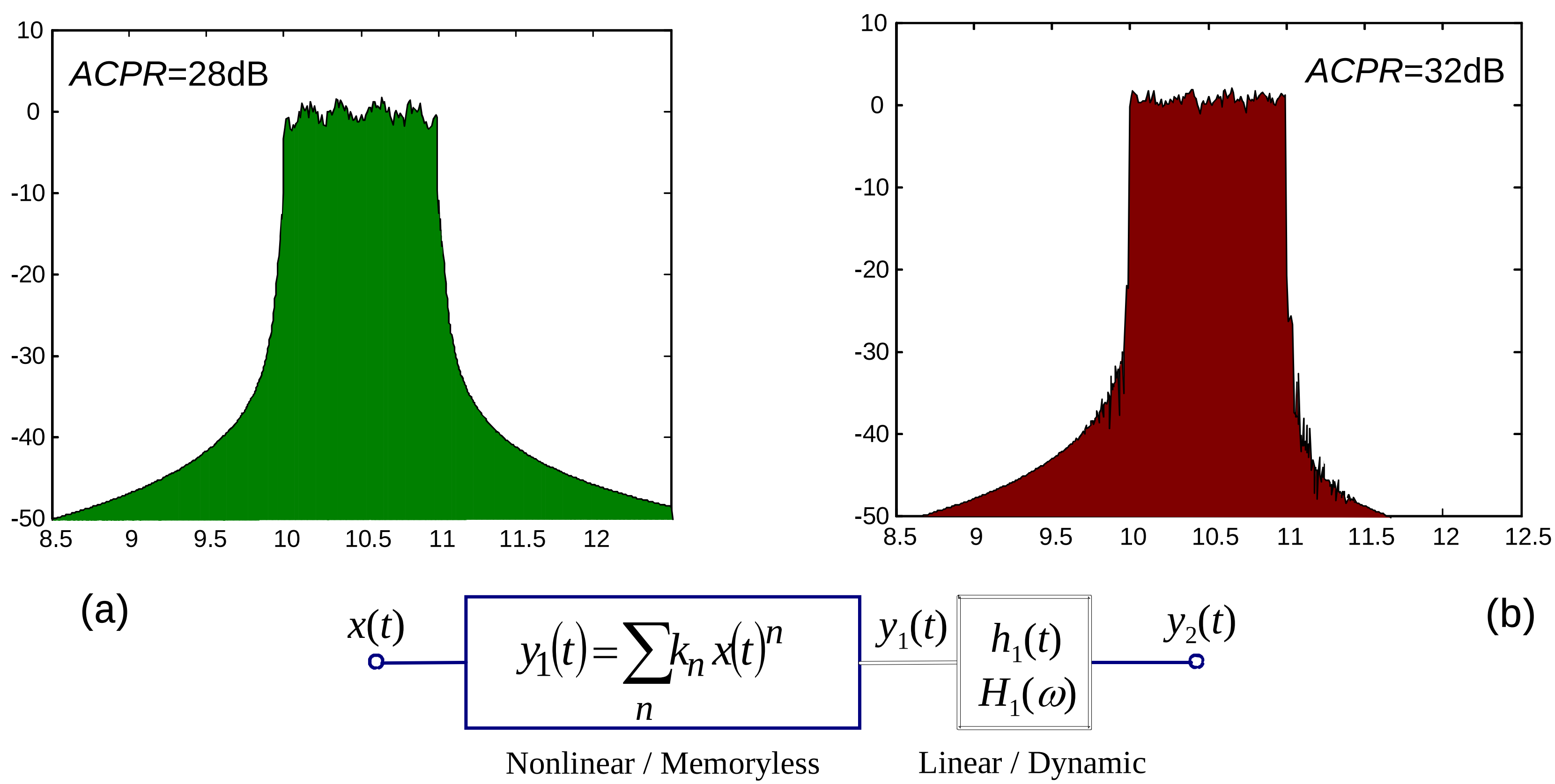
Response of the Hammerstein Model to Multisines of Distinct *pdf*

Although different phase arrangements produce distinct power spectra in a memoryless nonlinearity, they generate equal integrated *ACPR* values.



2. The Role of Excitation's Statistics on Nonlinear Dynamic Systems' Response

Now, although the wide-band output filter would keep the entire spectra, the narrow-band linear band-pass filter can reshape the spectrum side lobes, and thus generate quite different integrated output *ACPR* values.



Presentation Outline

1. Excitation Design for Behavioral Model Extraction

2. The Role of Excitation's Statistics
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3. Multisine Design for Behavioral Model Validation

4. Conclusions

3. Multisine Design For Behavioral Model Validation

Response of General Nonlinear Dynamic Systems to Arbitrary Signals

The reason for these discrepancies can be traced to the way the nonlinearity generates the spectral regrowth.

Since the multisine is periodic, there is a common frequency separation between the many different tones. So, the general multisine expression:

$$x(t) = \sum_{k=1}^{K} A_k \cos\left(\omega_k t + \phi_k\right); \quad A_k = A : \forall k$$

implies that all tone frequencies can be expressed by: $\omega_k = \omega_0 + k\Delta\omega$

and so, any (e.g., 3rd order) output spectral line at ω_x will be given by all mixing products verifying:

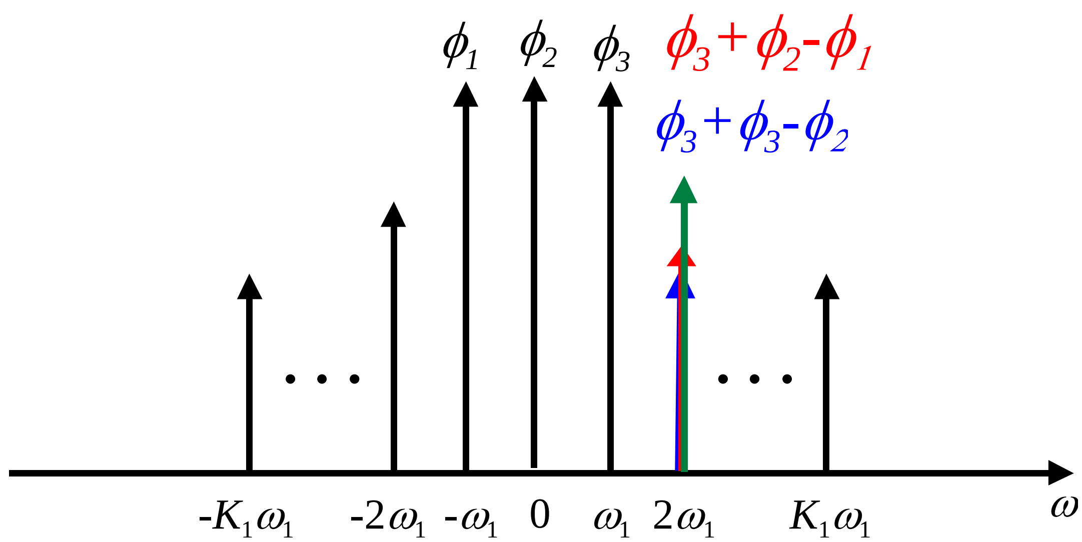
$\omega_x = \omega_{k_1} + \omega_{k_2} - \omega_{k_3}$ in which $x = k_1 + k_2 - k_3$

3. Multisine Design For Behavioral Model Validation

Response of General Nonlinear Dynamic Systems to Arbitrary Multisines

Each of these mixing products has a phase of: $\phi_X = \phi_{k_1} + \phi_{k_2} - \phi_{k_3}$

and so, the resulting voltage wise addition depends on the possible correlation between these ϕ_x.



3. Multisine Design For Behavioral Model Validation

Response of General Nonlinear Dynamic Systems to Arbitrary Multisines

Each of these mixing products has a phase of: $\phi_x = \phi_{k_1} + \phi_{k_2} - \phi_{k_3}$

and so, the resulting voltage wise addition depends on the possible correlation between these ϕ_x.

Thus, in the context of nonlinear dynamic systems, signal excitations can no longer be completely specified by their moments (pdf), as in memoryless systems:

$$m_n(x) = E\left\{x^n\right\} = \int_{-\infty}^{\infty} x^n \, pdf_x(x)\, dx$$

3. Multisine Design For Behavioral Model Validation

Response of General Nonlinear Dynamic Systems to Arbitrary Multisines

If signal excitations can not be completely specified by their moments (*pdf*), they can't be either specified by the second order joint statistics as in linear dynamic systems:

$$R_{xx}(\tau) = E\{x(t)x(t+\tau)\} \quad \longleftrightarrow \quad S_{xx}(\omega) = E\left\{X(\omega)X(\omega)^*\right\}$$

because this signal metric is blind to the signal' phases !

3. Multisine Design For Behavioral Model Validation

Response of General Nonlinear Dynamic Systems to Arbitrary Multisines

As expected from the polynomial structure of the Volterra-Wiener model, multisines must now be specified by their higher-order joint signal statistics:

$$R_{xx}(\tau) = E\{x(t)x(t+\tau)\} \quad \longleftrightarrow \quad S_{xx}(\omega) = E\left\{X(\omega)X(\omega)^*\right\}$$

$$R_{3x}(\tau_1,\tau_2) = E\{x(t)x(t+\tau_1)x(t+\tau_2)\} \quad \longleftrightarrow \quad S_{3x}(\omega_1,\omega_2) = E\left\{X(\omega_1)X(\omega_2)X(\omega_1+\omega_2)^*\right\}$$

$$\vdots \qquad\qquad \vdots$$

$$R_{nx}(\tau_1,\ldots,\tau_{n-1}) = E\{x(t)x(t+\tau_1)\ldots x(t+\tau_{n-1})\} \quad \longleftrightarrow$$

$$\longleftrightarrow \quad S_{nx}(\omega_1,\ldots,\omega_{n-1}) = E\left\{X(\omega_1)\ldots X(\omega_{n-1})X(\omega_1+\ldots+\omega_{n-1})^*\right\}$$

3. Multisine Design For Behavioral Model Validation

Response of General Nonlinear Dynamic Systems to Arbitrary Multisines

So, the desired multisine must meet the $pdf_x(x)$, the *PSD*:

$$S_{xx}(\omega) = E\left\{X(\omega)X(\omega)^*\right\}$$

and the higher order statistics, e.g.:

$$S_{xxxx}(\omega_1, \omega_2, \omega_3) = E\left[X(\omega_1)X(\omega_2)X(\omega_3)X(\omega_1 + \omega_2 + \omega_3)^*\right]$$

This guarantees that the spectral regrowth is approximated at least for the order of the statistics considered.

(J. Pedro and N. Carvalho, IEEE T-MTT'2005)

3. Multisine Design For Behavioral Model Validation

$$S_{x\dots x}(\omega_1,\dots,\ \omega_n) = E\left[X(\omega_1)\dots X(\omega_n)X(\omega_1+\dots+\omega_n)^*\right]$$

Discretized in K tones, these higher-order statistics require n-dimensional matrix approximations (of K^n points), which can not be obtained by a single multisine of K tones (K phases).

A Multisine of K tones has not enough number of *Degrees of Freedom* !

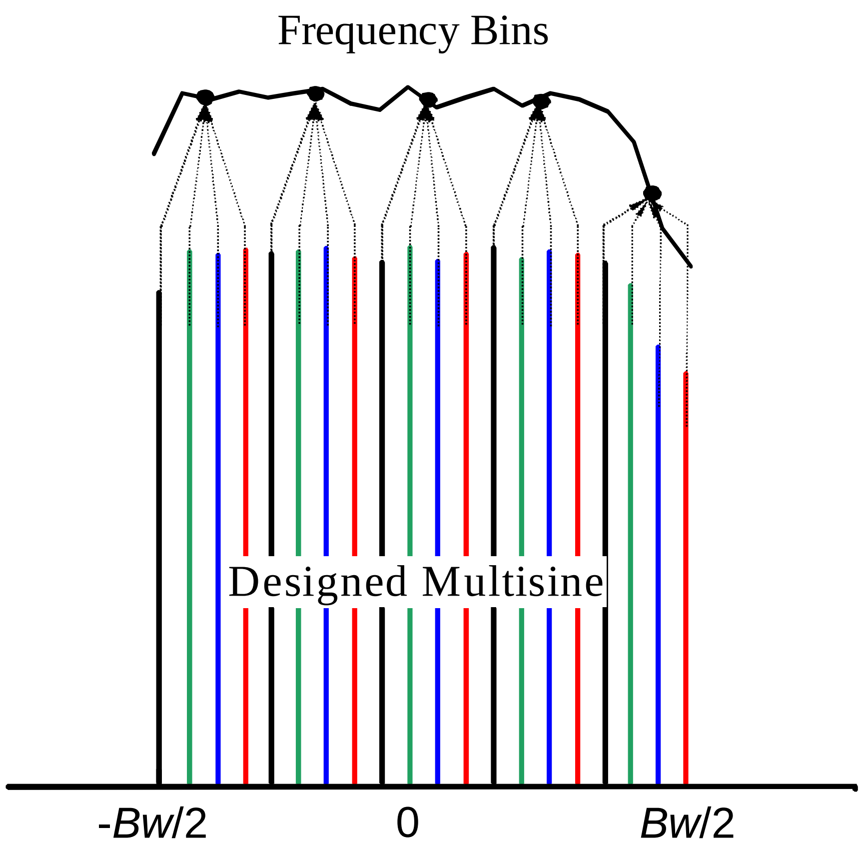
Two possibilities:

1 – An Ensemble of various Multines of K tones each.

2 – A single Multisine with many more tones.

3. Multisine Design For Behavioral Model Validation

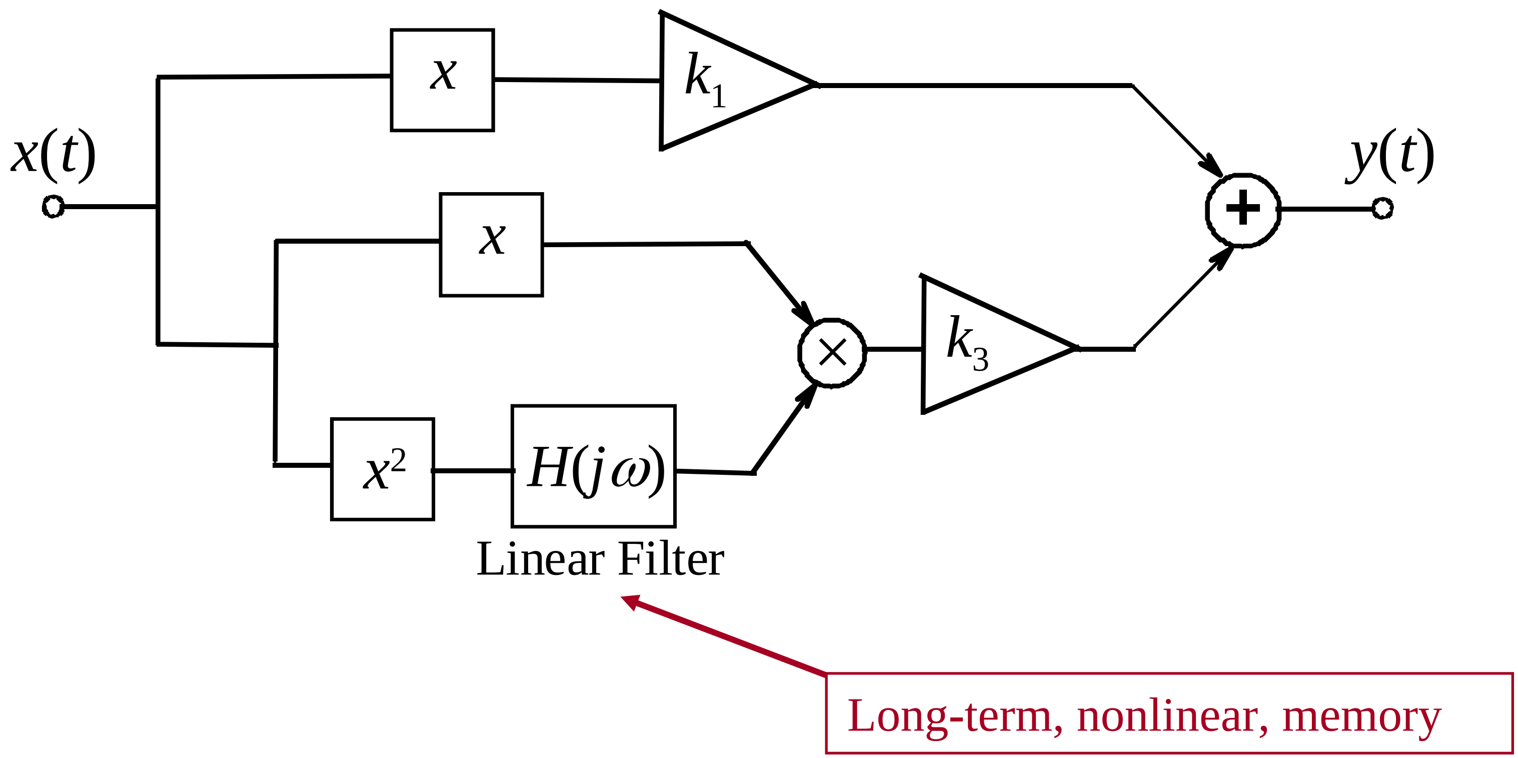
If a single multisine is the goal, then the number of tones must be increased, but the $S_{x\ldots x}(\omega_1,\ldots,\omega_n)$ error evaluated for a smaller number of bins.



3. Multisine Design For Behavioral Model Validation

Multisine Design Example for Nonlinear Dynamic System Excitation

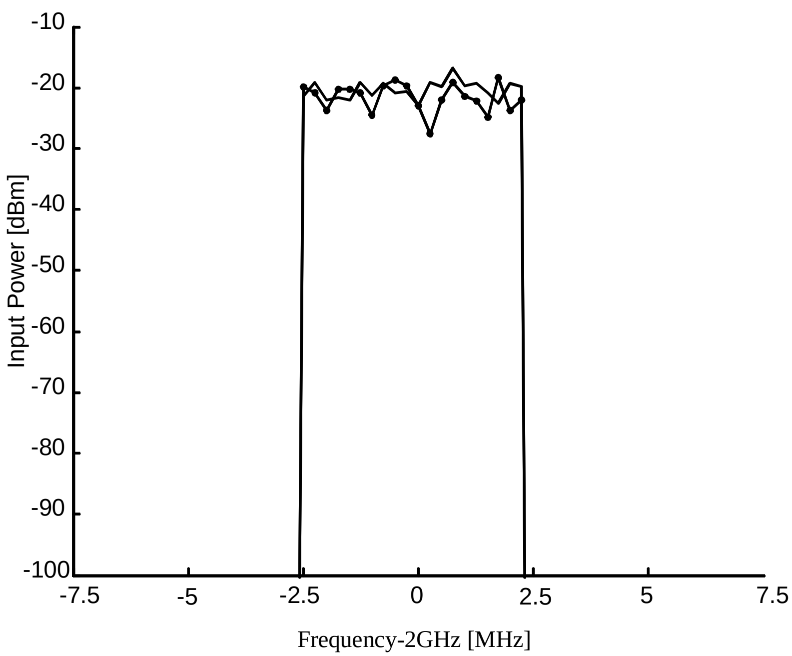
Consider the nonlinear dynamic system, which can be tuned to present certain properties independently:

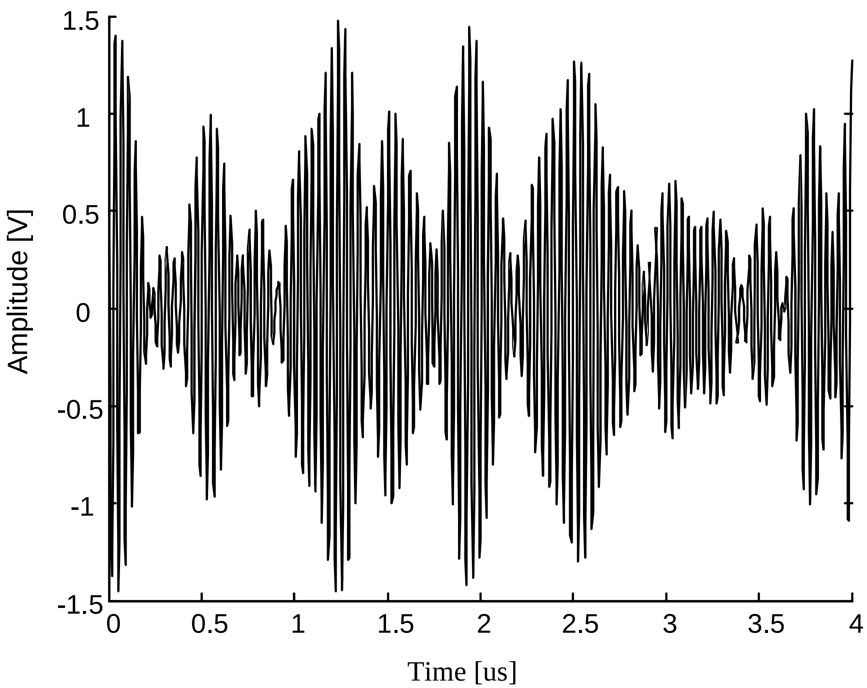


3. Multisine Design For Behavioral Model Validation

Multisine Design Example for Nonlinear Dynamic System Excitation

Consider also an input signal with a pre-determined higher order statistics, for which the multisine is to be synthesized.

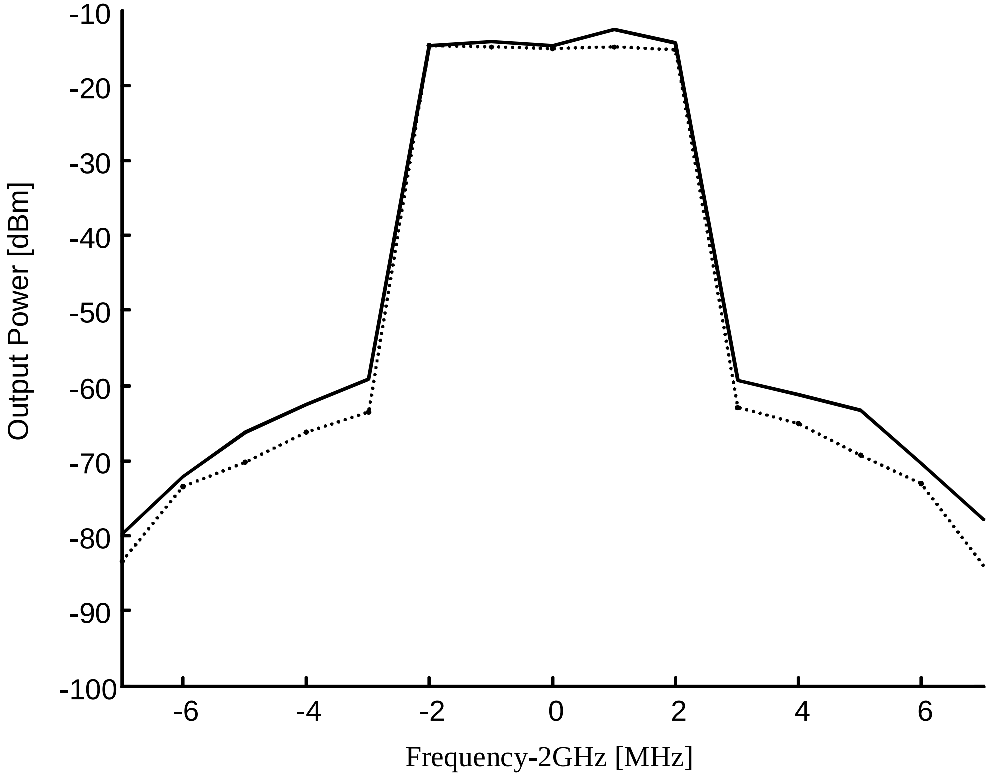




3. Multisine Design For Behavioral Model Validation

Multisine Design Example for Nonlinear Dynamic System Excitation

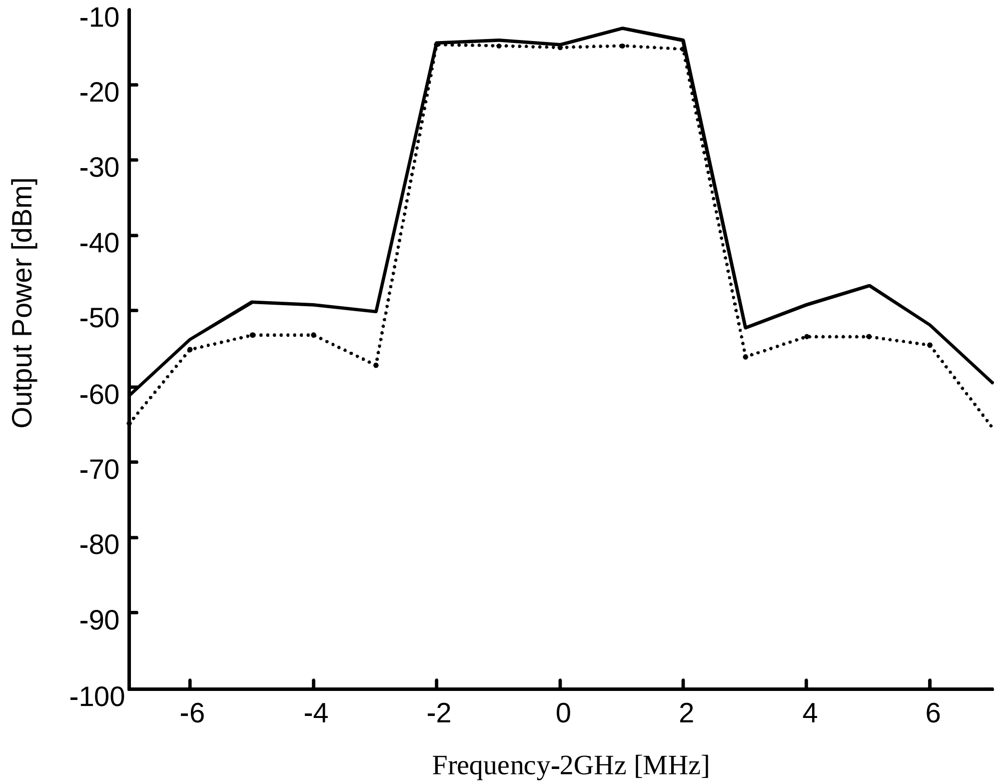
1st Case – A Nonlinear Memoryless System:



3. Multisine Design For Behavioral Model Validation

Multisine Design Example for Nonlinear Dynamic System Excitation

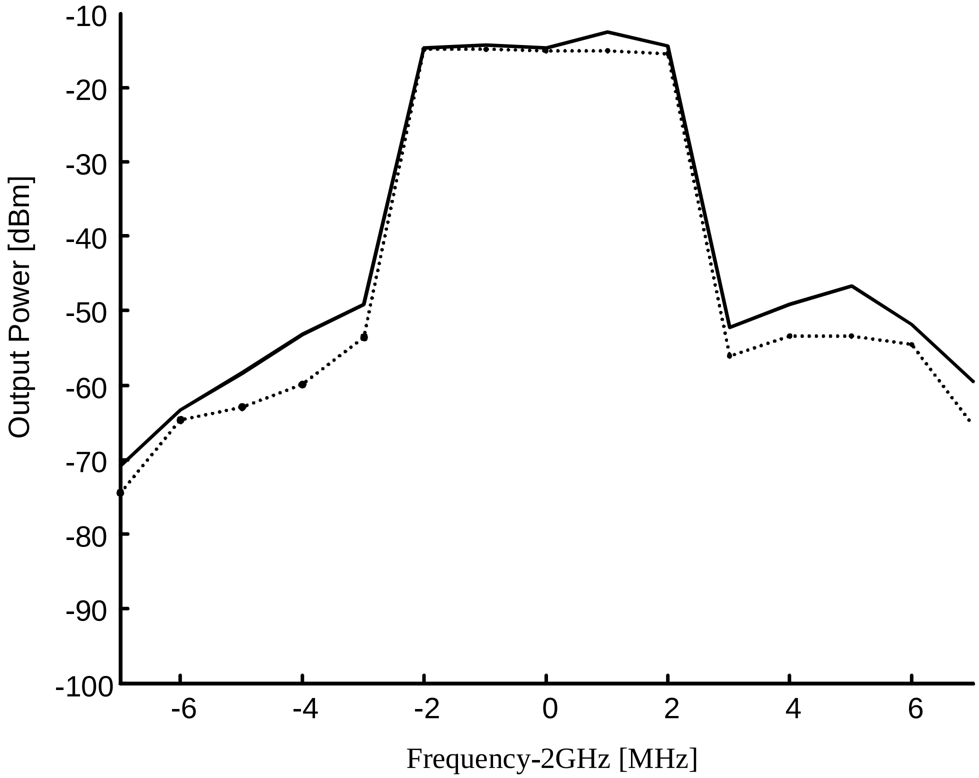
2nd Case – A Nonlinear Dynamic System with Memory at the Base-Band:



3. Multisine Design For Behavioral Model Validation

Multisine Design Example for Nonlinear Dynamic System Excitation

3rd Case – A Nonlinear Dynamic System with Memory at the Base-Band and 2nd Harmonic:



Presentation Outline

1. Excitation Design for Behavioral Model Extraction

2. The Role of Excitation's Statistics
on Nonlinear Dynamic Systems' Response

3. Multisine Design for Behavioral Model Validation

4. Conclusions

5. Conclusions

1. Only very complex test signals like *White Gaussian Noise* can provide all the needed information about the system.

2. Multisines are very promising stimuli, but require deep care in selecting the tone spacing and the amplitude and phase sets.

3. Signal's *pdf* play a determinant role as a signal metric, and thus for excitation design.

4. For nonlinear dynamic systems, the signal's *pdf* is incomplete.
   Joint *pdf*'s of the present input and past samples (within the memory span) are necessary to uniquely determine the response.

5. Conclusions

5. Complete excitation information can be identified by the higher order statistics - *Higher Order Auto-Correlations and Power Spectral Density Functions* - which provide information of both the amplitude and the phase.

6. In the particular context of nonlinear behavioral model extraction or validation with band-limited white Gaussian noise, either an ensemble of K-tone evenly spaced multisines of randomized phases, a single S-tone ($S>>K$) evenly spaced multisine of randomized phases or even a single K-tone multisine of un-commensurate frequencies can be used.

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