BULLETIN

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# Pedro Nunes Lectures

*Pedro Nunes Lectures* is a new initiative organized by CIM, in collaboration with SPM (Sociedade Portuguesa de Matemática), with the support of the Fundação Calouste Gulbenkian, for promoting short visits to Portugal of outstanding mathematicians. Each visitor is invited to give two or three lectures in Portuguese universities about recent developments in Mathematics, its applications and cultural impact.

*Pedro Nunes Lectures* are addressed to a wide audience covering broad mathematical interests, particularly PhD students and young researchers. They constitute an opportunity for students and, especially prospective students to foster acquaintance with outstanding mathematicians. The *Pedro Nunes Lectures* promote the cooperation between Portuguese universities and are a high level complement of PhD programs in Mathematics.

## Contents

## Inaugural Pedro Nunes Lecture

The first sessions of *Pedro Nunes Lectures* will be given by professor Luis Caffarelli (University of Texas at Austin) at the universities of Porto and Coimbra. More information in http://www.cim.pt/?q=pnl_caffarelli.

Professor Luis Caffarelli is Sid W. Richardson Foundation Regents Chair at the University of Texas at Austin, and has been awarded in 2009 the AMS Leroy P. Steele Prize for Lifetime Achievement. He is also the Director of Mathematics at the University of Texas at Austin of the CoLab-UTAustin/Portugal partnership program.

- ***Phase transition and minimal surfaces for non local operators***. July 8, 2009 - Departamento de Matemática da Universidade do Porto.

  **Abstract:** Movement by mean curvature, i.e, when a surface evolves with normal speed proportional to its mean curvature, appears in the modelling of phase transition phenomena, for instance as a limit of phase field models In the case of slow decay of long range interactions, the corresponding limiting transition surface moves proportionally to an "integral version" of mean curvature. We will describe this phenomena, and the geometric properties of the corresponding "integral minimal surfaces".

- ***Fully non linear equations for nonlocal diffusions***. July 15, 2009 - Departamento de Matemática da Universidade de Coimbra.

  **Abstract:** Fully non linear equations arise in optimal control and game theory, when one is able to optimize a (continuous) diffusion process (or has incomplete information about it). A complete theory of existence and regularity for these equations was developed in the eighties thanks to the remarkable contributions of Krylov and Safanov (the Harnack inequality) and of Evans-Krylov (the Evans-Krylov regularity theorem) In a series of papers, in collaboration with Luis Silvestre, we developed the parallel theory in the case of discontinuous (Levy type) diffusion. I plan to present the main steps of the theory, and give an idea of our proof of the Evans Krylov theorem in this case.

## Call for Proposals

**For ESF Research Conferences to be held in 2011**

The European Science Foundation invites scientists to submit proposals for high-level research conferences to take place in 2011 (and 2012 for Mathematics) within the framework of its Research Conferences Scheme in the following scientific domains:

- Molecular Biology;
- Brain, Technology and Cognition;
- Mathematics;
- Physics/Biophysics and Environmental Sciences;
- Social Sciences and Humanities.

**Submission deadline:** 15 September 2009, midnight.

For more information please contact the ESF Conferences Unit at conferences-proposals@esf.org.

(see http://www.esf.org/activities/esf-conferences/call-for-proposals.html)

# ESF and Mathematics in Europe

The European Science Foundation (ESF), http://www.esf.org, is an independent organisation based in Strasbourg aiming to Advance European research in all areas and to explore new directions of research at the European level. The ESF covers Humanities, Life, Earth and Environmental Sciences, Medical Sciences, Physical and Engineering Sciences and Social Sciences and represents an important interface through its membership, currently 78 different research organisations in 30 countries, extending beyond the borders of the European Union.

The activities of ESF that are more relevant to Mathematics, that is currently among the Physical and Engineering Sciences, are Research Networking Programmes, ESF Research Conferences, the Forward Looks and Exploratory Workshops. These are aimed to provide foresight and advice on science, research infrastructure and science policy issues.

For instance, a recent ESF Exploratory Workshop on "Curves Coding Theory and Cryptography", was held in Marseille, France last March. According to its abstract, "Algebraic curves entered into coding theory in the 1980's with Goppa's introduction of algebraic geometric codes. The proposal of elliptic curves for use in cryptography by Koblitz and Miller in 1985 culminated new elliptic curve-based cryptographic U.S. government standards in 2005. This workshop united researchers actively working on the computational aspects of curves, in order to explore interdisciplinary research and applications to coding theory and cryptography." This topic was recently highlighted in the ESF website as "Mathematical advances strengthen IT security", that recognised the potential importance of the mathematical theory of elliptic curves as a leading candidate for more efficient cryptography than the RSA cryptosystem, introduced by Rivest, Shamir, and Adlement in 1977. Quoting David Kohel, the convenor of the ESF workshop "the size of the parameters (essentially the key size) for elliptic curve cryptography (ECC) needed to ensure security (under our current state of understanding) is much lower for ECC than for RSA or ElGamal (another alternative cryptographic method)", so the advantage of elliptic curve cryptography lies in its immunity to the specialised attacks that have eroded the strength of RSA, with the result that smaller keys can be used to provide a given level of protection.

**Research Networking Programmes**

In 2009 ESF has 9 Programmes strongly related to Mathematics, among the 31 Programmes in the list of Physical and Engineering Sciences, with the following titles and durations:

- Interactions of Low-Dimensional Topology and Geometry with Mathematical Physics (2009-2014);
- New Frontiers of Infinity: Mathematical, Philosophical and Computational Prospects (2009-2014);
- Optimization with PDE Constraints (2008-2013);
- Harmonic and Complex Analysis and its Applications (2007-2012);
- Quantum Geometry and Quantum Gravity (2006-2011);
- Automata: from Mathematics to Applications (2005-2010);
- Advanced Mathematical Methods for Finance (2005-2010);
- Methods of Integrable Systems, Geometry, Applied Mathematics (2004-2009);
- Global and geometrical aspects of nonlinear partial differential equations (2004-2009).

According to the ESF guide-lines, the "Research Networking Programmes are "open" activities. Principal participants within a Programme, e.g. Steering Committee members, are expected to network with colleagues in other research groups to ensure that opportunities in a Programme's activities are known and are open to all eligible participants. New participants to a Programme from participating countries can be coopted during the lifetime of the Programme upon decision of the Steering Committee".

Although ESF activities have now more than thirty years, the situation towards the Mathematical Sciences is now different and has much improved in recent years. There is no possible comparison between the current activities in Mathematics and the situation in the begin of the nineties, when one of the first five years Scientific Programmes in this area was successfully proposed through an Iberian initiative on "Mathematical Treatment of Free Boundary Problems" and launched in January 1993, when the ESF had 54 member institutions from only 20 countries. FBPNews, the Newsletter of ESF mathematical programme was available in the web since 1994 and can still be consulted at http://newsletter.fbpnews.org/.

**ESF Research Conferences**

The European Science Foundation (ESF) and the European Mathematical Society (EMS), supported by the European Research Centres on Mathematics (ERCOM), have agreed to co-sponsor a series of Scientific Conferences, within the framework of the ESF Research Conferences Scheme. The conferences generally last for four or five days and up to 150 participants and invited speakers may attend. Chairs select participants from applications received as a result of publicising the conferences. A conference fee is charged to participants. The Series is known as 'ESF-EMS-ERCOM Mathematics Conferences', and aims at the highest scientific level with respect to topics and choice of participants. The Conferences intends to bring together participants and experts in mathematics to discuss topics that are of major importance to the scientific community in Europe. Conferences in the series are supposed to be hosted in and co-sponsored by selected ERCOM institutions participating in the Scheme, including CIM.

For 2010 the following ESF Mathematics Conference in partnership with EMS and ERCOM have been announced: Algebraic Methods in Dynamical Systems, Bedlewo, Poland, 16.5 - 21.5; Teichmueller Theory and Its Interactions in Mathematics and Physics, Bellaterra, Barcelona, Spain, 28.6 - 3.7.

**Forward Look on Mathematics and Industry**

With the purpose "to explore ways of stimulating and/or intensifying the collaboration between Mathematics and Industry" the ESF has started last April in Rome a new Forward look, that was proposed by Mario Primicerio (U. Florence) on behalf of the Applied Mathematics Committee of the EMS. This project intends to identify common issues, questions, and "good practices" between Mathematics and Industry in order to envisage strategies for a stronger interaction of mathematicians with large and medium size companies aimed at technological advancement. The project that should be completed in 2010, will build on the results of the OECD 2008 report on Mathematics and Industry (see: http://www.oecd.org/dataoecd/47/1/41019441.pdf) by focussing on the specificities of the European context.

The activity of the Forward Look, that is coordinated by the proposer and current president of the Applied Mathematics Committee of the EMS is structured in three working groups dedicated to "Training and career development", "Academia-Industry interface" and "Opportunities and challenges", that are coordinated, respectivly by Magnus Fontes (U. Lund), by Volker Mehrmann (T. U. Berlin) and Yvon Maday (U. Paris VI). More information can be found in the Forward Look website http://www.ceremade.dauphine.fr/FLMI.

**Towards a European Virtual Library in Mathematics**

An ESF Preparatory Meeting for exploring the creation of an European infrastructure for Mathematics, focused on digitization, access to research journals, doctoral dissertations and bibliographical databases was organised in Santiago de Compostela, Spain, 13-14th March 2009 http://www.usc.es/ esfmaths/. Following several joint activities between the European countries participating in the WDML (World Digital Mathematical Library http://www.wdml.org/) digitization project, a certain number of workshops and meetings were held in the last years, including Berlingen (2002, 2003), Gottingen (2003), Stockholm (2004), Aveiro (2006), Prague and Birmingham (2008).

Many European mathematical societies, national libraries and documentation centres have been collaborating in order to prepare digitization projects at a European level, including the so-called DML-EU initiative under the auspices of the EMS. In particular, the EMS Committee on Electronic Publishing (EPC/EMS) is keeping the Society "abreast with the development of electronic tools of doing mathematics such as electronic publishing, archiving and communication" and keeping reviews on the progress in these areas and making suggestions helping to build the appropriate electronic infrastructures. This issues involve topics such as the need for standardization and coordination, identification of intellectual property rights, the conflict of interests among stakeholders, technical standards, metadata, long term preservation and the importance of bibliographical databases, such as, Zentralblatt MATH or MathSciNet.

The Santiago de Compostela preparatory meeting, with more than thirty participants, had a programme organised under the coordination of Manuel de Léon (PESC/ESF), Pavel Exner, Thierry Bouche and Enrique Macías-Virgós, from the EPC/EMS. The workshop aimed to survey the current activities in Europe, and brief reports of national initiatives aligned with the European Virtual Library of Mathematics where presented, namely from Bulgaria, Czech Republic, France, Greece, Italy, Poland, Serbia, Spain and United Kingdom.

The conclusions of this workshop helped to prepare a European perspective on the topic and contributed to the preparation of another future collaborative initiative of the ESF with Mathematics in Europe. In particular, it is also expected that it may be useful to the EMS that is facing the challenge of the new FP7 opportunity on "Infrastructures for Mathematics and its interfaces in science, technology and society at large".

# An Interview with Jean-Éric Pin

*Let us start with your initial academic path. Your first university degree is in Mathematics and then you did your third cycle thesis[1] on automata, more precisely on Černý's Conjecture, a conjecture that is still unsolved...*

I did not start working on automata, it happened by chance. I started by attending a course on categories, which I did not like very much. Afterwards I followed a course on semigroups taught by Paul Dubreil. This was his last course and it was fairly elementary, but Dubreil suggested that the students attend the seminars given by Klaus Keimel, a German researcher living in France. I attended these conferences and once, Keimel and I met on the suburban train to Paris. At that time, I had done some work on semigroups by myself and I asked him for advice since I did not know whether it was worth publishing and how to do it. This work was published some years later under the horrible title *Holoïdes factoriels*. I also asked advice about my future studies and Klaus Keimel told me that Prof. Dubreil was about to retire, and suggested that I move to Schützenberger's school on automata. According to Keimel, Marcel-Paul Schützenberger was a genius, an opinion to which I fully subscribe. The following year there were two courses on the same day, one on the algebraic theory of automata, managed by Jean-François Perrot, and another one on the theory of context-free languages, managed by Maurice Nivat. I took the first course, together with no more than four other students. This course was taught by Jean-François Perrot, Dominique Perrin, Gérard Lallement, Jean Berstel and the tutorials were given by Jacques Sakarovitch and Jean-Michel Autebert, so we had the best lecturers of that time for just a few students. I learned a lot in this course and it was here that Perrot talked about Černý's Conjecture. The conjecture was easy to understand and quite fascinating, so I decided to work on it.

*Were you so naive then as to think that you could solve the problem easily?*

Not really. What happened was that, during my DEA[2], I solved a particular case and Perrot encouraged me to pursue this direction. In the second year of the third cycle thesis, I became interested in varieties of semigroups and varieties of languages and this topic ultimately became my main topic of interest.

*Do you feel that during the time you were preparing your two Ph.D. theses you had a master?*

Perrot, my supervisor, taught me a lot of things. I learned from him how to write a paper and organize a conference presentation. He took me to a conference in Italy at the end of the first year of my Ph.D. On this occasion, he introduced me to several people, notably Antonio Restivo and Aldo De Luca, that I met for the first time there. All of this turned to be very important for the future of my career. Aldo and Antonio and I remain very good friends. We met up just two weeks ago in Belgium.

Jean-Éric Pin

*And Schützenberger?*

I was not a direct student of Schützenberger, but, of course, I learnt a lot reading his articles. I have to say that Perrot helped me to start reading Schützenberger,

[1] Until 1984, the Ph.D. programme in France was composed of two theses: the first one was called *thèse de troisième cycle* and the second one *thèse d'état*.

[2] *Diplôme d'études approfondis*, a French degree corresponding to a master's degree.

whose style is rather peculiar. Perrot's claim was that Schützenberger was acting like a fox rubbing out its track with its tail. Therefore, reading Schützenberger was sometimes quite demanding for a young student. But when you read an article really in full detail, then at some point you become so familiar with its content that you have the feeling it is your article. There are actually very few papers that I read in such a depth in my life, but they include some of Schützenberger's and also an article by Wolfgang Thomas on the connections between automata and logic that I read a few years later. Schützenberger had a singular personality and this was a handicap for me to a closer relationship with him. Thus I was mostly influenced by Perrot, Perrin and Berstel. I only met Schützenberger from time to time, but I learned a lot from his papers. And he was also the president of the jury for the defense of my thesis.

*But did you discuss his work with him?*

There are a certain number of questions that I discussed with him, but these discussions were rather peculiar because he was on a much higher level than me and there were some sentences that I could not even understand at the time. But it became easier when I became more expert in automata theory. I remember that when I explained him my ideas about ordered semigroups, he got really interested and suggested that I read some related articles.

*In your huge list of publications, there are some papers in which you approach finite automata in a purely combinatorial way and other papers where the main tool is logic, but most of them are about finite semigroups, the algebraic counterpart of finite automata. Why? Is that just a matter of taste?*

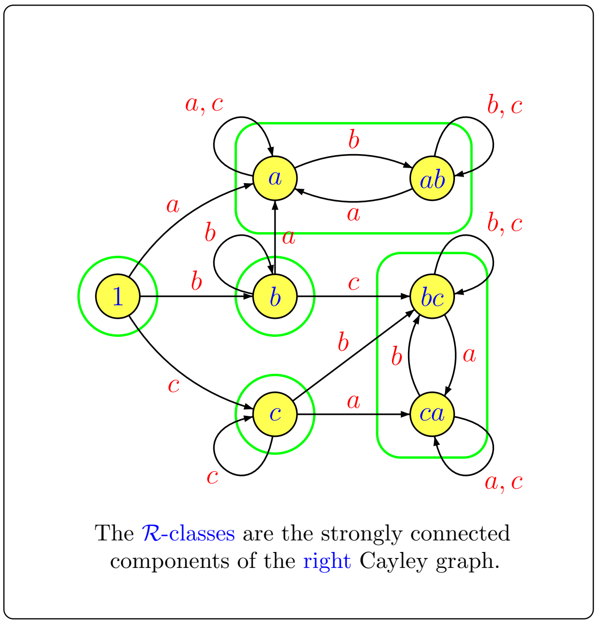
This is a matter of taste. Semigroups lured me even before I started my research. I always liked algebra very much. I read a first course on algebra by Roger Godement during the holidays immediately after the high school final exams and algebra became my favorite topic. Next I became interested in logic and more specifically, in model theory. Schützenberger, who worked with McNaughton, apparently never got interested in logic, I don't know why. Neither was it a conversation topic with Perrin and Berstel at that time. But it changed later. My interest in logic started when I studied the paper by Wolfgang Thomas I mentioned earlier. I really wanted to understand this article and I knew absolutely nothing about logic, even the basic definitions. I then asked one of my colleagues, Michel Parigot, a logician, for a reference book on logic and finite model theory. He recommended the first chapter of the Handbook of Mathematical Logic edited by Jon Barwise, which turned out to be an excellent advice. I read this chapter carefully and actually attempted to read other chapters as well. After that, I really got interested in the relationship between languages and logic. Even if my preference goes to algebra as I said before, I really enjoy the connections with logic. As any mathematician, I like the interaction between different areas of science that are apparently far apart and I have always looked for them.

*In the late eighties you began using profinite tools, which allowed you to find remarkable results...*

This is an interesting story. At that time Jorge Almeida was working on profinite monoids, but he preferred the approach by implicit operations, which hides a bit of the topological aspect. Independently, I read a paper by Christophe Reutenauer, called *Une topologie du monoïde libre*, which contained very interesting ideas that I decided to explore. In 1983, Stuart W. Margolis was an invited professor at the University Paris 6 and he spent these nine months at my place. We worked together in several directions and wrote several papers about inverse semigroups and semigroups with commuting idempotents. On this occasion, Stuart told me about the Rhodes Conjecture. This conjecture proposed an algorithm to compute the group radical of a finite monoid. I remember that I got the idea of my first crucial result on this subject in 1984, in the kitchen of Howard Straubing: I realized that computing the group radical of a finite monoid amounts to computing the topological closure of a regular language for the pro-group topology. This was the first of a long series of results and I wrote several papers about this subject. The first one was entitled *Topologies for the free monoid* and was published in the Journal of Algebra. This paper proposed a conjecture that one could compute the topological closure of a regular language by a simple algorithm, and discussed its consequences for the Rhodes Conjecture. It took over four years to be published and other papers about the same subject that were submitted later on were actually published much earlier. The next paper was entitled *A topological approach to a conjecture of Rhodes* and was published in the Bulletin of the Australian Mathematical Society. It gave a complete proof that the topological conjecture implies the strong form of the Rhodes Conjecture. Finally, Reutenauer and I reduced the topological conjecture to a conjecture on the free group: if $H_1, \ldots, H_n$ are finitely generated subgroups of the free group then the set $H_1 H_2 \cdots H_n$ is closed in the profinite topology. This latter conjecture was proved by Ribes and Zalesskii in 1992. But Rhodes' Conjecture was also proved by Ash using different arguments, actually a few years earlier.

*Later you got into the equational part of the varieties and you, along with Pascal Weil gave, for instance, an equational characterization of the Mal'cev products of two varieties of finite semigroups...*

I have been working on the Mal'cev product since the time of my Ph.D. I was studying some variants of the concatenation product, such as the unambiguous product, and Pascal Weil brought the equational part to me. I remember the precise moment I discovered one of the key arguments of this paper. I have to confess it was during a talk by Denis Thérien at the NATO School at the University of York in 1993. I suddenly had the intuition that Imre Simon's Factorization Forest Theorem was the technical tool we needed, although I did not remember precisely its statement. Victoria Gould was kind enough to comply with my surprising urgent request to get a copy of Simon's paper and I could verify that my intuition was right. Simon's theorem is nowadays considered to be a major combinatorial tool in semigroup theory.




Computation of an $\mathscr{R}$-class using the right Cayley graph of a semigroup.

*Unlike Jorge Almeida, you never worked heavily on combinatorics of profinite words...*

That is true. I am certainly less attracted by combinatorics on words than Schützenberger, Berstel, Perrin, etc. I always felt more comfortable with algebra. I use combinatorial results, such as the Factorization Forest Theorem, when I need them, but I generally prefer algebra to combinatorial arguments. It is just a matter of taste.

*For a long time, the algebraic study of recognizable languages relied on Eilenberg's theory of varieties inspired by some results of the 60's characterizing classes of languages in terms of semigroups, such as those of Schützenberger and Simon. Now, some people working in this area, such as Straubing, Thérien and yourself, were also interested in classes of languages that are not necessarily varieties, although they can be studied algebraically. Do you think that the interesting remaining open problems in this area are only the very difficult ones, like the decidability of the group complexity or the decidability of the dot-depth hierarchy?*

No, not at all, this is a flourishing area and there are plenty of interesting open problems, old and new. The recent paper *Duality and equational theory of regular languages*, by Mai Gehrke, Serge Grigorieff and myself, goes far beyond the classical context of the varieties. The classes of languages considered in this paper are more general than Eilenberg varieties and the theory developed in this paper also applies to infinite words, words over linear orders, tree languages, etc. By the way, exciting results on tree languages were recently obtained by Mikołaj Bojańczik, Zoltan Esik, Luc Segoufin, Howard Straubing, Igor Walukiewicz, Pascal Weil, etc. Concerning the main open questions in the area, some of them, such that the decidability of the dot-depth hierarchy, can be viewed not only from the perspective of the algebra, but also from the perspective of logic, and therefore they can be treated in the theory of finite models. The profinite approach also opens up fascinating perspectives on the classification of languages. I recently proposed to study the Wadge hierarchy associated with some profinite uniformly continuous functions and Pedro Silva and I just founded a non-commutative p-adic analysis. The connection with Fraïssé-Ehrenfeucht games is also on the way. There are also some very nice connections with duality theory, symbolic dynamics, combinatorial group theory or tropical geometry. There is now a continuum between topology, algebra, logic and automata theory. Top researchers of the new generation, like Bojańczik and Walukiewicz, who were both trained as logicians, are now convinced of the power of the algebraic approach. This makes me very optimistic for the future of this field.

*When did you start being interested in infinite words?*

My first paper on this topic dates back to 1984. A few years later, Dominique Perrin suggested me one day to write a book about infinite words with him, and without really thinking about it, I agreed. But it took us over fifteen years to complete this book!

*You are now quite interested in the work that you, Mai Gehrke and Serge Grigorieff started together about languages and dualities, two subjects that seemed apparently far apart. How did that start?*

At a conference in Nashville in 1996, I gave a talk about varieties and profinite topologies. Mai Gehrke, who was in the audience, mentioned she was interested in this topic. We started discussing its connection with nonstandard analysis but we didn't go very far at that time. We met again ten years later by pure chance. In November 2005, I was looking for a paper on the internet and I stumbled upon Mai's homepage, but the link to the paper I was looking for was broken. I wrote Mai an e-mail to warn her of the broken link and she answered she was curious to know why I was interested in her paper. In the course of the discussion that followed, I mentioned that I was looking for an expert in spectral topologies. I received an immediate and very enthusiastic answer from Mai, explaining this was one of her favorite topics. This is the way it started. In 2006, we were expecting an invited professor for a three month position at my research group but he had to decline, due to the late arrival of the official approval. I then asked Mai if she was interested in this position and she said yes. Thus in June 2006, Mai, Serge Grigorieff (a colleague of mine) and I started to work together. Mai had realized that the work I talked about in Nashville had a duality flavor, but she knew nothing about automata and Serge Grigorieff and I knew nothing about dualities. Thus our collaboration started by giving each other an introductory course on our favourite topics. But after a few weeks, we could understand each other and we started to make fast progress. It took us another year to publish our results in a short article that won a best paper award at ICALP 2008. We are now writing its complete version, and some other papers are on their way.

*Are these results another way to see things?*

Exactly that. The duality between regular languages and profinite words was known to Jorge Almeida for a long time. The novelty is the use of this duality to obtain an equational theory for any lattice of regular languages. This is particularly appealing for all the classes of regular languages defined by a fragment of logic closed under conjunctions and disjunctions, because this means that, in principle, an algebraic study is possible for these classes.

*Among your many results do you have a favorite one?*

My favourite result is probably the topological approach to the Rhodes Conjecture I mentioned earlier. But I also like the concept of ordered syntactic monoid that I introduced in 1995. It is a very simple idea, but it has far reaching consequences. I also like the result on dualities, but it is too early to measure its consequences.

*Are you still interested in Černý's Conjecture?*

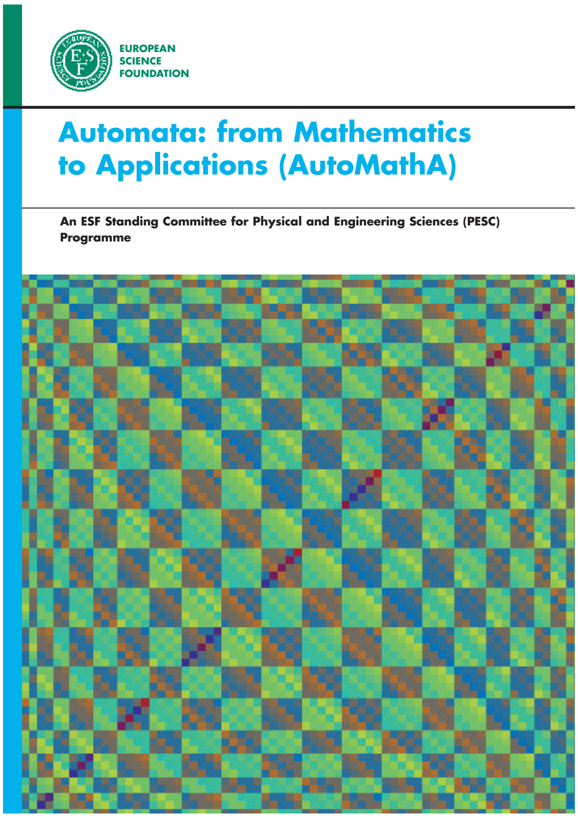
Let me first recall this fascinating conjecture for the reader: *If an n-state automaton is synchronizing, there exists a synchronizing word of length* $\leq (n-1)^2$. I am still interested in the conjecture, but I am no longer working on it. However, I am still receiving requests to talk about this topic in seminars and workshops. The only thing I have done in recent years was an article with Stuart Margolis and Mikhail Volkov, published in 2004. In my opinion, there have been two major results in recent years. The first one, due to Jarkko Kari, gives a counter-example to the extension of the Černý Conjecture that I proposed in my thesis. This is an important result because it more or less kills any hope of interpreting the upper bound $(n-1)^2$ as the dimension of some vector space. The other major result is Volkov's result on automata preserving a chain of partial orders.

*In your opinion, why is Cerný's Conjecture still a conjecture?*

It is not unlikely that Cerný's Conjecture is false, but finding a counterexample might be difficult. One may have to work with large automata and testing whether a relatively small automaton is synchronizing may already exceed the capacity of a computer. The best known upper bound is still cubic and did not improve since 1982, although the conjecture has been proved in many particular cases. However, we are still very far from a solution and it might well be a very difficult combinatorial problem.

*You worked at the Bull company for two years at the beginning of the 90's. How did it happen?*

This story is similar to that of the book on infinite words that I wrote with Perrin. I was a member of the National Council of CNRS, and another member of this council was a engineer at the Bull Research Center. He was taking the opportunity of seeing many CVs of researchers, to hire a researcher from time to time for the Bull Research Center. One day, jokingly, I asked him when he would hire me, but to my surprise, I got a concrete job offer as an answer! This was the beginning of the story, because Bull was interested in someone with my profile and I got interested in the experience. At that time, the research department at Bull consisted mostly of young people, including Ph.D. students and numerous top-level researchers, but very few senior researchers. One part of my work was to play this role of scientific management. An important thing which I did was to initiate a seminar. At the beginning, people were skeptical and I was only allowed to set up a monthly seminar, but after three months, it had been sufficiently successful to become a weekly seminar. There was an external speaker every two weeks, and each other week we had a speaker from Bull. This turned out to be very important to promote the activities of the Bull Research Center to the academic world and to keep permanent contact with researchers from outside. One thing made me very happy once: a colleague of mine working at the Ministry told me after his visit that his opinion about the Bull Research Center had completely changed, from a strongly negative to a very positive opinion. During my second year, Bull started to have serious financial problems and my colleagues found themselves in a delicate situation since the research center was shrinking day after day. But as far as I know, all of them managed to find a position elsewhere in companies or universities. And for some of them, their new connections with the academic world certainly helped. The most successful of them, Dominique Bolignano, founded his own company, Trusted Logic, with the initial support of Bull and INRIA. This company is now a world leader in embedded security solutions. Three months after I left Bull, I became the head of LITP[3], a joint research unit of the CNRS and the University, and my experience at Bull, which included some management courses, helped me a lot in this task.




Leaflet cover of the networking programme of the European Science Foundation AutoMathA chaired by J.-E. Pin (the picture is from Jorge Almeida and represents the action of Thue-Morse operator on the cyclic group of order 70 - see the feature article by J. Almeida in the CIM Bulletin n. 14 from June 2003).

*Let us talk a little about your laboratory, LIAFA. What is the initial academic profile of the members of LIAFA?*

Most of the French members come from the Grandes Écoles, mainly from the Écoles Normales Supérieures and a few from École Polytechnique or other engineer schools. Many members also come from foreign countries. Some have a degree in Mathematics, others, notably among the youngest members, have a degree in Computer Science, but also often a good background in Mathematics.

*What is the reason for such a number of foreigners?*

This is due to the scientific policy of the department, in particular mine when I was the head from 2003 to 2008. There are two kinds of positions. The selection process for the permanent research positions offered by the CNRS is a national competition. Our department is not involved in this process, except that the candidates have to express their wish to be appointed in LIAFA. High level research departments like LIAFA are very attractive to candidates so we have each year some good recruits. Foreigners are entitled to apply for a CNRS position and as a consequence, several of our full time researchers at LIAFA are non-natives. There are also university positions, such as Assistant Professor and Professor, and for these positions, especially for Professor, local people are usually discouraged. During the last twelve years, only one Assistant Professor from LIAFA was promoted Professor in the department. But many became professors elsewhere and thus there is a high turnover in the department.

*You have been a member of several scientific evaluation or advisory committees in France, and also in other countries, such as Portugal. In your opinion, are there bad relationships between people from theoretical computer science and people from applied computer science?*

I don't have this feeling, at least at the level of people. It is true however, that nowadays there is a strong tendency, in France and elsewhere, to favor short-term research. Killing research with no immediate application will have disastrous consequences in the long-term, and perhaps sooner. The other strong tendency is to focus on fashionable keywords. For instance, the forthcoming French-Spanish collaboration programme PICASSO selected the following topics: biomedicine, biotechnology, energy and climate change, nanotechnologies. For the French-Portuguese Pessoa programme, it is announced that "a thematization will be committed as from 2011 in order to reinforce the structuring character of the cooperation". "Structuring character" is one of these fancy keywords which, like "synergy", are mandatory in any cooperation application…

*Do you have the same opinion about the European Commission?*

The European Commission has kept some important support for fundamental research. For instance, the

[3]Laboratoire d'Informatique Théorique et Programmation, later called Laboratoire d'Informatique Algorithmique: Fondements et Applications (LIAFA).

Future and Emerging Technologies Open Scheme is a flexible tool for exploratory research, where one can submit proposals with a component of a fundamental nature. The European Research Council (ERC) offers advanced grants for both young and senior researchers. These grants are very selective, which is not a bad thing. The drawback is that writing applications is time-demanding.

*Let us talk now about your connections with Portugal. When did they start?*

I do not remember exactly when they started. It certainly begun with some early correspondence between Jorge Almeida and me, but I am not sure about the year. Later, I met Gracinda Gomes in a conference in Szeged, Hungary, in 1987. The cooperation between Jorge's and Gracinda's groups and my own group developed over the years. Three portuguese students did their Ph.D. in Paris and I participated in the juries of several other portuguese students. We collaborated in many scientific projects, either between France and Portugal or at the European level. I was also a member of the scientific committee of several semigroup conferences organized in Portugal and I was co-organizer, together with Gracinda Gomes and Pedro Silva, of the Thematic Term "Semigroups, Algorithms, Automata and Languages" held in Coimbra in May, June and July 2001. Finally, I am the chair of the programme "Automata: from Mathematics to Applications" (AutoMathA), a very successful research networking programme of the European Science Foundation gathering 15 European countries, including France and Portugal. One of the highlights of this programme has been a two week school organized in 2008 by Gracinda Gomes in Lisbon. The thematic term and the 2008 school were both completely successful and it is my opinion that training young students in high level research also has its "structuring character".

*In France there are several research centers in theoretical computer science, some of them quite big, LIAFA being a very good example. In Portugal there are none at that level. In your opinion, does Portugal need one?*

I would not say that. A few emblematic people, like Jorge Almeida and Pedro Silva in Porto for instance, or Gracinda Gomes in Lisbon, suffice to create a solid research group. Two or three dynamic people are enough to set up a research lab, as long as they get sufficient support to attract good researchers and Ph.D. students. Of course, money is needed to run such a center, since it is essential to send people to international conferences and to collaborate with other universities, both in Portugal and abroad, to invite researchers to give lectures in seminars, to hire researchers, to maintain a library and computer facilities, etc. Large research groups certainly have a broader international dimension, but the advantages over a strong small group are limited. There is now a tendency to create larger centers to improve the ranking of universities in various evaluations and comparing systems, such as the Academic Ranking of World Universities. But one should be very pragmatic about the policy of research centers. I know of some small, very active research groups. As long as their activity is excellent, why should one change their structure? Quarrels between different groups are a danger for large centers, increase of bureaucracy is another one. If these two hazards can be avoided, if each group has enough resources to avoid fighting, I have no objection to large centers, but the most important thing is research activity.

Jean-Éric Pin with Gracinda Gomes at Centro de Álgebra da Universidade de Lisboa in June 2009.

*You are a very active person. You can manage a lot of things at the same time. For instance, you do research very actively, you give many talks abroad, you supervise Ph.D. students, you are a member of several committees and boards of examiners, you are editor of four scientific journals, you were the director of your laboratory from 1994 to 1997 and from 2003 to 2008, etc. How can you do all these?*

I do not think I have been doing so many things at the same time and plenty of people I know are much busier than I am. I have been primarily a full time researcher, apart from a 14 year period when I taught part-time at École Polytechnique. Concerning the management of research units, LITP first and LIAFA later, the key word is *delegate*. Each time you can entrust a task or a responsibility to another person, do it. For the main decisions, it is a good idea to listen to people and to look for their advice and support. This can be time consuming, but it is really worth for important decisions. This way, I was able to reach a common agreement most of the time. Further, several decisions were taken by specialized committees, notably for hiring people. But then again I tried to reach a common agreement by using only fair arguments. Once the decision was taken, I also used to explain it to the members of the unit. On a more personal level, I am a well organized person regarding computer files and e-mail. Further, unlike some colleagues of mine, I did not hesitate to decline some invitations (programme committees, cocktails, etc.).

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Jean-Éric Pin is Directeur de Recherches at the CNRS in the Laboratoire d'Informatique Algorithmique: Fondements et Applications (LIAFA), of CNRS and University Paris 7, and a member of the Automata and Applications team. He works on theoretical computer science and he is well known for his wide contribution to the area, including many breakthroughs and original ideas. Born in 1953, Jean-Éric Pin got his university degree in Mathematics from École Normale Supérieure de Cachan and his Ph.D. degree in Mathematics from University Paris 6 in 1981. He has always worked on theoretical computer science, with a particular emphasis on its connections with algebra. He has been a researcher at CNRS since 1979. He has also been a professor at École Polytechnique from 1992 to 2005 and a research engineer at the Bull Corporate Research Center from 1991 to 1993. His research is mainly devoted to the algebraic theory of finite automata via the study of finite semigroups, in particular varieties of finite semigroups. He has always looked for relationships between different areas of Mathematics and his work also includes papers about logic, topology or combinatorics. Jean-Éric Pin is the author of over 130 scientific publications, including two undergraduate books, two research books and 14 book chapters. Twenty-two students have received their Ph.D. under his supervision. He earned the IBM France Scientific Prize in Computer Science in 1989 and a Best Paper Award at the conference ICALP 2008. He is a member of the editorial board of four scientific journals and he has a wide experience as a member of programme committees as well as management positions. To give some examples, he has been the head of the Laboratoire d'Informatique Théorique et de Programmation (LITP, from which LIAFA originated), of the Laboratoire d'Informatique Algorithmique: Fondements et Applications (LIAFA) and scientific director of the European Research Consortium in Informatics and Mathematics (ERCIM). He is chairman of the European Science Foundation (ESF) programme AutoMathA (2005-2010). This interview with Professor Pin is intended to give an overview of his life as a researcher with its many facets.

*Interview by Mário Branco (University of Lisbon)*

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1 - ERCOM is an European Mathematical Society (EMS) committee consisting of Scientific Directors of European Research Centres in the Mathematical Sciences, that aims to contribute to the unity of Mathematics, from fundamentals to applications. The 2009 annual meeting took place March 13 and 14, 2009 at the Institut Mittag-Leffler, Djursholm, Sweden, and the next one will take place March 12 and 13, 2010 at the International Centre for Mathematical Sciences, Edinburgh, UK. The full list of ERCOM centres can be found at http://www.ercom.org.

2 - The purposes of ERCOM are:

- to constitute a forum for communications and exchange of information between the centres themselves and EMS;
- to foster collaboration and coordination between the centres themselves and EMS;
- to foster advanced research training on a European level;
- to advise the Executive Committee of the EMS on matters relating to activities of the centres;
- to contribute to the visibility of the EMS;
- to cultivate contacts with similar research centres within and outside Europe.

# Hydrodynamic limit of particle systems

Patrícia Gonçalves

*Centro de Matemática da Universidade do Minho, Campus de Gualtar*
*4710-057 Braga, Portugal*
patg@math.uminho.pt

## Abstract

In these notes I present a classical particle system, namely exclusion type processes evolving on the one-dimensional lattice. I consider both settings, the nearest neighbor jumps of symmetric and asymmetric rates and long-jumps. For nearest neighbor jumps, for the symmetric and asymmetric case, one obtains the particle density evolving according to the heat equation and the inviscid Burgers equation, respectively, while for the long-jumps one gets to the fractional heat equation. From this global behavior I present a simple argument to obtain the limit distribution for the position of a tagged particle from the joint distribution of the empirical density of particles plus the current through the origin.

## 1 Introduction

The study of Interacting Particle Systems goes back to the late seventies and were introduce by Spitzer. The goal was to understand the macroscopic temporal evolution of physical systems through the underlying microscopic dynamics, ie the dynamics between the molecules constituting the physical system. The scenario is the following, first one supposes to have two scales for space and time and to have, for example, a fluid or a gas evolving in a certain volume. The idea is to split this volume into a certain number of cells and in each one of these cells one can have a random number of molecules that move according to a fixed rule, a probability transition rate. For details on the formal definition of Interacting Particle Systems, I refer the reader to [13].

The microscopic behavior of a physical system is very hard to obtain in a reasonable way, since the number of molecules is huge, typically of the Avogadro's number and in order to have some meaningfully description some simplifications have to be assumed. In this theory it is supposed to have a stochastic motion of molecules instead of a deterministic one, and with this assumption a probabilistic analysis of the system can be performed. The underlying motion relies on having each molecule waiting a random time and each one of them performing a random walk subjected to local restrictions. So, **Interacting Particle Systems** consist in a random motion of a collection of particles, each one waiting an exponential random time after which it moves from one cell to another, according to a probability transition rate. Probabilistic speaking, since the random times are variables with exponential law, these processes belong to the class of Markov processes and since the microscopic space is discrete these processes have compact space state.

In the **Hydrodynamic Limit** theory one is interested in deducing the macroscopic hydrodynamic equation that governs the temporal evolution of some physical quantity of interest, see [12] and [17]. So, for processes in which the microscopic dynamics conserves a macroscopic thermodynamic quantity, as for example, the density or energy, I will deduce the partial differential equation that governs the temporal evolution of this quantity of interest, through the random motion between particles. This partial differential equation is known as the **hydrodynamic equation** of the particle system.

In these review I will concentrate on particle systems which are of **exclusion type** and **ergodic**. The exclusion type means that, at each cell one has at most one particle per site, but nevertheless one could also consider more general Interacting Particle Systems in which one can have any number of particles per site, called Zero-Range processes, see [11], [12] and references therein, but in order to keep the presentation simple and capture the main ideas I will consider only exclusion type models of ergodic kind. The ergodicity property means that one can split the state space of the process into a disjoint set of invariant pieces, namely the hyperplanes with a fixed number of particles, but each one of them being a unique ergodic piece in the sense that fixing any two configurations on the same hyperplane it is possible to get from one to the other by allowed jumps of the dynamics. Anyway, it is also possible to consider more general particle systems in which the invariant pieces split into the ergodic component plus an isolated set of blocked configurations - these systems belong to the class of kinetically constrained lattice gases, which are of very well interest in the physics community since they model, for example, the liquid/glass transition. In these models particles can only move from on cell to the other if there is a certain number of particles in the cells at their vicinity, otherwise they are blocked [6]. So, for these systems there is a phase transition, since for higher densities of particles each hyperplane is a unique erdogic piece, but below a critical density each hyperplane splits into the irreducible component plus a number of blocked configurations. In [6] it was first proved the hydrodynamic limit for a non ergodic particle system of **gradient type**. The result for non-gradient systems is still open as well as for the case of non-erdogic gradient systems for which below the critical density each hyperplane splits into the irreducible component, plus isolated blocked configurations, plus a set in which there is a mixture of the behavior in each one of this sets, lets say to have a path of possible moving configurations that get blocked after a certain number of jumps. For details on the universe of this kind of constrained models, we refer the reader to [6], [17] and references therein.

After having the hydrodynamic result, one has the knowledge about the global macroscopic behavior of the system. Now, one can focus the attention on a single **tagged particle** and analyze its motion. Since each particle performs a random walk, it is known that if instead having a system with an arbitrary number of particles, we consider it to have just one, then the limit distribution of the position of this tagged particle is given by a **Brownian motion** or a **Levy process**, depending on the properties of the probability transition rate. But what about the limit distribution for one fixed or tagged particle when the system is in the presence of more than one particle? It is known since the work of Robert Brown that for a transition rate with finite second moment, the movement of a single particle in a random medium is given by a Brownian motion. In this setting the motion of a single particle is influenced by the position of the other particles in the system, but can one get to the Brownian motion in the limit as well, or do other processes come along as the Levy processes? Here I am going to present, an argument which allows to deduce the limit distribution for the position of a tagged particle, through the global analysis of the system in the equilibrium setting. Recently there have been obtained partial results about the limit distribution for the position of a tagged particle when the system is out of equilibrium, for details see for example [8], [9], [10] and references therein. When we are restricted to one-dimensional systems with jumps to neighboring sites, the initial order of particles is preserved and the main idea is to relate the position of the tagged particle with the current of particles through a fixed bond together with the empirical density of particles in a certain box. From this relation the limit distribution for the position of a tagged particle is an easy consequence of the joint Central Limit Theorem (C.L.T.) for the current and the empirical measure. In these notes I will define different dynamics for particle systems, that lead us to very different hydrodynamic equations and for which the limit distribution for the position of a tagged particle, one will get to the Brownian motion, to the Fractional Brownian motion and to a Levy process.

Here follows an outline of this review. On the second section I introduce the particle systems by means of their generators and describe its invariant measures. On the third section the empirical measure is introduced and I present an heuristic argument to get to the hydrodynamic equation from the explicit definition of the microscopic dynamics. On the fourth section is stated the hydrodynamic limit result for the processes considered here. On the fifth section I present the C.L.T. for the empirical measure and in the following section the current through the origin is defined and its C.L.T. is stated. On the last section I present the C.L.T. for the position of a tagged particle.

## 2 The Particle Systems

Here I consider the most classical example of an Interacting Particle System: the **Exclusion process**. In order to capture the fundamental ideas behind the hydrodynamic limit theory I will restrict this exposition to one-dimensional particle systems evolving on $\mathbb{Z}$. For more general processes and larger state space we refer the reader to [13].

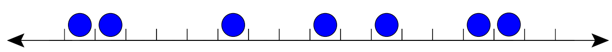

Figure 1: one possible configuration of the Exclusion process

At first one fixes a probability $p(\cdot)$ on $\mathbb{Z}$ and each particle, independently from the others, waits a mean one exponential time, at the end of which being at the site $x$ it jumps to $x+y$ at rate $p(y)$.

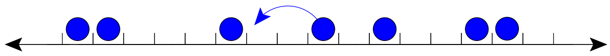

Figure 2: a jump possible jump occurring at rate $p(-2)$

For this process there can be at most one particle per site, and if a clock rings and a particle attempts to jump to an occupied site, the jump is suppressed in order to respect the exclusion rule and after that the clock restarts.

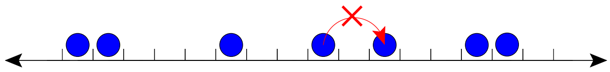

Figure 3: a forbidden jump

The space state for this Markov process is $\Omega = \{0,1\}^{\mathbb{Z}}$ in such a way that a 1 means to have the site occupied while the 0 denotes that it is empty. Now, we give the precise definition of the process by means of its generator.

The Exclusion process is denoted as the Markov process $\eta_t \in \Omega$ with generator given on local functions $f : \Omega \to \mathbb{R}$ by

$$\mathscr{L} f(\eta) = \sum_{x,y\in\mathbb{Z}} \eta(x)(1-\eta(x+y))p(y)[f(\eta^{x,x+y}) - f(\eta)],$$

where

$$\eta^{x,x+y}(z) = \begin{cases} \eta(z), & \text{if } z \neq x, x+y \\ \eta(x+y), & \text{if } z = x \\ \eta(x), & \text{if } z = x+y \end{cases}.$$

I recall here that a core for the operator $\mathscr{L}$ is the set of local functions, ie functions defined on the state space of the process that depend on the configuration $\eta$ only through a finite number of coordinates $\eta(x)$, see [13] for a proof of this result. So, in this process configurations are denoted by $\eta$, so that $\eta(x) = 0$ if the site $x$ is vacant and $\eta(x) = 1$ otherwise, as mentioned above. In these notes I consider three different kind of dynamics:

- Symmetric Simple Exclusion Process (ssep), for which the probability transition rate is given by $p(1) = p(-1) = 1/2$, ie jumps occur to neighboring sites at the same rate.
- Asymmetric Simple Exclusion Process (asep), for which the probability transition rate is given by $p(1) = 1 - p(-1) = p > \frac{1}{2}$, ie jumps also occur to neighboring sites but with a drift to the right.
- Long-jump Exclusion Process, for which the probability transition rate satisfies

  $$p(x,y) = |y-x|^{-(1+\alpha)}, \alpha \in (0,2),$$

  ie jumps occur from any site $x$ to $y$, but the further the distance the smaller the probability of jumping.

Following the Boltzmann ideas from Statistical mechanics the first step to do when one analyzes the temporal evolution of a macroscopic thermodynamical quantity of a physical system, is to obtain the knowledge of its **invariant states**. For particle systems, the invariant states are translated as invariant measures of the system. So in this setting, $\mu$ is an invariant measure of the system, if starting the process from $\mu$, ie if the distribution of $\eta_0$ is $\mu$, then for any time $t$, the distribution of the system at time $t$, ie the distribution of $\eta_t$ is again given by $\mu$ - this means that the trajectory of the measure distributions is constant in time and equal to $\mu$.

Now we describe a set of **invariant measures** for the processes considered above. Fix $0 \leq \rho \leq 1$ and denote by $\nu_\rho$ the Bernoulli product measure on $\Omega$ with density $\rho$, ie its marginal at the site $x$ is given by:

$$\nu_\rho(\eta : \eta(x) = 1) = \rho.$$

So, for any site $x$, $\eta(x)$ has Bernoulli distribution of parameter $\rho$ and since $\nu_\rho$ is a product measure $(\eta(x))_{x\in\mathbb{Z}}$ are independent random variables. It is known that $(\nu_\rho)_\rho$ with $\rho \in [0,1]$ is a family of invariant measures for the exclusion process. I note here that this family is homogeneous (since the marginal at the site $x$ does not depend on $x$) and translation invariant (since it is invariant by the shift application).

## 3 Hydrodynamic equation

In this section I deduce the hydrodynamic equation for two of the processes described above by means of the random microscopic dynamics. For simplicity I present here the computations for the Simple Exclusion Process: symmetric and asymmetric rates. For details on the hydrodynamic limit for the Long-jump process we refer the reader to [7], but the hydrodynamic equation for this process is the fractional heat equation $\partial_t \rho(t,u) = -(-\Delta)^{\alpha/2}$, where $-(-\Delta)^{\alpha/2}$ is the fractional Laplacian.

Now, I introduce the **empirical measure** associated to the Markov process $\eta_\cdot$. For each configuration $\eta$, denote by $\pi^N(\eta, du)$ the measure given by

$$\pi^N(\eta, du) = \frac{1}{N} \sum_{x\in\mathbb{Z}} \eta(x) \delta_{\frac{x}{N}}(du)$$

and define the process of empirical measures by $\pi_t^N(\eta, du) = \pi^N(\eta_t, du)$. Here $\delta_u$ is the Dirac measure at $u$.

In the sequence I present an heuristic argument to obtain the conservation law that describes the temporal evolution of the density of particles. Any of the dynamics introduced above, does not create or destroy particles, it simply move particles according to some pre-determined rule and as a consequence the number of particles is a conserved quantity. So, the **density of particles** is the **thermodynamical quantity** of interest for these processes.

From the classical theory of Markov processes it is known that for a test function $H$

$$M_t^{N,H} =< \pi_t^N, H > - < \pi_0^N, H > - \int_0^t \mathscr{L} < \pi_s^N, H > ds$$

is a martingale with respect to the natural filtration. Note that $< \pi_t^N, H >$ denotes the integral of $H$ with respect to the measure $\pi_t^N$. For a particle system with generator $\mathscr{L}$ and whose dynamics conserves the number of particles $\mathscr{L}(\eta(x)) = W_{x-1,x}(\eta) - W_{x,x+1}(\eta)$, where for a site $x$ and a configuration $\eta$, $W_{x,x+1}(\eta)$ denotes the instantaneous current between the sites $x$ and $x+1$, namely it is the difference between the jump rate from $x$ to $x+1$ and the jump rate from $x+1$ to $x$. This **gradient property** allows us to perform a summation by parts and write down the martingale as

$$\begin{aligned} M_t^{N,H} &= < \pi_t^N, H > - < \pi_0^N, H > \\ &- \int_0^t \frac{1}{N^2} \sum_{x\in\mathbb{Z}} \nabla^N H\Big(\frac{x}{N}\Big) W_{x,x+1}(\eta_s) ds, \end{aligned} \tag{3.1}$$

where $\nabla^N H$ denotes the discrete derivative of $H$.

From this point on, we split the argument depending on the behavior of the expectation of the current. For the asep, the instantaneous current between the sites $x$ and $x+1$ is given by $W_{x,x+1}(\eta) = p\eta(x)(1-\eta(x+1)) - q\eta(x+1)(1-\eta(x))$ and its expectation with respect to the invariant measure $\nu_\rho$ equals to $(p-q)\rho(1-\rho)$ which is non-zero for $\rho \in (0,1)$. For the ssep, the instantaneous current between the sites $x$ and $x+1$ is given by $W_{x,x+1}(\eta) = \frac{1}{2}\Big(\eta(x) - \eta(x+1)\Big)$ and its expectation with respect to the invariant measure $\nu_\rho$ vanishes. This property of the expectation of the instantaneous current is crucial to the following conclusions.

We proceed by closing the integral part of the martingale as a function of the empirical measure. Since, for the asep, the expectation of the current does not vanish, by re-scaling time by $tN$ and performing a change of variables, one gets to

$$\begin{aligned} M_{tN}^{N,H} &= \frac{1}{N} \sum_{x\in\mathbb{Z}} H\Big(\frac{x}{N}\Big)\Big(\eta_{tN}(x) - \eta_0(x)\Big) - \\ &\int_0^t \frac{1}{N} \sum_{x\in\mathbb{Z}} \nabla^N H\Big(\frac{x}{N}\Big) W_{x,x+1}(\eta_{sN}) ds. \end{aligned}$$

Now we introduce the notion of **conservation of local equilibrium**. Physical reasoning suggests that due to the huge number of particles, physical systems may not present a global equilibrium picture, but microscopically by the interaction among particles its reasonable to assume that for a macroscopic time $t$ the system is **locally in equilibrium**. This means, loosely speaking, that the expectation of $\eta_{tN}$, is close to the expectation of $\eta(0)$ with respect to the equilibrium measure of the system, but with parameter predicted by the hydrodynamic equation:

$$E(\eta_{tN}(x)) \sim E_{\nu_{\rho(t,x/N)}}[\eta(0)] = \rho(t, x/N).$$

Applying expectation with respect to the distribution of the system at the microscopic time $tN$ to the martingale above and since this martingale vanishes at time 0, it holds that

$$\begin{aligned} &\frac{1}{N} \sum_{x\in\mathbb{Z}} H\Big(\frac{x}{N}\Big)\Big(\rho(t,x/N) - \rho(0,x/N)\Big) = \\ &\int_0^t \frac{1}{N} \sum_{x\in\mathbb{Z}} \nabla^N H\Big(\frac{x}{N}\Big) \tilde{W}(\rho(s,x/N)) ds \end{aligned}$$

Taking the limit as $N \to +\infty$, $\rho(t,u)$ is identified as a **weak solution** of the hyperbolic conservation law:

$$\begin{cases} \partial_t \rho(t,u) + \nabla \tilde{W}(\rho(t,u)) = 0 \\ \rho(0,\cdot) = \rho_0(\cdot) \end{cases} \tag{3.2}$$

where $\tilde{W}(\rho) = E_{\nu_\rho}[W_{0,1}] = (p-q)\rho(1-\rho)$. This partial differential equation is known as the **inviscid Burgers equation.**

Now we analyze the case for the ssep. Since for this process $W_{x,x+1}(\eta) = \frac{1}{2}\Big(\eta(x) - \eta(x+1)\Big)$, this allows us to perform a double summation by parts in the integral part of the martingale and write (3.1) as:

$$\begin{aligned} M_t^{N,H} &= < \pi_t^N, H > - < \pi_0^N, H > - \\ &\int_0^t \frac{1}{N^2} \sum_{x\in\mathbb{Z}} \Delta^N H\Big(\frac{x}{N}\Big) \frac{1}{2} \eta_s(x) ds. \end{aligned}$$

We note here that particle systems for which the instantaneous current can be written as a function minus its translation are called **gradient systems**. The ssep is an example of a gradient system since $W_{x,x+1}(\eta) = \frac{1}{2}\eta(x) - \frac{1}{2}\eta(x+1)$. Now, following the same argument as above, by re-scaling time by $tN^2$, performing a change of variables and by the local equilibrium assumption, when taking the limit as $N \to +\infty$, $\rho(t,u)$ is a **weak solution** of the well known **heat equation**:

$$\begin{cases} \partial_t \rho(t,u) = \frac{1}{2}\Delta \rho(t,u) \\ \rho(0,\cdot) = \rho_0(\cdot). \end{cases} \tag{3.3}$$

# 4 Law of Large Numbers for the empirical measure

In order to introduce the notion of hydrodynamic limit we have to fix some notation. Let $\rho_0 : \mathbb{R} \to [0,1]$ be an initial profile and denote by $(\mu_N)_{N\geq 1}$ a sequence of probability measures defined on $\Omega$. Assume that a time 0, the system starts from a initial measure $\mu_N$ that is associated to the initial profile $\rho_0$:

**Definition 1.** *A sequence $(\mu^N)_{N\geq 1}$ **is associated to** $\rho_0$, if for every continuous function $H:\mathbb{R}\to\mathbb{R}$ and for every $\delta>0$*

$$\lim_{N\to+\infty}\mu^N\Big[\eta:\Big|\frac{1}{N}\sum_{x\in\mathbb{Z}}H\Big(\frac{x}{N}\Big)\eta(x)-\int_{\mathbb{R}}H(u)\rho_0(u)du\Big|>\delta\Big]=0.$$

Note that the term on the left hand side of the expression above, corresponds to the integral of $H$ with respect to $\pi_0^N$. Thus the above definition corresponds to asking that empirical measure at time 0 satisfy a law of large numbers, namely that the sequence $\pi^N(\eta,du)$ converges in $\mu_N$-probability to $\rho_0(u)du$.

The goal in hydrodynamic limit consists in showing that, if at time $t=0$ the empirical measures are associated to some initial profile $\rho_0$, then at the macroscopic time $t$ they are associated to a profile $\rho_t$ which is the solution (in some topology) of the corresponding hydrodynamic equation. In other words, the aim is to prove that the random measures $\pi_\cdot^N$ converge in probability to the deterministic measure $\rho(t,u)du$, which is absolutely continuous with respect to the Lebesgue measure and whose density evolves according to the hydrodynamic equation.

Since the work of Rezakhanlou in [16], it is known that for the asep starting from a sequence of measures $(\mu_N)_N$ associated to a profile $\rho_0(\cdot)$ and some additional hypotheses (see [16] for details) under the **hyperbolic time scale** $tN$

$$\pi_{tN}^N\xrightarrow[N\to+\infty]{}\rho(t,u)du,$$

in $\mu^N S_N(t)$-probability, where $\rho(t,u)$ is the entropy solution of (3.2) and $S_N(t)$ is the semigroup associated to the generator of the asep.

For the ssep, the hydrodynamic limit can be derived by the entropy method since the local equilibrium convergence holds for this process, see [12] for details. Under the **parabolic time scale** $tN^2$, it holds that

$$\pi_{tN^2}^N\xrightarrow[N\to+\infty]{}\rho(t,u)du,$$

in $\mu^N S_N(t)$-probability, where $\rho(t,u)$ is the weak solution of (3.3) and $S_N(t)$ is the semigroup associated to the generator of the ssep.

Probabilistic speaking, hydrodynamic limit is a **Law of Large Numbers** for the empirical measure associated to a Markov process starting from a general set of initial measures. A natural question that follows has to do with the fluctuations of this measure around the equilibrium state. "Does a Central Limit Theorem holds?" and "How is the behavior of the limit process?" In order to analyze the C.L.T. for the empirical measure, we consider the simplest case in which the process is equilibrium, ie the initial measure is $\nu_\rho$. This will be developed in the next section. The scenario out of equilibrium is much harder to obtain and few cases are know, here we leave this issue out of discussion.

# 5 Central Limit Theorem for the empirical measure

In this section we state the C.L.T. for the empirical measure for the simple exclusion process: ssep and asep. Let $\mathscr{S}(\mathbb{R})$ denote the Schwartz space of test functions. Fix $\rho$ and an integer $k$. Denote by $\mathscr{Y}_\cdot^N$ the density fluctuation field, ie a linear functional acting on functions $H\in\mathscr{S}(\mathbb{R})$ as

$$\begin{aligned}\mathscr{Y}_t^N(H) &= \sqrt{N}\Big[<\pi_t^N,H>-\mathbb{E}_{\nu_\rho}<\pi_t^N,H>\Big]\\ &= \frac{1}{\sqrt{N}}\sum_{x\in\mathbb{Z}}H\Big(\frac{x}{N}\Big)(\eta_t(x)-\rho).\end{aligned}$$

For an integer $k\geq 0$, let $\mathscr{H}_k$ be the Hilbert space induced by $\mathscr{S}(\mathbb{R})$ and $<f,g>_k=<f,K_0^k g>$, where $<\cdot,\cdot>$ denotes the inner product of $L^2(\mathbb{R})$, $K_0=x^2-\Delta$ and denote by $\mathscr{H}_{-k}$ its dual. Let $D(\mathbb{R}^+,\mathscr{H}_{-k})$ (resp. $C(\mathbb{R}^+,\mathscr{H}_{-k})$) be the space of $H_{-k}$-valued functions, right continuous with left limits (resp. continuous), with the uniform weak topology, by $Q_N$ the probability measure on $D(\mathbb{R}^+,\mathscr{H}_{-k})$ induced by $\mathscr{Y}_\cdot^N$ and $\nu_\rho$.

**Theorem 5.1** (G. [4])**.** *Fix an integer $k>2$. Let $\eta_\cdot$ be the asep evolving on the time scale $tN$, starting from the invariant measure $\nu_\rho$ and $Q_N$ be the probability measure on $D(\mathbb{R}^+,\mathscr{H}_{-k})$ induced by $\mathscr{Y}_\cdot^N$ and $\nu_\rho$. Denote by $Q$ the probability measure on $C(\mathbb{R}^+,\mathscr{H}_{-k})$ corresponding to a stationary Gaussian process with mean 0 and covariance given by*

$$E_Q[\mathscr{Y}_t(H)\mathscr{Y}_s(G)]=$$
$$\rho(1-\rho)\int_{\mathbb{R}}H(u+(p-q)(1-2\rho)(t-s))G(u)du$$

*for every $0\leq s\leq t$ and $H$, $G$ in $\mathscr{H}_k$. Then, $(Q_N)_N$ converges weakly to $Q$.*

For the asep the limit density field satisfies

$$\mathscr{Y}_t(H)=\mathscr{Y}_0(H)-\int_0^t\mathscr{Y}_s((p-q)(1-2\rho)\nabla H)ds,$$

ie $\mathscr{Y}_t$ satisfies:

$$d\mathscr{Y}_t=(p-q)(1-2\rho)\nabla\mathscr{Y}_t dt.$$

In this case we obtain a simple expression for $\mathscr{Y}_t$ given by $\mathscr{Y}_t(H)=\mathscr{Y}_0(T_tH)$ with $T_tH(u)=H(u+(p-q)(1-2\rho)t)$, which is the semigroup associated to $(p-q)(1-2\rho)\nabla$. Restricted to $\mathscr{F}_0$ (the $\sigma$-algebra on $D([0,T],\mathscr{H}_{-k})$ generated by $\mathscr{Y}_0(H)$ and $H$ in $S(\mathbb{R})$) $Q$ is a Gaussian field with covariance given by $E_Q(\mathscr{Y}_0(G)\mathscr{Y}_0(H))=\rho(1-\rho)<G,H>$. In this case the limit density field at time t is just a translation (by the characteristics velocity) of the initial limit density field, so the systems does only depends on the initial configuration of the system.

For the ssep a more sophisticated process appears in the limit, it is known as an Onrstein-Uhlenbeck process and the temporal evolution depends highly on the initial configuration of the system but also on the randomness of the dynamics.

**Theorem 5.2** (Ravishankar [15]). *Fix an integer $k > 3$. Let $\eta_\cdot$ be the ssep evolving on the time scale $tN^2$, starting from the invariant measure $\nu_\rho$ and $Q_N$ be the probability measure on $D(\mathbb{R}^+, \mathscr{H}_{-k})$ induced by $\mathscr{Y}^N_\cdot$ and $\nu_\rho$. Denote by $Q$ be the probability measure on $C(\mathbb{R}^+, \mathscr{H}_{-k})$ corresponding to a stationary mean zero generalized Ornstein-Uhlenbeck process with characteristics $\mathfrak{A} = 1/2\Delta$ and $\mathfrak{B} = \sqrt{\chi(\rho)}$. Then $(Q_N)_N$ converges weakly to $Q$.*

For the ssep the limit density field satisfies

$$d\mathscr{Y}_t = \frac{1}{2}\Delta\mathscr{Y}_t dt + \sqrt{\chi(\rho)}\nabla dB_t$$

where $B_t$ is a Brownian motion.

For the C.L.T. for the long-jump process I refer the reader to [7].

# 6 Current fluctuations

In this section I introduce the notion of flux or current of particles through the origin. Let $J_{-1,0}(t)$ be the number of particles that jump from the site $-1$ to 0 minus the number of particles that jump from the site 0 to $-1$ during the time interval $[0, t]$. Since

$$J_{-1,0}(t) = \sum_{x \geq 0} \Big(\eta_t(x) - \eta_0(x)\Big),$$

the current can be written in terms of the density fluctuation field as

$$\frac{1}{\sqrt{N}}\Big\{J_{-1,0}(t) - \mathbb{E}_{\nu_\rho}[J_{-1,0}(t)]\Big\} = \mathscr{Y}^N_t(H_0) - \mathscr{Y}^N_0(H_0),$$

where $H_0$ is the Heaviside function, $H_0 = 1_{[0,+\infty)}$. Using this relation and the C.L.T. for the empirical measure, a C.L.T. for the current through the origin can be obtained.

**Theorem 6.1** (G. [4]). *Let $\eta_\cdot$ be the asep evolving on the time scale $tN$ and starting from $\nu_\rho$. Then*

$$\frac{1}{\sqrt{N}}\Big(J_{-1,0}(tN) - \mathbb{E}_{\nu_\rho}(J_{-1,0}(tN))\Big) \xrightarrow[N \to +\infty]{} \sigma_a(J)\mathscr{B}_t$$

*where $(\sigma_a(J))^2 = \rho(1-\rho)|(p-q)(1-2\rho)|$ and $\mathscr{B}_t$ is the standard Brownian motion.*

**Theorem 6.2** (Arratia [1], De Masi-Ferrari [2], Peligrad-Sethuraman [14]). *Let $\eta_\cdot$ be the ssep evolving on the time scale $tN^2$ and starting from $\nu_\rho$. Then*

$$\frac{1}{\sqrt{N}} J_{-1,0}(tN^2) \xrightarrow[N \to +\infty]{} \sigma_s(J)\mathscr{W}^H_t$$

*where $(\sigma_s(J))^2 = \sqrt{\frac{2}{\pi}}\rho(1-\rho)$ and $\mathscr{W}^H_t$ is the Fractional Brownian motion of Hurst parameter $H = 1/4$.*

# 7 Tagged Particle

Now we want to prove the C.L.T. for a single tagged particle that we suppose to be initially at the origin:

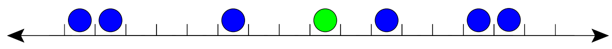

Figure 4: tagged particle at the origin

For that, let $\eta$ be a configuration of $\Omega$ such that $\eta(0) = 1$. For the other sites $x$ consider $\eta(x)$ distributed according to the invariant measure of the system, denoted by $\nu_\rho$. This means that now we are starting the process from the measure $\nu_\rho$ conditioned on configurations with a particle at the origin:

$$\nu^*_\rho(\cdot) = \nu_\rho(\cdot|\eta(0) = 1).$$

This measure is no longer an invariant measure of the system, since the clock at the origin can ring and the particle initially at the origin can move to an empty site according to the transition probability rate $p(\cdot)$.

In order to keep track of the position of this particle, let $X_t$ denote the position at time $t$ of the tagged particle initially at the origin ($X(0) = 0$).

Since in the one-dimensional setting and for nearest neighbor jumps the order of particles is preserved, this allows us to obtain a simple relation between the position of the tagged particle, the current through the origin and the empirical density of particles as:

$$\Big\{X(t) \geq n\Big\} = \Big\{J_{-1,0}(t) \geq \sum_{x=0}^{n-1} \eta_t(x)\Big\}$$

**Theorem 7.1** (Ferrari-Fontes [3], G. [4]). *Let $\eta_\cdot$ be the asep evolving on the time scale $tN$, starting from $\nu^*_\rho$ and let $X(tN)$ denote the position at time $tN$ of the particle initially at the origin. Then*

$$\frac{1}{\sqrt{N}}\Big(X(tN) - \mathbb{E}_{\nu^*_\rho}(X(tN))\Big) \xrightarrow[N \to +\infty]{} \sigma_a(X)\mathscr{B}_t$$

*where $(\sigma_a(X))^2 = |p-q|(1-\rho)$ and $\mathscr{B}_t$ is the standard Brownian motion.*

**Theorem 7.2** (Arratia [1], De Masi-Ferrari [2], Peligrad-Sethuraman [14], G.-Jara [5]). *Let $\eta_\cdot$ be the ssep evolving on the time scale $tN^2$ and starting from $\nu_\rho^*$ and let $X(tN^2)$ denote the position at time $tN^2$ of the particle initially at the origin. Then*

$$\frac{1}{\sqrt{N}}X(tN^2) \xrightarrow[N\to+\infty]{} \sigma_s(X)\mathscr{W}_t^H,$$

*where $(\sigma_s(X))^2 = \sqrt{\frac{2}{\pi}\frac{1-\rho}{\rho}}$ and $\mathscr{W}_t^H$ is the Fractional Brownian motion of Hurst parameter $H = 1/4$.*

When one considers the long-jump process the relation above between the position of the tagged particle, the current and the density of particles does not hold since now particles can move to sites arbitrarily away from each other - so the order of particles is no longer preserved. Anyway for this process the C.L.T. for the tagged particle is achieved by considering the process seen from an observer sitting on the position of the tagged particle, namely $\xi_t(x) = \eta_t(x + X(t))$. For this new process the position of the tagged particle becomes the number of shifts of the system, which can be written as a martingale plus an additive functional of the Markov Process, for details see [8]. For a transition rate $p(\cdot)$ homogeneous and regular of degree $\alpha$, ie such that there exists a function $q : \mathbb{R}\backslash\{0\} \to \mathbb{R}$ of class $C^2$ such that $p(x) = q(x)$ for any $x \in \mathbb{Z}\backslash\{0\}$ and such that $q(\lambda u) = \lambda^{1+\alpha}q(u)$ for any $\lambda \neq 0$ and $u \in \mathbb{R}\backslash\{0\}$ it was proved in the equilibrium case that:

**Theorem 7.3** (Jara [8]). *Let $\eta_\cdot$ be the exclusion process with homogeneous regular transition rate of degree $\alpha$, evolving on the time scale $tN^\alpha$, starting from $\nu_\rho^*$ and let $X(tN^\alpha)$ be the position at time $tN^\alpha$ of the particle initially at the origin. Then*

$$\frac{X_{tN^\alpha}}{N} \xrightarrow[N\to+\infty]{} (1-\rho)\mathscr{Z}_t,$$

*where $\mathscr{Z}_t$ is the Levy process whose characteristic function is given by $-\log E[\exp i\beta\mathscr{Z}_t] = t\psi(\beta)$ with*

$$\psi(\beta) = \int_{\mathscr{R}}(1 - e^{i\beta u})q(u)du$$

*and $q$ as above.*

In fact this theorem as stated holds for a general class of transition rates, which includes the jump-rate defined in the setting of this review, namely $p(x, y) = |y-x|^{-(1+\alpha)}$ with $\alpha \in (0, 2)$.

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Article

# Some Notes on the History of Branching Processes, from my Perspective[4]

Peter Jagers

*Mathematical Sciences, Chalmers and University of Gothenburg*
jagers@chalmers.se

Few fields within the mathematical sciences cherish their past like branching processes. Ted Harris's classical treatise from 1963 opens by a terse but appetising two-page flashback. Three years later, David Kendall's elegant overview was published, and like Charles Mode in his monograph (1971), I could borrow from that for the historical sketch opening in my 1975 book, but also add some observations of my own.

At that time we all knew that the French, notably de Candolle and Bienaymé, had considered the nobility and family extinction problem, before Galton publicized it. I also speculated about connections between Bienaymé and the demographer Benoiston de Châteauneuf, who had been studying old French noble families. The plausibility of such contact was corroborated by Chris Heyde and Eugene Seneta in their "I. J. Bienaymé: Statistical Theory Anticipated", where they also showed that Bienaymé was not only first at formulating the mathematical problem, but indeed knew its solution already in 1845. The original publication has not been found, though, but as pointed out by Bru, Jongmans, and Seneta (cf. also the recent monograph by Iosifescu *et al.*) there is a proof in a treatise by A. A. Cournot, published only two years after Bienaymé's communication. Though this is not explicitly stated, it seems plausible that Cournot reproduces Bienaymé's argument. (The book by Iosifescu and co-authors also presents an intriguing discussion by Bienaymé arguing that the limited size of mankind (in the mid XIX:th Century!) should show that human mean reproduction must have varied above and below one in historical time.)

There are good reasons for branching processes to keep its heritage alive. Not only is the background in the frequent disappearance of family names, even in growing populations, picturesque and easily understood, it is also something that could not have been explained by prevailing - and long dominating - deterministic population theory. Indeed, it provides convincing arguments for a stochastic population theory, and not only for "small" populations. Further, in spite of its alluring disguise, the family extinction problem concerns an important and basic feature of population development, viz. the frequent extinction of family lines and, as a consequence, ubiquitous shared ancestry.

It also tells a story of interplay between mathematics, natural science, culture, and society. Indeed, listen to Galton's classical formulation in Educational Times 1873, initiating modern theory:

"PROBLEM 4001: A large nation, of whom we will only concern ourselves with adult males, $N$ in number, and who each bear separate surnames colonise a district. Their law of population is such that, in each generation, $a_0$ per cent of the adult males have no male children who reach adult life; $a_1$ have one such male child; $a_2$ have two; and so on up to $a_5$ who have five. Find (1) what proportion of their surnames will have become extinct after $r$ generations; and (2) how many instances there will be of the surname being held by $m$ persons".

Rarely does a mathematical problem convey so much of the flavour of its time, colonialism and male supremacy hand in hand, as well as the underlying concern for a diminished fertility of noble families, paving the way for the crowds from the genetically dubious lower classes.

It also exhibits a mathematical theory initiated not by mathematicians but by a broad *savant*, Francis Galton, a polyhistor well versed in mathematics but primarily if anything, a biologist. We see an example falsifying both extremist views on science, that of a pure science, and in particular mathematics, devoid of political meaning and implications; and that degrading science and scientific development into a purely social phenomenon. Indeed, in branching processes, they all meet: pure mathematical development, biology, physics, and demography, and the concoction is spiced to perfection by the social and cultural context in which it is formed.

As is well known, Watson determined the extinction probability as a fixed point of the reproduction generation function $f$. He observed that 1 is always such

[4]From a lecture at the Oberwolfach workshop on random trees in January 2009. To be published in a specialized journal

a fixed point, $f(1) = 1$, and from this he and Galton (1874) concluded, seemingly without hesitation: “All the surnames, therefore, tend to extinction in an indefinite time, and this result might have been anticipated generally, for a surname lost can never be recovered, and there is an additional chance of loss in every successive generation. This result must not be confounded with that of the extinction of the male population; for in every binomial case where $q$ is greater than 2 we have $t_1 + 2t_2 + \cdots + qt_q > 1$, and, therefore an indefinite increase of male population”.

It is strange that so intelligent a couple as Galton and Watson (the latter turned clergyman but had been second Wrangler at Cambridge, carried on mathematics and physics as a Rector and even was awarded an honorary D. Sc. by his *Alma Mater*) could have presented, and even believed in such verbiage. It is even stranger that it took more than fifty years to rectify it, in particular since Bienaymé had already published a correct statement of the extinction theorem. I always thought the reason simply was that people of the time just did not notice, or bother about, such a mathematical trifle. But according to Heyde and Seneta, “its implications were strongly doubted” already at the time of publication.

And indeed, I checked an (almost) contemporary and non-mathematical criticism quoted by them, by a Swedish historian or political scientist, Pontus Fahlbeck. He was a commoner who married a baroness and became the author of a monumental two-volume treatise on the Swedish aristocracy (1898, 1902). There he gives a correct, verbal description of the relation between growth of the whole versus frequent extinction of separate family lines, and writes - somewhat condescendingly or intimately, it may seem: “*Galton*, who with characteristic curiosity considered the question, has tried to investigate to what extent families ... must die out, with the help of a competent person”. Fahlbeck then recounts examples considered by Galton, showing that “the tendency is the extinction of all”. (The account is not completely lucid.) This is followed by a sequel of questions, and a reassuring answer:

“If this course of events be based on a mathematical law, then it should be as necessary, or not? And what then about our general conclusions, that no necessity forces extinction? Is there not in this a contradiction, which if both arguments are right, as they undoubtedly are, leads to what philosophers call an antinomy? However, mathematical calculations, as applied to human matters, may seem unrelenting but are actually quite innocuous. The necessity lies buried in them like an electrical current in a closed circuit, it cannot get out and has no power over reality”¨ (pp. 133-135, my translation).

As you know, it was another polyhistor, J. B. S. Haldane, chemist, physiologist, geneticist statistician, and prolific political writer in the New Statesman as well as the Daily Worker (He was a notable member of the intellectual British left of the 30’s and 400’s, beautifully described by Doris Lessing, among others) who got things basically right, although the really correct formulation was printed slightly later (Steffensen, 1930). If the mean number of children is less than, or equal to, one then Galton and Watson were right, but if it exceeds one, then there is another smaller fixed point, which yields the correct extinction probability $q < 1$.

In many realistic situations, however, this extinction probability though less than one, will be large together with the mean reproduction $m = f'(1)$. Indeed, values of 0.75 and 2, respectively, e.g., are obtained for realistic reproductive patterns among human males, or for that sake females, in historic times, and similarly among animals in wild life.

Lecturing in Peking in October 2008, I met with a cute illustration of this, which may well have occurred to some of you. In the *China Daily* I read that Kung Te-chen, who was the 77:th great...grandson of Confucius (Kung Fu-tse) had died on Taiwan at the age of 89. Yes, same surname inherited from father to son for more than 75 generations. Since Confucius (500 B.C.), China’s population has undergone a tremendous growth, but as we all know, there are few Chinese family names. Indeed, Wikipedia tells us that three surnames (partly different in different parts of the country) are carried by some 30% of the population. In Korea the situation is even more extreme; half the population has one of the names Kim, Lee, or Park.

Thus, branching processes were born out of a social demographic context. Its first fundamental result, the extinction theorem, has relevance far beyond that, in explaining homogeneity in large populations, as well as (part) of the more than frequent extinction in the course of natural evolution. Indeed, 1991 the palaeontologist David Raup claimed that 99.99% of all species, ever existing on our earth, are extinct now.

When branching processes reappear in scientific literature, between the great wars, the impetus comes from genetics (Fisher and Haldane) and biology more generally. Haldane deduces his approximation for the survival probability, still very important for the consideration of fresh, slightly fitter mutants in a resident population. In Russia, Kolmogorov coins the term *branching process* itself.

After World War II, the nuclear age arrives. In Stalin’s Moscow, Kolmogorov and his disciples, people like the Yaglom twin brothers and B. A. Sevastyanov, try to pursue their research as a purely mathematical undertaking. But of no or little avail. Sevastyanov’s thesis was classified, while being written, and since he himself was not deemed reliable he was not allowed to keep it. Every morning a KGB officer opened a safe in the library and handed it out to its author, who continued writing on it until five, when he had to give it back.

Kolmogorov and others, including I believe Sacharov, protested, and finally the ban was lifted (Sevastyanov, 1999). Things had become easier than in the 30's.

In the United States, Ted Harris was employed by the Rand Corporation, an integral part of the military-industrial complex, and his work on electron-photon cascades and Galton-Watson processes with continuous type space (energy) was clearly inspired by nuclear physics. But both he and Sevastyanov saw themselves as mathematicians, though working on a pattern relevant for natural science. Sevastyanov even takes a rather purist stance; I have heard him saying that mathematics is nothing but mathematics, a somewhat unexpected opinion from a mathematician who is neither an algebraist nor a topologist, not even a pure analyst, and who actually after his thesis worked several years in the secret military part of the Steklov Institute, the so called "Box". Maybe it comes naturally to someone who has devoted his life to mathematics in the overly politicised Soviet Union.

With such leaders, it is not surprising that the 50's and 60's was an era of mathematisation. Time structure was added to the simply reproductive branching process in what Bellman and Harris called *age-dependent* processes, depicting populations where individuals could have variable life spans, but split into a random number of children at death, independently of age. Truly age-dependent branching processes were introduced by Sevastyanov, the reproduction probabilities possibly affected by the mother's age at splitting.

The processes thus arising were not Markovian in real time, but could be analytically treated using renewal properties, and the then remarkable renewal theory, which had recently been established by Feller and others. Another development retained the Markov property, but viewed population evolution as occurring in real time, thus establishing connection to the elementary birth-and-death processes that were flourishing in semi-applied literature.

These approaches however remained in a sort of physical world, far from animal or even plant population dynamics, in the sense that they all considered childbearing through *splitting* only, like fission, cell division, or molecular replication. Or, for the classical Galton-Watson process, there was the alternative interpretation of disregarding time, and just count generations, as though they did not overlap in real time. The only exception were the models from the-birth-and-death sphere, where exponentially distributed life spans allowed alternative interpretations. That also lead to the first model of populations where individuals could give birth *during* their lives, Kendall's generalised birth-and-death process (1948).

The first monographs, Harris's from 1963 and Sevastyanov's from 1971, as well as Athreya's and Ney's from 1972, however stayed firmly in the tradition of physical splitting. Branching processes remained separated from the deterministic differential equations, matrix, and Lotka-Volterra tradition of population dynamics and mathematical demography. It was the rise of point process theory that rendered the formulation of general branching processes natural, so as to depict populations where individuals can give birth repeatedly, in streams of events formed by a point process, and possibly even of various types. 1968 time was ripe, and Crump's and Mode's article and mine appeared simultaneously in the winter 68-69. Mine was also part of my Ph.D. thesis, defended in October, fortunately. In those times in Sweden, formal originality was insisted upon, in a somewhat square manner, and in spite of the enormous friendliness of my polite Japanese opponent, Kiyosi Ito himself, I might not have been let through, had the stern local mathematics professors known that some Americans had done the same, sceptical towards probability theory, as they were. The status of probability within mathematics has certainly changed since then!

The advent of general branching processes meant that branching processes now embraced virtually all mathematical population theory. The dominating mathematical population framework since more than a century was the *stable population theory*, dating back to Quetelet and Lotka. Its real father or forerunner was, however, Euler who deduced its main findings, the exponential increase of population size and how the ensuing stable age distribution is determined by survival and reproduction rates, already around 1750. As I pointed out in my 1975 book, Euler even used rapid population growth as an argument against those incredulous who would not believe that the sons of Adam could have filled the earth during the 5000 years since Adam and Eve were ousted from Eden. Nevertheless, his contributions seem long forgotten in the demographic and mathematical biology communities.

Stable population theory is deterministic but based upon a probabilistic formulation of individual life events. All its findings could now be strictly proven in terms of general branching processes, and basic concepts like average age at childbirth given an interpretation. Furthermore, the stabilisation of population composition could be brought one step further: stable population theory had only considered the distribution of age in old populations. Age is what could be called an individual property: it is your age and nobody else's. In a population there are however also important relational properties.

My research into this area started in a quaint manner. In my youth, Gothenburg had a well-known doctor caring for the city's alcoholics. Now that he had retired in the late 70's he took up a research idea that he had toyed with for some time. He had made the observation that an astonishing proportion of his patients were first-born.

Studying the literature, he found that not only Gothenburg alcoholics, but also poets, statesmen, and people suffering from various mental disorders had been found often to be first-born. Galton had even claimed that the first-borns were the motor of history, since they were more often "men of note". The retired doctor realised that the apparent overrepresentation of first-borns could be an artefact, and performed a primitive but adequate simulation experiment. He drew the family trees of an invented but realistic population on a long white paper table cloth. Towards the end of the paper roll, he then sampled individuals at random, or at least haphazardly. Many were first-borns. Now he wanted to discuss with me.

I found the probability of being first-born in an old single-type supercritical general branching process. It is $E[e^{-\alpha\tau}]$, where $\alpha$ is the Malthusian parameter and $\tau$ mother's age at her first bearing. Since the Malthusian parameter equals $\ln 2$ divided by the doubling time and the latter is usually larger than age at first bearing, we see that the probability of being first-born tends to be larger than 0.5, even in populations with large broods or families (Jagers, 1982). (In an old critical population the probability of being first-born is larger than one over the expected sibship size, due to Jensen's inequality.)

The important is, however, that being first-born is not a property of your own life and birth-time. It concerns your relation to your sibship. Thus, this simple observation led on to an investigation of how the whole pedigree, family structure, and type distribution in multi-type populations stabilise during exponential growth. A strict framework for general branching processes in abstract type spaces was formulated, related to branching tree ideas due to Neveu and Chauvin (1986). In these, type distribution and ancestry, and hence mutational history could be traced backward in a Markov renewal structure. Our group published a whole sequel of papers on these topics during the 80's and 90's, and indeed a final (?) attempt to popularise the admittedly heavy theory by restriction to discrete time quite recently (Jagers and Sagitov, 2008). Stable pedigrees and backward times was virgin land, the only exceptions being the investigations by Bühler into the family structure of Bellman-Harris processes (1971) and later by Joffe and Waugh into kin numbers in Galton-Watson processes (1981 and 1982).

In the mean time, deterministic population dynamics had advanced through work by eminent mathematicians like Odo Diekmann and Mats Gyllenberg, inspired by the biologist Hans Metz. They had realised that the differential equations formulations they had been brought up with were becoming a straitjacket, and turned to semigroups of positive operators, yielding a theory corresponding to the Markov renewal theory of expectations of multitype general branching. However, they took a further step, considering the feedback loop individual → population → environment → individual. Through this theory, *structured population dynamics*, they were able to analyse the fascinating new ideas that Metz and his followers had advanced to explain evolution, under the name of *adaptive dynamics*.

This was a new challenge to branching processes, and is being met in a series of path-breaking papers by Sylvie Méleard and her co-workers. We have also tried to formulate models investigating the consistency of adaptive dynamics, and in particular the problem of *sympatric speciation*, i.e. how successive small mutations can lead to new species, and their coexistence - but with less success so far.

The general problem of interaction in population dynamics is elusive. On one hand, the very concept of population builds upon individuals in some sense being the agents, those changing the population by their actions. The branching process idea is to make this vague idea of "individual initiative" precise by sharpening it into the requirement of stochastic independence between individuals. This is proper as an idealisation, but obviously takes us far from reality. In special cases this can be remedied, as in the models considered by Méleard and Champagnat and Lambert, or in the population size dependence studied by Kersting, Klebaner, and others, which allows an understanding of the linear growth occurring in the famous polymerase chain reactions, PCR, (cf. Haccou *et al*). But a real liberation from independence, replacing it by exchangeability in some form, e.g., remains out of reach.

This overview has centred on my own interests, branching processes as a form of theoretical biology. In particular it has focussed on the supercitical case, which was the main interest of my own expansive youth. My recent papers, quite suitably, deal with the path and time to extinction (2007). However, most of the revival branching processes and related areas experienced in the 90's, and which continues to this day has a different character. Mainly it is purely mathematical; partly it is inspired by computer algorithms. The whole area of superprocesses and measure-valued Markov branching processes, would belong to the former realm, whereas random trees though certainly belonging to pure mathematics also has drawn upon both phylogenetics and computer science. But these are areas where others have much more insight than I, and I leave it to you to comment upon the impressive growth of these fields during the past three or so decades.

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# Didactics of Mathematics as a Mathematical Discipline:

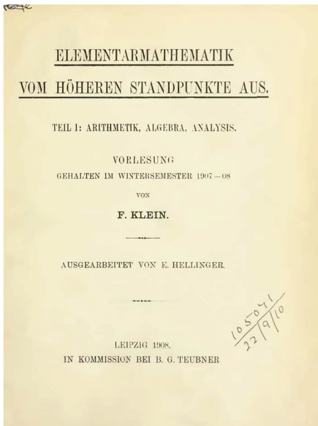
*A Portuguese reflection upon a Kleinean challenge*

Elfrida Ralha, CMAT, Universidade do Minho
Jaime Carvalho e Silva, CMUC, Universidade de Coimbra
José Manuel Castanheira, DME, Universidade da Madeira
Suzana Nápoles, CMAF, Universidade de Lisboa

The famous German mathematician Felix Klein (Dusseldorf, 25th April, 1849 - Gottingen, 22nd June, 1925) is well known both as a distinguished researcher (non-euclidean geometry, group theory and theory of functions) and as a remarkable teacher. According to Hedrick and Noble[5], *he combined familiarity with all the fields of mathematics and the ability to perceive the mutual relations of these fields; and he made it his notable function, as a teacher, to acquaint his students with mathematics, not as isolated disciplines, but as an integrated living organism... He endeavored to reduce the gap between the school and the university, to rouse the schools from the lethargy of tradition, to guide the school teaching into directions that would stimulate healthy growth; and also to influence university attitude and teaching toward a recognition of the normal function of the secondary school.*

Klein was appointed professor at Erlangen[6] in 1872 and he joined the Technische Hochschule, at Munich, in 1875, and the University of Leipzig, in 1880. By 1886 he accepted a chair at the University of Gottingen and there he established a research center which became a role model for the best mathematical research centers all over the world: he promoted weekly discussion meetings, he arranged for a mathematical reading room with a mathematical library and he even managed to give fame to the journal *Mathematische Annalen* and to convince David Hilbert to join in his research center. By then the distinction between pure and applied mathematics began to fade away and the intellectual partnership between Klein and Hilbert proved to be, in Gottingen, a decisive orientation for others in exploring the relationship among mathematics, science in general and technological disciplines. From then on, Klein became also interested in mathematical instruction in schools and in 1905 he was to play a decisive role in formulating a plan recommending that the mathematical concept of function as well as the basics of differential and integral calculus should be taught in secondary schools. Many countries around the world gradually implemented this recommendation and, by 1908, Felix Klein became elected the first chairman of the International Commission on Mathematical Instruction (ICMI), at the Rome International Mathematical Congress.

By 1908, Klein's famous notes "Elementarmathematik vom höheren Standpunkte aus" (Tome I) were published, in Leipzig, for the first time.


ELEMENTARMATHEMATIK
VOM HÖHEREN STANDPUNKTE AUS.
TEIL I: ARITHMETIK, ALGEBRA, ANALYSIS.
VORLESUNG
GEHALTEN IM WINTERSEMESTER 1907–08
VON
F. KLEIN.
AUSGEARBEITET VON E. HELLINGER.
LEIPZIG 1908.
IN KOMMISSION BEI B. G. TEUBNER

In this volume dedicated to "Arithmetik, Algebra and Analysis" Klein would then write on its preface that:

*The new volume which I herewith offer to the mathematical public, and specially to the teachers of mathematics in our secondary schools, is to be looked upon as a first continuation of the lectures "Über den mathematischen Unterricht an den höheren Schulen"... At that time our concern was with the different ways in which*

[5]E. R. Hedrich (Vice President and Provost) and C. A. Noble (Professor of Mathematics, Emeritus), from the University of California, were the translators (from German to English) and the authors of the preface of Klein's books on "Elementary Mathematics from an advanced standpoint".

[6]Felix Klein is particularly linked to his "Erlangen Program" which he first presented, to a restricted audience, when he was appointed as professor for the Faculty of Philosophy and for the Senate of the University of Erlangen, in Germany.

*the problem of instruction can be presented to the mathematician. At present my concern is with developments with subject matter of instruction.* ***I shall endeavor to put before the teacher, as well as the maturing student, from the view-point of modern science, but in a matter as simple, stimulating and convincing as possible, both the content and the foundations of the topics of instruction, with due regard for the current methods of teaching.***

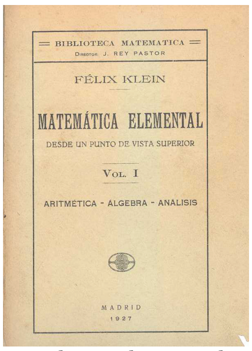
Two other volumes on the same theme of "Elementarmathematik vom höheren Standpunkte aus" were to follow: Part II, on "Geometry" (1909) and Part III, published posthumously by Springer, on "Precision and Approximation Mathematics" (1928).

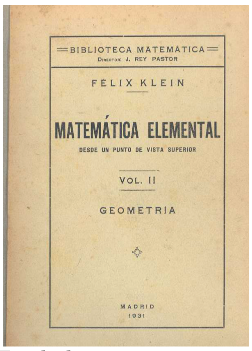


Several German editions of parts I and II were published and for the 3rd edition, Felix Klein, himself, would incorporate more material to the original text, adding the following justification to its preface:

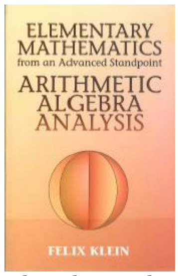
*Two volumes on "Elementary Mathematics from an Advanced Standpoint"... were to bring the attention of secondary school teachers of mathematics and science the significance for their professional work of their academic studies... But during the sixteen years which have elapsed since the first publication,* ***science has advanced, and great changes have taken place in our school system****, changes which are still in progress. This fact is provided for in the appendices which have been prepared.*

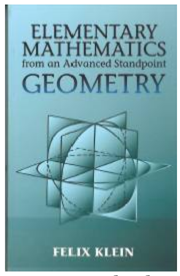
Meanwhile several translations of this work were taking place, from which we enhance the Spanish version (Madrid, 1927 and 1931) which was coordinated under the direction of Rey Pastor:



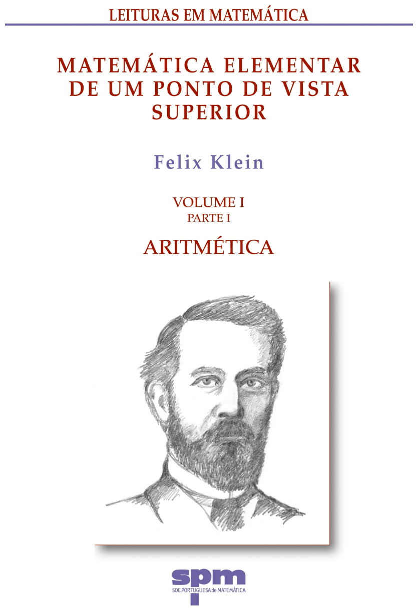




And we also enhance the English version, translated from the 3rd German Edition by E. R. Hedrick and C. A. Noble (Tome I, in 1932, and Tome II, in 1939) and published by The MacMillan Company, New York. By 2004 Dover Publications offered us an unabridged republication of this translation.




On the one hand, we keep in mind that, after one hundred years, this unique work is still a remarkable text for both mathematicians and mathematics educators and we are pleased to acknowledge that the Centro de Álgebra and the Centro de Matemática e Aplicações Fundamentais, from the University of Lisbon, are preparing the first Portuguese translation which is to be presented as the next three issues (the first of them, on *Aritmética*, is now completed) of the collection "Leituras em Matemática", from Sociedade Portuguesa de Matemática.




On the other hand if, in the very words of Klein himself, many changes, in our school systems, had occurred in sixteen years, what can we say about such a reality in this new century? A century that has, for example, enthusiastically embraced the so-called Modern Mathematics and, not long after that, has completely banned such teaching principles and mathematical content from our schools.

Wishing to reflect upon some ideas for the teaching and learning of mathematics in the XXI century we share - **through the organization of a Workshop on Didactics of Mathematics as a Mathematics Discipline, next October, at the University of Madeira** - our worries with a specialized panel of mathematicians, historians of mathematics and mathematics educators who celebrating the 100th anniversary of Klein's publications, in a partnership between IMU

(International Mathematical Union) and ICMI, have decided to re-visit Klein's ideas for, in the words of Michèle Artigue:

*(It is intended as) a stimulus for mathematics teachers, so to help them make connections between the mathematics they teach, or can be asked to teach, and the field of mathematics, while taking into account the evolution of this field over the last century.*

And, according to Bill Barton:

*The objectives are (· · ·)* ***to produce a readable but professional book that conveys the connectedness, growth, relevance, and beauty of the discipline of mathematics from its big ideas to the cutting edge of research and applications.***

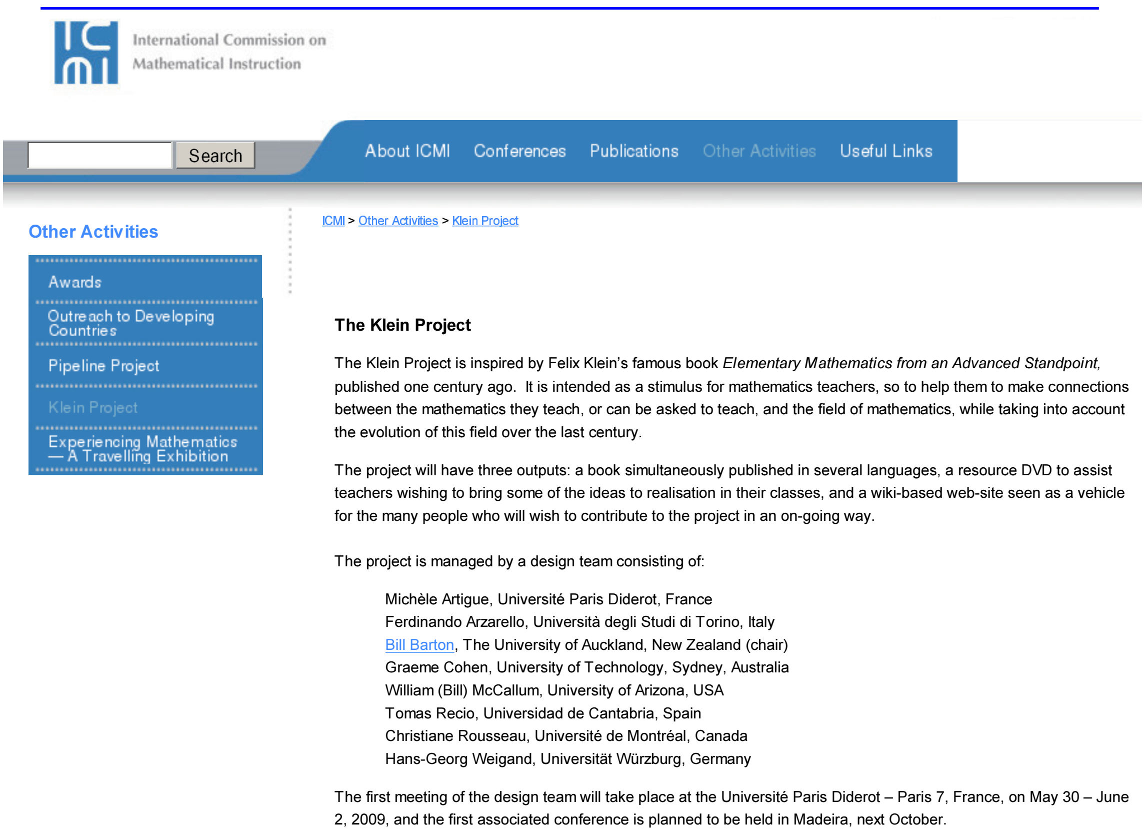
As referred in the ICMI web site `http://www.mathunion.org/index.php?id=805`, the Klein Project will have three outputs: a book simultaneously published in several languages, a resource DVD to assist teachers wishing to bring some of the ideas to realisation in their classes, and a Wikipedia-based web-site seen as a vehicle for the many people who will wish to contribute to the project in an on-going way.

Within the prospective languages of the Klein project, a Portuguese version of those outputs from its beginning will serve directly the community of mathematics teachers in Portugal, in Brazil, in Africa or in the rest of the world that use Portuguese as their primary language, making it the 5th language in the world by number of native speakers.

The Mathematics Education of the present times is, undoubtedly, affected by many factors, some of which Klein himself might had never dreamed of, but the partnership among those who deal with mathematics and with education is, 100 years after Felix Klein defended it, still in need of some debate. For example it might be useful to reflect upon:

- Which "advanced" mathematics is more prone to be introduced in our actual school curricula?
- Which mathematics is, nowadays, more exciting and more alive?
- Which influences, both within mathematics and externally, are we getting in our mathematics classes?
- Which ways do contemporary mathematics educators prefer?
- Within a universal discipline such is mathematics, which educational factors are national and which are international?
- What kind of knowledge can we withdraw from past experiences?

In summary: what are the new mathematical challenges to be faced by our school systems in a global society where, according to MSC (Mathematical Subject Classification), ninety seven categories/mathematical fields (with several thousands of sub-fields) are nowadays to be taken into consideration?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- Awards
- Outreach to Developing Countries
- Pipeline Project
- Klein Project
- Experiencing Mathematics — A Travelling Exhibition

ICMI > Other Activities > Klein Project

**The Klein Project**

The Klein Project is inspired by Felix Klein's famous book *Elementary Mathematics from an Advanced Standpoint,* published one century ago. It is intended as a stimulus for mathematics teachers, so to help them to make connections between the mathematics they teach, or can be asked to teach, and the field of mathematics, while taking into account the evolution of this field over the last century.

The project will have three outputs: a book simultaneously published in several languages, a resource DVD to assist teachers wishing to bring some of the ideas to realisation in their classes, and a wiki-based web-site seen as a vehicle for the many people who will wish to contribute to the project in an on-going way.

The project is managed by a design team consisting of:

Michèle Artigue, Université Paris Diderot, France
Ferdinando Arzarello, Università degli Studi di Torino, Italy
Bill Barton, The University of Auckland, New Zealand (chair)
Graeme Cohen, University of Technology, Sydney, Australia
William (Bill) McCallum, University of Arizona, USA
Tomas Recio, Universidad de Cantabria, Spain
Christiane Rousseau, Université de Montréal, Canada
Hans-Georg Weigand, Universität Würzburg, Germany

The first meeting of the design team will take place at the Université Paris Diderot – Paris 7, France, on May 30 – June 2, 2009, and the first associated conference is planned to be held in Madeira, next October.

# Coming Events

## July, 02-04, 2009: BCAM-CIM Workshop on Applied Mathematics,

Hotel Indautxu, Bilbao, Spain.

Organizers

José Miguel Urbano (CMUC/UC) and Enrique Zuazua (BCAM).

Aims

The Basque Center for Applied Mathematics (BCAM) and the Centro Internacional de Matemática (CIM) organize a joint workshop in applied mathematics in Bilbao. The workshop is intended to give the opportunity for a limited number of researchers to meet and discuss the state of the art and establish new research projects within applied mathematics.

Invited Speakers

Miguel Escobedo, EHU-UPV (Bilbao).

Irene Fonseca, Carnegie Mellon U. (Pittsburgh).

Manuel Luna, Universidad de Sevilla (Sevilla).

Ander Murua, EHU-UPV (San Sebastián).

Francisco Palacios, UPM (Madrid).

José F. Rodrigues, CIM (Coimbra).

Julio Rossi, FCyN and UBA (Buenos Aires).

Alexis F. Vasseur, University of Texas (Austin).

Juha Videman, IST (Lisboa).

Peicheng Zhu, BCAM/Ikerbasque (Bilbao).

Carlos Mora-Corral, BCAM (Bilbao).

Vincent Lescarret, BCAM (Bilbao).

Aurora-Mihaela Marica, BCAM (Bilbao).

Gonçalo Pena, CMUC (Coimbra).

For more information about the event, see

http://www.bcamath.org/bcam-cim-2009

## July, 07-10, 2009: Workshop on Quantum Effects in Biological Systems (QuEBS 2009),

Instituto Superior Técnico, Lisboa.

Organizers

Yasser Omar (IST, SQIG-IT) and Masoud Mohseni (MIT)

Aims

The identification and study of quantum mechanical phenomena in biological systems is an emerging area of interdisciplinary research spanning physics, chemistry, biology, and quantum information science. In this workshop, we intend to explore the interplay between quantum coherence and environmental effects in both driven and undriven biomolecular systems. In particular, the role of quantum dynamical coherence and decoherence in excitonic energy transfer within chromophoric systems, such as photosynthetic complexes, will be addressed.

Moreover, the existence of quantum entanglement in the dynamics of such systems will also be discussed. Furthermore, this workshop could provide new insight for engineering artificial photosystems, such as quantum dots and dendrimers, to achieve optimal energy transport by exploiting their environmental effects, with potential applications for the development of more efficient solar cells or photosensors.

The workshop will be constituted by both invited and contributed talks, and will include plenty of time for brainstorming and discussions on this novel area.

Invited Speakers

Alan Aspuru-Guzik (Harvard U)

Hans Briegel (U Innsbruck)

Greg Engel (U Chicago)

Theodore Goodson (U Michigan)

Susana Huelga (U Hertfordshire)

Ian Mercer (UC Dublin)

Alexandra Olaya-Castro (UC London)

Martin Plenio (Imperial College)

Gregory Scholes (U Toronto)

Robert Silbey (MIT)

For more information about the event, see

http://sqig.math.ist.utl.pt/lqcil/quebs09

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**July, 2009: Pedro Nunes Lectures**, by Luis Caffarelli. Please see page 2 and

http://www.cim.pt/?q=pnl_caffarelli

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**July, 2009: Kinetics and statistical methods for complex particle systems**,

Complexo Interdisciplinar da Universidade de Lisboa, Portugal

An initiative of the UTAustin-Portugal program in Mathematics in co-operation with CIM.

**Summer school**: July 13-18, 2009

**Workshop**: July 20-24, 2009.

ORGANIZERS

M. C. Carvalho (Universidade de Lisboa).

Fabio Chalub (Universidade Nova de Lisboa).

Irene Gamba (University of Austin).

Diogo Gomes (Universidade Técnica de Lisboa).

Rui Vilela Mendes (Universidade de Lisboa).

Robert Pego (Carnegie Mellon University).

AIMS

This two weeks event will consist of a summer school of 5 lectures of 4-hour mini courses during the first week, and a follow up with a second week holding a conference featuring talks at a more advanced level. The initiative will focus on analytical and numerical issues related to dynamical properties associated to non conservative interactive particle systems where non-equilibrium statistical asymptotic states are a signature of their complexity. This area of research have been emerging in the last decade as a follow up of recent studies to kinetic systems that models the evolutions of probabilities distributions into non-classical states where classical macroscopic models fail. New simulations that incorporate stochasticity, multi-scale and approximations to nontrivial diffusion limits will be addressed. The meetings will discuss connections to probability and stochastic theory in connection to natural and social sciences.

SUMMER SCHOOL LECTURERS

Eric Carlen (Rutgers University, USA).

Pierre Degond (University P. Sabatier, France).

Irene M. Gamba (University of Austin, USA).

Markos Katsoulakis (University of Massachusetts, USA)

Robert Pego (Carnegie Mellon University, USA).

For more information about the event, see

http://kinetic.ptmat.fc.ul.pt/

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**September, 11, 2009: Jornada SPM/CIM/CMAT "Dia das Equações"**

Rio Douro, Portugal.

This event, on board of a "rabelo" boat, will be devoted to partial differential equations.

ORGANIZERS

Lisa Santos, Assis Azevedo and Fernando Miranda (CMAT, Universidade do Minho)

INVITED SPEAKERS

Eugénio Rocha (Universidade de Aveiro).

Filipe Oliveira (Universidade Nova de Lisboa).

José Miguel Urbano (Universidade de Coimbra).

Juha Videman (Instituto Superior Técnico).

Miguel Ramos (Universidade de Lisboa).

For more information about the event, see

http://www.cim.pt/?q=spm_cim_jornada_DDE

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**September 24-26, 2009: International Conference on History of Astronomy in Portugal: theories, institutions, practices**,

Museu da Ciência, Universidade de Lisboa, Portugal

AIMS

The United Nations 62nd General Assembly, in order to celebrate the 400th anniversary of Galileo's first telescopic observations, has declared 2009 as the International Year of Astronomy (IYA2009). It is intended with this celebration to make widely known the importance of Astronomy as a science and as a technique. Among the different strategies proposed to reach this aim, the Portuguese National Committee of the

IYA2009, formed by the Portuguese Society of Astronomy, emphasizes the need to promote events related to the history of Astronomy.

This is particularly adequate to the Portuguese situation, as in our country there is still a lack of knowledge about the History of Astronomy, which we also can see in the lack of bibliography in Portuguese about this theme. This is fully admitted in the National Proposal of a Plan of Activities prepared by Portuguese National Organization Committee IYA2009 (*National Proposal of a Plan of Activities for the IYA2009*, preliminary version of 23/07/2007, 3.1.5, p. 7).

As in Portugal the history of astronomy is deeply tied up to the history of mathematics, a meeting on the history of astronomy is an excellent opportunity for our mathematics institutions to support IYA2009, allowing scholars and Portuguese researchers on the history of astronomy not only the opportunity of discussing these matters among themselves but also to listen and to talk to some of the best non- Portuguese researchers in this area, contributing to include Portugal in the international net of history of astronomy researchers. Some of the most important mathematics research centres in Portugal are sponsoring this conference: CMAF, CMUC, CMUP and CIM. Also the Portuguese Society of Mathematics and the Portuguese Society of Astronomy are supporting this conference.

Organizers

Luís Saraiva (CMAF, MCUL, U. Lisboa)

Carlos Sá (CMUP, U. Porto)

António Duarte-Leal (CMUC, U. Coimbra)

Speakers

Michael Hoskin (U. Cambridge)

Sérgio Nobre (Unesp, Rio Claro)

José Chabas (U. Pompeu Fabra, Barcelona)

José Vaquero (U. Extremadura)

Henrique Leitão (FCUL, CIUHCT)

Luis Tirapicos (MCUL)

Jim Bennett (History of Science Museum, U. Oxford)

Roberto de Andrade Martins (Unicamp, Brasil)

Carlos Ziller Camenietzsky (U. F. Rio de Janeiro)

Ugo Baldini (U. Pádua)

Heloisa Gesteira (MAST, Rio de Janeiro)

Fernando Figueiredo (U. Coimbra)

Isabel Malaquias (U. Aveiro)

Pedro Raposo (OAL)

Helmuth Malonek/ T. Costa (U. Aveiro)

Vitor Bonifácio (U. Aveiro)

Paulo Crawford/Ana Simões (FCUL, CIUHCT)

For more information about the event, see

http://www.cim.pt/?q=events

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**October 1-4, 2009: Didactics of Mathematics as a Mathematical Discipline (a XXIst century Felix Klein's follow up)**,

Universidade da Madeira, Funchal.

Organizing Committee

Elfrida Ralha (Universidade do Minho).

Jaime Carvalho e Silva (Universidade de Coimbra).

Suzana Nápoles (Universidade de Lisboa).

José Manuel Castanheira (Universidade da Madeira).

Local Organizers

Elsa Fernandes (Universidade da Madeira).

Sandra Mendonça (Universidade da Madeira).

Aims

For more information about the event, see page 24 and

http://glocos.org/index.php/dm-md/

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**November 23-24, 2009: The Mathematics of Darwin Legacy**,

Complexo Interdisciplinar da Universidade de Lisboa,

Portugal

See last page and http://www.cim.pt/Darwin2009/

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For updated information on these events, see http://www.cim.pt/?q=events

# EDUCATIONAL INTERFACES BETWEEN MATHEMATICS AND INDUSTRY

The International Commission on Mathematical Instruction (ICMI) and the International Council for Industrial and Applied Mathematics (ICIAM) are jointly launching the EIMI Study as part of the series of ICMI Studies. It will seek to better understand the connections between innovation, science and mathematics and to offer ideas and suggestions on how education and training can contribute to enhancing both individual and societal developments. The Study will examine the implications for education at the intersection of these two communities of practice – industrialists and mathematicians. We wish to emphasise that there should be a balance between the perceived needs of industry for relevant mathematics education and the needs of learners for lifelong and broad education in a globalised environment. The Study aims at broadening the awareness: of the integral role of mathematics in society; of industry with respect to what mathematics can and cannot realistically achieve; of industry with respect to what school and university graduates can and cannot do realistically in terms of mathematics; and of mathematics teachers and educators with regard to industrial practices and needs with respect to education. The Study also aims: to enhance the appropriate usage of mathematics in society and industry; to attract and retain more students, encouraging them to continue their mathematical education at all levels; and to improve mathematics curricula at all levels of education. To achieve these aims, ten content areas, each one with several questions, are suggested:

**1. The Role of Mathematics – Visibility & Black Boxes.** People are rarely aware of the importance of mathematics in modern technologies. The use of mathematics in modern society should be more visible questioning: How can mathematics, especially industrial mathematics, be made more visible to the public at large? — How can mathematics be made more appealing and exciting to students and the professionals in industry? — How can mathematics serve a progressive rather than a restrictive role in education and training for the workplace? — To what extent is it necessary or desirable to describe the inner workings of black boxes? What are the social implications of not explaining the inner workings of black boxes?

**2. Examples of Use of Technology and Mathematics.** Modern workplaces are characterised by the use of different types of technology including Mathematics in fields as diverse as the chemical industry, oil exploration, medical imaging, micro- and nanoelectronics, logistics & transportation, finance, information security, and communications and entertainment. What are insightful examples of the role of technology in showing and/or hiding mathematics in the workplace? — Does the existence of special types of technology and the hiding of mathematics from the view of the user imply a change in the mathematical demands on the user? How? — Do old competencies like estimation of results and reading of different scales become obsolete when using modern technology? Or, do they become more important? — What are the social and political consequences of the 'crystallising' and 'hiding' of mathematics in black boxes?

**3. Communication and Collaboration.** In the workplace, mathematics is seldom undertaken as an individual activity. Mathematical work, mostly on modelling and problem solving, is almost always a group activity and frequently the groups involved are made up of individuals with diverse expertise and expectations: How to identify which societal and/or industrial problems should be worked on? — How to better communicate within multi-disciplinary working groups? — How to communicate the underlying mathematics to the problem owners and/or general public? — How to achieve greater quantitative literacy among school leavers, workers, and the general population?

**4. The teaching and learning of Industrial Mathematics – Making Industrial Mathematics more visible.** Who decides what will be explained and to whom? — How to decide the level of explanation for various groups? — How to organise teaching and learning in order to make industrial mathematics visible – if this is wanted/necessary? — How much is it appropriate to explain for educational purposes in order to generate interest and excitement without overwhelming the learner?

**5. Using Technology and Learning with Technology: Modelling & Simulation.** Using a new technology usually requires special efforts to become acquainted with it, to develop routines and practice. This can be an obstacle to switching to a more modern technology as long as the older one still "does the job". On the other hand, change and innovation are necessary in industry. How should one decide on the level of detailed mathematics expected to be taught/learned in a given vocational black box situation? — How can mathematics help the transfer of technological procedures and/or solutions between different fields of industry? — What criteria should be used to judge the appropriateness of simulation in the teaching & learning of industry related practice? — How can one compensate for the "standardising effects" of any technology that is in widespread use?

**6. Teaching and Learning for Communication and Collaboration.** Communication and collaboration form an integral and important part of the industrial use of mathematics. Because of their importance in industry, it is desirable to have these skills taught and learned in all parts of education and training, questioning: What communication skills are specific to mathematics? — Are there specific skills for use in relation to industrial mathematics? — How do we teach mathematics as a second language?

**7. Curriculum and Syllabus Issues.** A partnership between mathematics and industry requires adjustments of the mathematics curriculum. This can also impact the teaching of mathematics in general, questioning: What are the (dis)advantages of identifying a core curriculum of mathematics for industry within the general mathematical curriculum at various levels and for various professions? — What are useful ways to introduce mathematics for industry into vocational education? — What are the (dis)advantages of creating specific courses on mathematics for industry vs. including the topic in the standard mathematical courses at various levels? — What are the (dis)advantages of treating mathematics for industry as an interdisciplinary activity or as part of the traditional mathematics syllabus?

**8. Teacher Training.** Teachers must be trained in new mathematical content, pedagogy and assessment and to recognise the presence of mathematics in society and industry. What level of understanding of this new content in relation to EIMI is appropriate for each grade level? — What are good practices that support this new direction in teacher training? — How to implement these changes in an efficient way?

**9. Good Practices & Lessons to be Learned.** In all sectors of education there are examples of good practice in relation to the Study. This Study would like to collect good examples of how to integrate industry into the educational process. Lessons to be learned from failures are of the same interest as those from successes.

**10. Research and Documentation.** National and trans-national documentation is widely missing in the field of mathematics and industry. Suggestions and contributions describing existing and future research and documentation of activities in the field of Educational Interfaces between Mathematics and Industry will be most welcome.

The ICMI/ICIAM joint Study on Educational Interfaces between Mathematics and Industry is designed to enable researchers and practitioners around the world to share research, theoretical work, projects descriptions, experiences and analyses. It consists of two main components: the Study Conference and the Study Volume.

1. The **Study Conference** will be held at the Fundação Calouste Gulbenkian in Lisbon, Portugal, on April 19-23, 2010. Participation will be by invitation only, based on a submitted contribution. Proposed contributions will be reviewed and selections made according to the potential to contribute to the advancement of the Study, with explicit links to the themes and approaches outlined in this Discussion Document. The Local Organising Committee is composed by Adérito Araújo (University of Coimbra), Assis Azevedo (University of Minho and CIM), António Fernandes (Technical University of Lisbon) and José Francisco Rodrigues (University of Lisbon and CIM).

2. The **Study Volume**, a post-conference publication, will appear in the New ICMI Study Series (NISS), published by Springer. Acceptance of a paper for the Conference does not ensure automatic inclusion in this book. A report on the Study will be presented during the 7th International Congress on Industrial and Applied Mathematics (ICIAM 2011, to be held on July 18-22, 2011, in Vancouver, Canada), as well as at the 12th International Congress on Mathematical Education (ICME-12), to be held in Seoul, Korea, on July 8-15, 2012.

The deadline for proposals of contributions to the study conference and to the study volume is **October 15, 2009** and should be done through the Study website http://www.cim.pt/eimi/

For further information please contact the two co-chairs of the Study, Alain Damlamian (Faculté de Sciences et de Technologie, Université Paris-Est, damla@univ-paris12.fr) or Rudolf Sträßer (Institut für Didaktik der Mathematik, Justus-Liebig-Universität Gießen, Rudolf.Straesser@math.uni-giessen.de)

**References:**
OCDE 2008 Report: http://www.oecd.org/dataoecd/47/1/41019441.pdf
EIMI 2008 - Discussion document: http://eimi.mathdir.org/wp-content/uploads/2009/03/eimi_dd-09.pdf

# THE MATHEMATICS OF DARWIN LEGACY

Lisbon, 23-24 November 2009

An INTERNATIONAL CONFERENCE

In the framework of the "Darwin year", CIM organizes a conference in collaboration with the European Society for Mathematical and Theoretical Biology at the University of Lisbon, on the occasion of the 150th anniversary of the publication of "On the Origins of Species by Means of Natural Selection".

The general aim of this two days high level conference is to present a general overview of the mathematical models of the evolution, including topics as Evolutionary Game Theory, Structured Evolution, Population Genetics, Probabilistic Models and Selection Theories, by a limited number of selected scientists, as well as to publish a collective book with the invited survey contributions.

INVITED SPEAKERS

Reinhard Bürger (U. Vienna, AT)
Warren J. Ewens (U. Pennsylvania, USA)
Mats Gyllenberg (U. Helsinki, FI)
Vincent Jansen (Royal Holloway, UK)
Sylvie Méléard (E. Polytechique, FR)
Hans Metz (U. Leiden, NL)
Masayasu Mimura (Meiji U., JP)
Jorge Pacheco (U. Lisboa, PT)
Benoit Perthame (U. Paris, FR)
Peter Taylor (Queens U., CA)
Peter Schuster (U. Vienna, AT)
Franjo Weissing (U. Groningen, NL)

POSTER SESSIONS

In order to encourage the participation in the conference, Poster Sessions will be organized, in particular for younger researchers that are encourage to apply, before the deadline of 30 September 2009, through the conference website:

http://www.cim.pt/Darwin2009.

SCIENTIFIC COMMITTEE

Fabio Chalub (U. N. Lisboa)
Peter Jagers (Chalmers U. Gothenborg)
Eva Kisdi (U. Helsinki)
José Francisco Rodrigues (U. Lisboa)

SPONSORS

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VENUE and SECRETARIAT

Complexo Interdisciplinar da Universidade de Lisboa, http://www.ciul.ul.pt/index_en.htm

Editors: Assis Azevedo (assis@math.uminho.pt)
Mário Branco (mbranco@ptmat.fc.ul.pt).
Address: Departamento de Matemática, Universidade de Coimbra, 3001-454 Coimbra, Portugal.

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