## MATHEMATICS AND WILD FIRES

by J. A. Ferreira\*; I. N. Figueiredo\*\*; and J. F. Rodrigues\*\*\*

In 2017 the center of Portugal, where the city of Coimbra is localised, was devastated by severe wild fires. Taking into account that Mathematics can play a significant role in predicting fire behaviour and spread, as well as, in defining suitable strategies to prevent and combat fires, the CIM (Centro Internacional de Matemática), in cooperation with CMUC (Centre for Mathematics of the University of Coimbra), CMAFcIO (Center for Mathematics, Fundamental Applications and Operations Research of the University of Lisbon) and APMTAC (Portuguese Association of Theoretical, Applied and Computational Mechanics) organized a small workshop, with about forty registered participants, that took place the 8-9 November 2018 at the Mathematics Department of the University of Coimbra. Alberto Bressan (Penn State University, USA) opened the meeting with a description of a new class of variational problems, seeking optimal strategies for the confinement of forest fires. Assuming the spreading of the wild fire can be controlled by constructing barriers, he addressed the existence (or non-existence) of a strategy that completely blocks the spreading of the fire, within a bounded domain by reducing it to an optimization problem, where one seeks to minimize the total value of the burned region, plus the cost of building the barrier. Deriving necessary conditions for optimality, nearly optimal strategies can be numerically constructed. Various open problems were also presented.

A generalization of the Richards' equations for the parametric spread of ellipse-borne wildfires, was present-



Figure 1.—A. Bressan — Dynamic blocking problems for a model of fire confinement

\* APMTAC & UCoimbra; \*

\*\* CMUC & UCoimbra;

\*\*\* CMAFcIO & ULisboa



ed as a Finsler geometric paradigm for wildfire spread modelling by Steen Markvorsen (Technical University of Denmark). He aimed to cover any type of (possibly time-dependent) ovaloid-borne wildfires in dimensions 2 and 3. Considering a given ovaloid at a given point in space-time as the local indicatrix, i.e. the local firelet, that is obtained by short (unit)time spread of a model fire from that point, ignited at that given time, and under the assumption of homogeneous (linearized) measures of fuel, wind, and topography, he considered a Finsler geometric framework. In this setting the generalized Richards' equations can be formulated as a time-dependent eikonal type Finsler-Hamilton-Jacobi equation, which can be solved and thence the corresponding wildfire spread problem - using results from the differential geometry of geodesic sprays and/or the control geometry of differential inclusions. Finally he also addressed the important problem of including the curvature of the front by modifying the Finsler eikonal equation into a second order equation, which (by construction) produces the observed initial increase in fire particle speed from point ignitions.

Gianni Pagnini (Basque Center for Applied Mathematics, Spain) talked on a versatile probabilistic approach for modelling fire-spotting. He based his approach on the weighted superposition of random fronts whose fluctuations of the position are distributed according to proper densities functions including the random effects of turbulent heat transfer and fire-spotting. In particular, a Gaussian density was considered for turbulence and a lognormal for the landing distance of the firebrands and a global sensitivity analysis for the proposed fire-spotting model has been performed. The proposed fire-spotting model has been implemented in WRF-Sfire, a full-fledged coupled fire-atmosphere model (http://www.openwfm.org/wiki/ WRF-SFIRE), using the Weather Research and Forecasting model (WFR). A test case has been performed and was discussed at the workshop.

The need of efficient fire prediction and fighting systems was emphasized by Wenceslao González Manteiga (Universidad de Santiago de Compostela, Spain), by using and developing statistics and optimization techniques. Reporting his research work with members of the MODESTYA group at his university, he described applied nonparametric inference techniques for spatial and spatio-temporal point processes to understand wildfire behavior, and operations research techniques for an optimal allocation of limited resources, such as aircrafts, in fire extinction. Working with a large number of data (more than 100.000 wildfires between 1999 and 2014) in Galicia (NW Spain), and a large number of covariates that may be in-

**Figure 3.—** M. A. Turkman — Statistical methods towards the construction of decision tools to assist wildfire management.





## Figure 2.

Left: G. Pagnini — Fire-spotting modelling for regional-scale wildfire simulators: a case study with WRF-Sfire.

Right: W. G. Manteiga — Using statistics and optimization techniques to fight against wildfires in Galicia.

volved in wildfire risk, which increase the computational demand and complexity of a Big Data scenario, his talk outlined the challenges found in the analysis of wildfires.

Maria Antónia Amaral Turkman (Universidade de Lisboa, Portugal) presented statistical methods towards the construction of decision tools to assist wildfire management, which were exemplified with the fire risk maps for 2018, based on satellite data of fire ignition and burned area in Portugal from 1988 to 2017. These maps were obtained in May 2018 and, for instance, spotted the Monchique area, in the South of Portugal, as a potential danger place, where in fact the greatest fires of 2018 in Portugal took place, and, in the North, the area of Caminha were prevention measures were effective. They are considered useful tools for decision makers to allocate firefighting capacities and to support fire/forest management decisions in space, according to the risks involved. Using data sources coming from satellite images and ground sources, and exploiting a Markovian structure for the fire incidence data, her team constructed a model able to capture, as much as possible, the strong spatio-temporal dependence structures in the fire incidence data, allowing at the same time for the introduction of any type of dynamic explanatory variables in the model. This was achieved through Bayesian hierarchical modeling techniques and simulation-based inference.

Dominique Morvan (Université Aix-Marseille, France) spoke on wildfires physics and modelling. Observation that the behavior of wildfires is governed by various physical mechanisms, at different scales in space (and time), ranged between less than 1mm (the flame) to larger than 100km (the plume), he stressed the numerical simulation

## Figure 4.—M. Rochoux — Overview and challenges of data-driven wildland fire spread modeling.







**Figure 5.**—Example of numerical simulation of a surface fire propagation in a grassland from Morvan's talk (left) and a view of the Mathematics and Wild Fires Workshop attendance (right).

of wildfires is a high challenging multiscale problem. Despite these difficulties, the resolution of some problems in fire safety engineering such as the propagation of a fire front through a wildland urban interface (WUI), needs to describe a fire at a relative local scale (few hundred meters), with a relatively high level of details. It is in this context, that a new class of fire models, referred in the literature as "fully physical models", has been proposed at the end of 90's. This approach, which is often referred in the literature as a multiphase formulation, consists of formulating the problem with the balance equations (mass, momentum, energy, etc.) of the coupled system formed by the vegetation and the surrounding atmosphere. A short presentation of what is a fully physical wildfires model and



**Figure 6.—**The table of the final session on challenges, open problems and scientific interaction (from left to right): W. G. Manteiga, M. A. Turckman, R. R. Linn, M. Rochoux, S. Markvorsen and J. F. Rodrigues.

some results obtained with this kind of approach was also given.

Although data-driven wildfire spread models are still at an early stage of development, Mélanie Rochoux (CECI, CNRS-CERFACS, France) provided an excellent overview of the current strategies and she highlighted some of the challenges and opportunities this new data assimilation approach offers. This most important aspect of wild fires modeling is a new approach that couples existing models and real-time observations of the wildland fire dynamics, with the objective of reducing the uncertainties in both model fidelity and input data. This approach, called "data-driven modeling" (or "data assimilation"), takes full advantage of the recent advances in remote sensing technology to improve forecasts of the wildland fire evolution.

Finally, all the speakers have participated at the Final Session on Challenges, Open Problems, Scientific Interaction, which was coordinated by José Francisco Rodrigues (Universidade de Lisboa) and had the special participation of Rodman Ray Linn, a well-known expert in computational models for wildfire behavior at the Los Alamos National Laboratory, USA. Rodman Linn made a short presentation plenty of mathematical opportunities in fire modeling and simulation, including the importance of the curvature in the fire front, the need to understand mathematically the breaking of the "wall of flames" and the influence of insects and of other ecological and environmental conditions in the variation of fire in forests.

W. Manteiga referred the interest of fire models for the insurance companies and shared his experience with the building of a COST proposal. J. F. Rodrigues asked for a model to describe the perturbation and breaking of telecommunications in the fire areas, a serious accident that occurred in the 2017 fires in Portugal. M. A. Turkman reported her experience in a current multidisciplinary national project on fire and M. Rochoux spoke of several open issues in fire modeling involving data sets, deep learning from images, data reduction and visualization.





**Figure 7.**—From left to right: A. Bressan, D. Morvan, M. Rochoux, W. G. Manteiga, M. A. Turckman, J. F. Rodrigues, S. Markvorsen, G. Pagnini, I. N. Figueiredo, J. A. Ferreira.

## Figure 8.—

Mathematization of Knowledge is a main theme of one of the 1969 Almada Negreiros' frescoes of the Department of Mathematics of the University of Coimbra.

Several participants also raised questions, emphasized the need of communication across disciplines and sharing data. S. Markvorsen also highlighted on the green board the significant phrase "Mathematization of Knowledge" that is written in a beautiful fresco by Almada Negreiros at the entrance of the Department of Mathematics of the University of Coimbra.

