

BULLETIN

INTERNATIONAL CENTER FOR MATHEMATICS

JUNE 2006

20

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COMING EVENTS

June 28-30: Workshop “From Lie Algebras to Quantum Groups”

ORGANIZERS

Helena Albuquerque and Joana Teles (University of Coimbra), Samuel Lopes (University of Porto).

AIMS

This workshop is a Satellite Conference of the International Congress of Mathematicians (ICM, Madrid, Au-

gust 22-30, 2006) and will bring together leading specialists in the topics of Lie algebras, quantum groups and related areas. It aims to present the latest developments in these areas as well as to stimulate the interaction between young researchers and established specialists.

The event will be held at the Department of Mathematics of the University of Coimbra.

KEYNOTE SPEAKERS

Helena Albuquerque (Univ. of Coimbra, Portugal)

Georgia Benkart (Univ. of Wisconsin-Madison, USA)
Alberto Elduque (Univ. of Zaragoza, Spain)
George Lusztig (MIT, USA)
Shahn Majid (Univ. of London, UK)
Carlos Moreno (Univ. Complutense de Madrid, Spain)
Michael Semenov-Tian-Shansky (Univ. of Bourgogne, France)

For more information about the event, see

www.aim.estt.ipt.pt/~jmmp/CIM/Lie/index.htm

July 1: 9th European Workshop on Applications and Generalizations of Complex Analysis

ORGANIZERS

Amílcar Branquinho and J. Carvalho e Silva (Univ. of Coimbra), Ana Foulquié and Maria Isabel Cação (Univ. of Aveiro).

AIMS

This workshop is satellite to the “12th European Intensive Course on Complex Analysis and Applications to Partial Differential Equations” to be held in both Universities of Coimbra and Aveiro from June 26 to July 7, 2006. It is intended to give an opportunity for discussions between junior and senior researchers from several European countries in various fields of mathematics related to Complex, Quaternionic and Clifford Analysis, like Algebra, Geometry, Numerical Analysis, Differential Equations, and Special Functions.

The event will be held at the CIM premises at the Astronomical Observatory of the University of Coimbra.

For more information about the event, see

www.mat.uc.pt/~ajplb/9th.htm

July 19-21: Workshop on Mathematics in Chemistry

ORGANIZERS

Jean-Claude Zambrini, J. Pinto Paixão and F. Bastos Pereira (University of Lisbon).

AIMS

To identify and discuss research problems in the area of the chemical sciences whose development is strongly dependent on mathematical techniques. To foster the collaboration between leading researchers in chemistry and mathematics.

Chemistry is an exact science since it relies on quantitative models that can be described and applied by using the mathematical language. For instance, the theory of chemical bonding and molecular structure, rates and equilibria of chemical reactions, molecular thermodynamics, relationships involving energy, structure and reactivity, modeling of solvation, are swarming with problems whose solutions require sophisticated mathematical techniques. Mathematics also plays a central role in many areas of “applied” chemistry and chemical engineering. Important examples include atmospheric chemistry, biochemistry, and the broad field of computer simulations. The development of faster and more accurate spectroscopic techniques, the design of molecular devices, biomolecular computers, and of new empirical methods to predict reliable chemical data, and the conception of more efficient chemical reactors are just a few of a vast number of other topics that have strong links to applied mathematics. A closer interaction between chemists and mathematicians may therefore lead to significant progress in many key problems in chemistry. The proposed workshop will foster that interaction since it will identify a number of important research issues which will benefit from a joint effort.

Intended Audience are Researchers and post-graduate students on mathematics or chemical sciences.

The event will be held at the Complexo Interdisciplinar of the University of Lisbon.

INVITED SPEAKERS

Sylvio Canuto (São Paulo, Brazil)

David C. Clary (Oxford, UK)

Irene Fonseca (Pittsburgh, USA)

James T. Hynes (Paris, France and Boulder, USA)

Claude Leforestier (Montpellier, France)

John A. Perdew (New Orleans, USA)

Piotr Piecuch (Michigan, USA)

Jean-Louis Rivail (Nancy, France)

Mário Nuno Berberan e Santos (Lisboa, Portugal)

João Aires de Sousa (Lisboa, Portugal)

António Varandas (Coimbra, Portugal)
Marcelo Viana (Rio de Janeiro, Brazil)
Hans-Joachim Werner (Stuttgart, Germany)
J. H. Zhang (New York, USA)

For more information about the event, see

www.math-chem.org

September 4-8: 3rd International Workshop on Mathematical Techniques and Problems in Telecommunications

ORGANIZERS

António Navarro (Univ. of Aveiro), Carlos Rocha and Carlos Salema (Technical Univ. of Lisbon).

AIMS

The goals are three fold. Firstly, to identify and possibly find solutions for a number of mathematical problems in the field of Telecommunications. Secondly to disseminate among telecommunications engineers some mathematical techniques which are not widely known in this community even if they are being applied in modern communication techniques. Thirdly, to improve mutual understanding and recognition between mathematicians and telecommunication engineers, heavy users of mathematical techniques in the field of engineering.

The intended audience includes telecommunications engineers, mostly those providing the problems and being introduced to new mathematical tools, and mathematicians, mainly providing solutions and being introduced to real life problems that may influence the direction of their research. A strong participation of young scientists, mainly those attending undergraduate degrees is also expected.

The event is a Satellite Conference of the ICM and will take place at the Polytechnic Institute of Leiria.

INVITED SPEAKERS

V. Poor (Princeton University, USA)
Cross-Layer Issues in Wireless Networks

J. Rosenthal (Notre Dame University, USA)
Encryption

J. C. Pedro (University of Aveiro, Portugal)
Mathematical Needs for Behavioural Modelling of Telecommunication Circuits and Systems

J. Craveirinha (University of Coimbra, Portugal)
A multiobjective routing optimisation framework for multiservice networks - a heuristic approach

C. Guillemot (INRIA, France)
Signal Processing And Compression

For more information about the event, see

www.mtpt.it.pt

September 11-15: 2nd Summer School on Mathematics in Biology and Medicine

ORGANIZERS

J. Carneiro, F. Dionísio, G. Gomes and I. Gordo (Gulbenkian Institute of Science, Oeiras).

AIMS

The aim of this event is to promote the use of mathematical modelling in biology and medicine. This will be accomplished by bringing some of international experts to give a short course on their area of expertise. The lecturing team combines researchers with a diversity of backgrounds in mathematics, biology and medicine, who will share their experience with the participants.

The school is aimed at postgraduate students from mathematics, physics, biology or medicine, who are motivated to develop biomathematical research approaches.

The role of mathematical formalisms in providing insight into biological and medical processes became apparent at the beginning of the 20th century. The approach has since increased in popularity, especially during the past 10-20 years. This new phase of expansion is, to a large degree, stimulated by new developments in molecular biology and computation. Appropriate mathematical models are in great demand in many areas of biology.

Given the high stands of mathematical and biomedical research in Portugal, it is disappointing that only a few research groups integrate the two disciplines. This can be promoted by organizing interdisciplinary activities as proposed here.

The school will include a broad range of areas in biology and medicine where mathematical modelling is established. We plan to include six courses covering several research areas such as evolution, populations genetics,

epidemiology, population biology, developmental biology and immunology. Each course will consist of three lectures.

The event will take place at the Gulbenkian Institute of Science, Oeiras.

SHORT COURSES

Developmental Biology

R. Azevedo (University of Houston, USA)

Neurobiology

C. Brody (Cold Spring Harbor, USA)

Immunology

D. Coombs (Univ. of British Columbia, Canada)

Evolutionary Biology

T. Day (Queens University, Ontario, Canada)

Epidemiology and Population Biology

S. Levin (Princeton University, USA)

Population Genetics and Disease Mapping

G. McVean (University of Oxford, UK)

For more information about the event, see

eao.igc.gulbenkian.pt/mbm2006

October 20-21: CompIMAGE: International Symposium on Computational Modelling of Objects Represented in Images

ORGANIZERS

João Manuel Tavares and Renato Natal Jorge (Faculty of Engineering, Univ. of Porto, Portugal).

AIMS

In our days the research related with objects modelling has been a source of hard work in several distinct areas of science as, for example, mathematics, mechanical, physics, informatics, etc. One major application of objects modelling is in medical area. For instance, it is possible to consider the use of statistical or physical procedures on medical images in order to model the represented objects. Its modelling can have different goals like 3D shapes reconstruction, organs segmentation in

3D or 2D images, etc. Examples of others applications are: temporal tracking of objects, tracking and analyses of objects deformation, objects recognition, objects simulation, etc.

The main goal of the CompIMAGE consists in the provision of a comprehensive forum for discussion on the current state-of-the-art in these fields. The Symposium will cover (but is not limited to): Image Processing and Analysis, Image Segmentation, Data Interpolation, Registration, Acquisition and Compression 3D Reconstruction, Objects Tracking, Motion and Deformation Analysis, Objects Simulation, Medical Imaging, Computational Bioimaging and Visualization.

The Symposium will bring together several researchers representing several fields related to Computational Vision, Computer Graphics, Computational Mechanical, Mathematics, Statistics, Medical Imaging, etc. The expertise will span a broad range of techniques, such as finite element method, modal analyses, stochastic methods, principal components analyses, independent components analyses, distribution models, etc.

The Symposium will be held at Hotel D. Luís, Coimbra.

INVITED SPEAKERS

Francisco J. Perales (Univ. Illes Balears, Spain)

Perceptual Users Interfaces. Virtual & Augmented Reality Applications

Hélder Araújo (University of Coimbra, Portugal)

Active and Catadioptric Vision Systems for Robotics Applications

Mario Forjaz Secca (Univ. Nova de Lisboa, Portugal)

MR Diffusion Tensor Imaging

Sónia Isabel Gonçalves (University of Lisbon, Portugal and Vrije Universiteit Medical Centre, Netherlands)

Multimodality in Brain Imaging: Methodologic aspects and applications

Hemerson Pistori (Dom Bosco Catholic Univ., Brasil)

Computer Vision and Digital Inclusion of Persons with Special Needs: Overview and State of Art

Tony Chan (UCLA, USA)

Capturing illusory objects in images via level set methods

For more information about the event, see

paginas.fe.up.pt/CompIMAGE

OTHER CIM EVENTS IN 2006:

WORKING AFTERNOONS SPM/CIM

CIM, Coimbra

A joint initiative of the Portuguese Mathematical Society (SPM) and the International Center for Mathematics (CIM). Next meetings:

October 7, 2006 - Numerical Analysis

Organizer: Maria Joana Soares (Univ. of Minho)

December 16, 2006 - Algebra

Organizer: Gracinda Gomes (Univ. of Lisbon)

For more information, see

www.spm.pt/investigacao/spmcim/spmcim.phtml

IBERIAN CONFERENCE IN OPTIMIZATION

November 16-18, CIM, Coimbra

CIM EVENTS FOR 2007

The CIM Scientific Council, in a meeting held in Coimbra on February 11, approved the CIM scientific programme for 2007.

The list of events is the following:

WORKSHOP ON
MATHEMATICAL CONTROL THEORY AND FINANCE

April 10-14, Instituto Superior de Economia e Gestão,
Lisboa

WORKSHOP ON EIGENVALUE PROBLEMS:
SOFTWARE AND APPLICATIONS

June 27-29, University of Porto

LISBON QUANTUM COMPUTATION, INFORMATION
AND LOGIC 2007

July 18-20, Instituto Superior Técnico, Lisboa

CIM / UC SUMMER SCHOOL:
TOPICS IN NONLINEAR PDES

July 22-28, University of Coimbra

WORKSHOP ON ROBOTICS AND MATHEMATICS

September 17-19, University of Coimbra

For updated information on these events, see

www.cim.pt/?q=events

CIM NEWS

MEETING OF THE GENERAL ASSEMBLY OF CIM

The General Assembly of CIM met on May 27, 2006,

during the morning, in the CIM premises at the Astronomical Observatory of the University of Coimbra.

In the afternoon of the same day, the members of the General Assembly had the opportunity to attend a talk by Doutor Jorge Barros Luís (Montepio Geral).

RESEARCH IN PAIRS AT CIM

CIM has facilities for research work in pairs and welcomes applications for their use for limited periods.

These facilities are located at Complexo do Observatório Astronómico in Coimbra and include:

- office space, computing facilities, and some secretarial support;
- access to the library of the Department of Mathematics of the Univ. of Coimbra (30 minutes away by bus);
- lodging: a two room flat.

At least one of the researchers should be affiliated with an associate of CIM, or a participant in a CIM event.

Applicants should fill in the electronic application form in

www.cim.pt/?q=research

CIM ON THE WEB

For updated information about CIM and its activities, see

www.cim.pt

NEWS FROM OUR ASSOCIATES

(from UA)

VIEWS ON ODES

June 21-24, 2006

Aveiro, Portugal

ORGANIZERS

Vasile Staicu (Chairman), Alexandre Almeida, António Caetano, Luís Castro, José Mendes, Eugénio Rocha and João Santos (all Univ. of Aveiro).

The conference “Views on ODEs” will celebrate the 65th birthday of Professors Arrigo Cellina (Milan, Italy) and James A. Yorke (Maryland, USA) and aims to bring together those enrolled in research activities related with ordinary differential equations, differential inclusions and their applications. The main topics are: Dynamical systems, Bifurcations, Invariant measures, Chaotic attractors, Prevalence, Population dynamics, Markov operators, Semigroups, Viscosity solutions, Hamilton-Jacobi equations, Hyperbolic systems, Optimal control and differential inclusions, Variational and topological methods.

INVITED SPEAKERS

Zvi Artstein (Weizmann Inst. of Science, Rehovot, Israel), Jean-Pierre Aubin (CREA - École Polytechnique, Paris, France), Stefano Bianchini (SISSA-ISAS, Trieste, Italy), Alberto Bressan (Penn State Univ., USA), Francis Clarke (Univ. Claude Bernard Lyon 1, France), Constantin Corduneanu (Univ. of Texas at Arlington, USA), Francesco S. De Blasi (Univ. degli Studi di Roma Tor Vergata, Italy), Rui Dilão (IST, Technical Univ. of Lisbon, Portugal), Helene Frankowska (CREA - École Polytechnique, Paris, France), Andrzej Lasota (Univ. of Silesia, Katowice, Poland), Jean Mawhin (Univ. Catholique de Louvain, Louvain-la-Neuve, Belgium), Stefan Mirica (Univ. of Bucharest, Bucharest, Romania), Josef Myjak (Univ. degli Studi dell’Aquila, Italy), Boris S. Mordukhovich (Wayne State Univ., Detroit, USA), Farruh Mukhamedov (Depart. of Physics, Univ. of Aveiro, Portugal), Rafael Ortega (Univ. of Granada, Granada, Spain), Mitsuharu Otani (Waseda Univ., Tokyo, Japan), Carlos Rocha (IST, Technical Univ. of Lisbon, Portugal), Biagio Ricceri (Univ. of

Catania, Catania, Italy), Nikolaos Papageorgiou (National Technical Univ. of Athens, Greece), Giulio Pinigiani (Univ. of Florence, Florence, Italy), Miguel A. F. Sanjuan (Univ. Rey Juan Carlos, Madrid, Spain), Susanna Terracini (Univ. of Milano, Bicocca, Italy).

For more information, see

www.divp-proj.org

(from UC)

12TH EUROPEAN INTENSIVE COURSE ON COMPLEX ANALYSIS AND APPLICATIONS TO PARTIAL DIFFERENTIAL EQUATIONS

June 26-July 7, 2006

Coimbra and Aveiro, Portugal

ORGANIZERS

Amílcar Branquinho and J. Carvalho e Silva (Univ. of Coimbra), Ana Foulquié and Maria Isabel Cação (Univ. of Aveiro).

This intensive course follows the eleven held at the Universities of Coimbra and Aveiro from 1995 to 2005. This intensive course will have a total of 40 hours of lectures and is at postgraduate level. Lecturers will have time available to discuss with the students. This course is organized by the Universities of Coimbra and Aveiro with the same goals as the ones organized under the Socrates/Erasmus Intensive Program of Higher Education, and is opened to all young mathematicians interested in Complex Analysis and its applications.

INVITED LECTURERS

Andrei Martínez Finkelshtein (Almería Univ., Spain)
The Riemann-Hilbert technique for Polynomials Orthogonal on the Unit Circle

Guillermo López Lagomasino (Carlos III Univ., Spain)
Multi-Orthogonal Polynomials

Irene Falcão (Minho Univ., Braga, Portugal)
*Numerical conformal mappings and generalizations:
part I*

Sebastian Bock (Bauhaus-Univ. Weimar, Germany)
*Numerical conformal mappings and generalizations:
part II*

There will be a Workshop on “Applications and Generalizations of Complex Analysis” on the 1st of July, sponsored by CIM (www.mat.uc.pt/~ajplb/9th.htm).

For more information, see

www.mat.uc.pt/~ajplb/12.htm

(from IST)

ICORS 2006
INTERNATIONAL CONFERENCE ON ROBUST
STATISTICS

July 16-21, 2006

Lisbon, Portugal

ORGANIZERS

Conceição Amado, Rosário Oliveira, Ana Pires and Isabel Rodrigues (Technical Univ. of Lisbon), Carla Pereira (Univ. of Porto), Peter Rousseeuw (Univ. of Antwerp), Manuela Souto de Miranda (Univ. of Aveiro).

The ICORS 2006 conference aims to join people working on robust statistics and related fields. It intends to bring together both leading experts and young researchers, thus creating a forum to cover recent progress and to stimulate exchanges among active researchers. The conference will also encourage informal contacts and discussions between participants.

Contributions to applied statistics are welcome as well as theoretical ones, introducing new problems related to or interacting with robust statistics.

Previous ICORS meetings were held in: Vorau (2001), Vancouver (2002), Antwerp (2003), Beijing (2004) and in 2005 in Jyväskylä.

INVITED SPEAKERS

Jorge Adrover (National Univ. of Córdoba, Argentina), Claudia Becker (Martin-Luther-Univ. Halle-Wittenberg, Germany), José Berrendero (Univ.

Autónoma de Madrid, Spain), Ana Bianco (Univ. of Buenos Aires, Argentina), Javier Cabrera (Rutgers Univ., USA), Andreas Christman (Vrije Univ. Brussel, Belgium), Christophe Croux (Catholic Univ. Leuven, Belgium), Laurie Davies (Univ. of Essen, Germany), Peter Filzmoser (Vienna Univ. of Technology, Austria), Ursula Gather (Univ. of Dortmund, Germany), Marc Genton (Texas A & M Univ., USA), Ivette Gomes (Univ. of Lisbon, Portugal), Alfonso Gordaliza (Univ. of Valladolid, Spain), Xuming He (Univ. of Illinois at Urbana-Champaign, USA), Christian Hennig (Univ. College London, UK), Alfio Marazzi (Univ. of Lausanne, Switzerland), Ricardo Maronna (National Univ. of La Plata, Argentina), Ivan Mizera (Univ. of Alberta, Canada), Stephan Morgenthaler (École Polytechnique Fédérale de Lausanne, Switzerland), Hannu Oja (Univ. of Tampere, Finland), Marco Riani (Univ. of Parma, Italy), Elvezio Ronchetti (Univ. of Geneve, Switzerland), Anne Ruiz-Gazen (Univ. of Toulouse I, France), Stefan Van Aelst (Ghent Univ., Belgium), Maria-Pia Victoria Feser (Univ. of Geneve, Switzerland), Victor Yohai (Univ. of Buenos Aires, Argentina), Ruben Zamar (Univ. of British Columbia, Vancouver, Canada).

For more information, see

www.math.ist.utl.pt/icors2006

(from UC)

WORKSHOP “GEOMETRIC ASPECTS OF
INTEGRABLE SYSTEMS”

July 17-19, 2006

Coimbra, Portugal

ORGANIZERS

Joana Nunes da Costa, Joana Teles and Raquel Caseiro (Univ. of Coimbra), Nenad Manojlovic (Univ. of Algarve), Patrícia Santos (Inst. Sup. Eng. Coimbra).

This is a satellite workshop of the ICM, and is intended to focus on the following topics: Hamiltonian Integrable Systems, Bi-Hamiltonian Systems and Poisson Geometry, Quantum Integrable Systems, Superintegrability, Separability, Reduction, Lie Algebroids in Mechanics.

In addition to the invited lectures there will be contributions from the participants. The scientific committee will review the applications for contributions from the participants. PhD students are invited to participate.

INVITED SPEAKERS

Francesco Calogero (Univ. Roma “La Sapienza”, Italy)

José Cariñena (Univ. Zaragoza, Spain)

Pantelis Damianou (Univ. of Cyprus, Nicosia, Cyprus)

Jean-Pierre Francoise (Univ. Pierre et Marie Curie, Paris 6, France)

Franco Magri (Univ. Milano, Bicocca, Italy)

Juan Carlos Marrero (Univ. of La Laguna, Spain)

Yavuz Nutku (Feza Gurst Institute, Turkey)

Orlando Ragnisco (Univ. Degli Studi Roma, Italy)

For more information, see

www.mat.uc.pt/~geomis

(from UP)

XVTH OPORTO MEETING ON GEOMETRY, TOPOLOGY AND PHYSICS

July 20-23, 2006

Porto, Portugal

ORGANIZERS

Miguel Costa, Carlos Herdeiro and João Nuno Tavares (Univ. of Porto), Marco Mackaay (Univ. of Algarve), José Mourão and Roger Picken (IST, Lisbon).

The aim of the Oporto meetings is to bring together mathematicians and physicists interested in the interrelation between geometry, topology and physics and to provide them with a pleasant and informal environment for scientific interchange. The XVth Oporto Meeting is a Satellite Meeting of the ICM.

As in previous years, the meeting is focussed on the short courses given by the main speakers, which are supplemented by seminars by other participants. The talks are at the advanced graduate or postdoctoral level, and should be of interest to all researchers wishing to learn about recent developments in the overlap between geometry, topology and physics.

MAIN SPEAKERS

Frank Ferrari (Univ. Libre de Bruxelles, Belgium)
Supersymmetric Gauge Theories, Matrix Models and Geometric Transitions

Dan Freed (University of Texas at Austin, USA)
The Geometry of Abelian Gauge Fields

Nicholas Manton (Cambridge University, UK)
Supersymmetric Systems - Investigation and Interpretation

Veeravalli Varadarajan (UCLA, USA)
Unitary representations of Super Lie Groups

Albert Schwarz (Univ. of California at Davis, USA)
Supergeometry and Number Theory

For more information, see

www.fc.up.pt/cfp/omgtp2006

(from UA)

COMMUNICATING MATHEMATICS IN THE DIGITAL ERA

August 15-18, 2006

Aveiro, Portugal

ORGANIZERS

Eugénio Rocha (Chairman, Univ. of Aveiro), António Batel, Carlos Ferreira and Joaquim Pinto (Univ. of Aveiro), José Borbinha (IST-Lisbon).

The Digital Era has brought dramatical changes to the way researchers search, produce, publish and disseminate scientific work, in particular, mathematicians. This process is still evolving due to improvements in Information Science (Information Architecture, Archiving, Long-term Preservation), new achievements in Computer Science Technologies (ex. XML, Data Mining, Clustering, Recovery, Visualization, Web Tools) and initiatives as DML, Open Access Journals (ex. DOAJ), Digitisation Projects (ex. EMANI, GDZ, JS-TOR, NUMDAM), Scientific Catalogues (ex. Google Scholar), or OS Digital Repositories (ex. EPRINTS, DSpace).

CMDE2006 is a Satellite Conference of the ICM and aims to bring together researchers from areas of Mathematics, Information Sciences and Computer Sciences, providing a forum for presenting and discussing new ideas that may contribute to improve paradigms/mechanisms of producing, searching and using scientific and technical scholarship in Mathematics.

The main topics of CMDE2006 include: Data Mining, Clustering and Recovery, Digital Libraries and Archiving Networks, E-Mathematics Resources, Electronic Publishing, Free and Open Source Initiatives, Information Representation and Visualization, International Copyrights and Author's Rights, Math-networking and Electronic Communication, Mathematics E-Learning, Metadata Models and Standards, Multimedia Tools, Retrodigitisation, and Web Searching.

Presentations will be divided in four categories: Philosophical Implications on Scholarly Publishing, Theoretical and Technical, Projects and Initiatives, Software and Demonstrations.

INVITED SPEAKERS

J. Borwein (CEIC/IMU chair)
T. Bouche (NUMDAM, mini-DML project, Gallica-Math, CEDRAM)
B. Cipra (JPBM 2005 Communication Awarded)
J. Ewing (American Mathematical Society Executive Director)
P. Ion (W3C HTML-Math Working Group co-chair)
J. Kiernan (IBM Almaden Research Center)
B. Wegner (ELibM/EMIS chair)

For more information, see

www.cmde2006.org

(from IST)

INTERNATIONAL SUMMER SCHOOL AND WORKSHOP OF OPERATOR ALGEBRAS, OPERATOR THEORY AND ITS APPLICATIONS

September 1-5, 2006

Lisbon, Portugal

ORGANIZERS

Amélia Bastos (Coordinator), António Bravo, Amarino Lebre, Lina Oliveira, Paulo Pinto, Ana Moura Santos, Frank-Olme Speck (all Technical University of Lisbon).

AIMS

Operator algebras are presently one of the dynamic areas of mathematics. Nowadays Operator Algebras and

Operator Theory play an important role in different areas of Mathematics and its Applications, particularly in Mathematical Physics and Numerical Analysis.

This event is being organized at Instituto Superior Técnico, Universidade Técnica de Lisboa, Portugal, and it is a Satellite Conference of the ICM. It consists of a summer school with short courses on Operator Algebras and its Applications to Mathematical Physics and Numerical Analysis, and a workshop that will bring together researchers in the Operator Algebras and Operator Theory areas.

The main topics of the workshop include: Crossed product C^* -algebras; C^* -algebras of operators on Hardy and Bergman spaces; Invertibility theory for non-local C^* -algebras; Von Neumann algebras; Approximate methods in operator algebras; Asymptotic properties of approximation operators; Toeplitz, Hankel, and convolution type operators and algebras; Symbol calculi; Invertibility and index theory; Operator Theoretical Methods in Diffraction Theory; Factorization Theory and Integrable Systems; Applications to Mathematical Physics.

INVITED SPEAKERS

SUMMER SCHOOL COURSES

S. Power (Lancaster Univ., UK)
Subalgebras of Graph C^ -Algebras*
Konrad Schmüdgen (Leipzig Univ., Germany)
 C^ -Algebras - Selected Topics*
B. Silbermann (Chemnitz Univ., Germany)
 C^ -Algebras and Asymptotic Spectral Theory*
H. Upmeyer (Marburg Univ., Germany)
Toeplitz Operator Algebras and Multivariable Complex Analysis

WORKSHOP PLENARY LECTURERS

Mikhail Agranovich (Moscow Institute of Electronics and Mathematics, Russia)
Florin Boca (Univ. of Illinois, USA)
Lewis Coburn (State Univ. of New York at Buffalo, USA)
David Evans (Cardiff Univ., UK)
Israel Gohberg (Tel Aviv Univ., Israel)
Yuri Karlovich (Univ. Autonoma del Estado de Morelos, Mexico)
Naum Krupnik (Univ. of Toronto, Canada)
Vladimir Manuilov (Moscow State Univ., Russia)
Nikolai Nikolski (Univ. of Bordeaux, France)

Vladimir Rabinovich (Instituto Politecnico Nacional, Mexico)

Steffen Roch (Technical Univ. Darmstadt, Germany)

Stefan Samko (Univ. of Algarve, Portugal)

Ilya Spitkovski (William and Mary College, USA)

For more information, see

woat2006.ist.utl.pt

(from UNL)

SCRA2006/FIM XIII
INTERNATIONAL CONFERENCE ON
INTERDISCIPLINARY MATHEMATICAL AND
STATISTICAL TECHNIQUES

September 1-4, 2006

Lisbon/Tomar, Portugal

LOCAL ORGANIZERS

Carlos A. Coelho (Chair) and João T. Mexia (Co-Chair) (New University of Lisbon), Luís M. Grilo and Luís Merca (Polytechnic Institute of Tomar), Teresa Oliveira (Open Univ.-Lisbon).

The Forum for Interdisciplinary Mathematics, the Department of Mathematics, New University of Lisbon and the Polytechnic Institute of Tomar, are proud to co-organize a four-day International Conference in Lisbon/Tomar, Portugal. This year's conference theme is Interdisciplinary Mathematical and Statistical Techniques. The major concentration of the conference's academic activities will be in mathematical and statistical sciences including, but not limited to, Actuarial and Financial Mathematics, Statistics and Applications, Biostatistics, Combinatorics, Computer and Information Sciences, Differential Equations, Distribution Theory and Near Exact Distributions, Environmental

Statistics, Experimental Designs, Extreme Values, Forest Mensuration Modeling, Forest Economics, Graph Theory, Linear Statistical Inference, Mathematical Economics, Mathematics, Multivariate Statistics, Non-parametric Statistical Inference, Operations Research, Probability/Stochastic Processes, Public Health, Quality Control, Reliability and Life Testing, Sampling, Semigroups, and partner areas. Those wishing to contribute outside these areas are welcome to submit their abstracts. Suggestions for further topics and proposals to organize a symposium should be sent to the organizers.

The conference will feature separately a section on student paper competition. A selection panel will judge the presentations and make recommendations for awards. In addition, the conference will devote a session on "Editors Round Table" to help scholars appreciate the "current trends and techniques" of scholarly publications.

INVITED SPEAKERS

C. R. Rao (Penn State Univ., USA)

Barry C. Arnold (Univ. of California, Riverside, USA)

Francine Blanchet-Sadri (Univ. of North Carolina, Greensborough, USA)

Carlos Braumann (Univ. of Évora, Portugal)

Tadeusz Calinski (Agricultural Univ. of Poznan, Poland)

Richard Davis (Colorado State Univ., USA)

Angela Dean (Ohio State Univ., USA)

Malay Ghosh (Univ. of Florida, Gainesville, USA)

Steven Gilmour (Queen Mary Univ., London, UK)

Ivette Gomes (Univ. of Lisbon, Portugal)

Samad Hedayat (Univ. of Illinois at Chicago, USA)

Benjamin Kedem (Univ. of Maryland, USA)

John Stufken (Univ. of Georgia, Athens, USA)

For more information, see

scra2006.southalabama.edu

PAST EVENTS - SCIENTIFIC REPORTS

FOLLOW-UP WORKSHOP ON MATHEMATICS AND THE ENVIRONMENT

Scientific Report

The Follow-Up Workshop on Mathematics and the Environment took place in the CIM headquarters in Coimbra on January 27-28, 2006.

The meeting was intended as an informal gathering of some of the participants of the 2004 *CIM Thematic Term on Mathematics and the Environment* and other scientists with interest in this topic.

The two scientific sessions (on Oceanography and Atmospheric Sciences) were each organized under the following format: an invited talk, three short communications and an open discussion (see the complete program on www.aim.estt.ipt.pt/~jmmmp/CIM/wme2006/program.htm). The main goal of the event has been to strengthen the connections and to plan future collaboration between mathematicians, geophysicists and engineers.

After the talks and the discussions it became clear that Oceanography and Atmospheric Sciences represent an immense land of opportunity for researchers and graduate students in Mathematics. Among the possible interactions we would like to highlight the following:

- modelling and derivation of systems of equations (asymptotic analysis);
- validation of boundary conditions;
- data analysis - model/observations (stochastic analysis);
- error propagation (numerical analysis);
- small scale phenomena;
- simulation of flows (computational mathematics).

These interactions may develop through the joint supervision of Master and PhD students and in the framework of national and international research projects.

The crucial role played by CIM was acknowledged and it was suggested that a web page gathering relevant and stimulating mathematical problems in the area of the environmental sciences would be kept in the CIM web site.

José Miguel Urbano (Universidade de Coimbra) and Juha Videman (Instituto Superior Técnico)

AVEIRO WORKSHOP ON GRAPH SPECTRA

Scientific Report

The theory of graph spectra is a strong field of research in mathematics and in several applied sciences (e.g. chemistry), combining different areas, like linear algebra, algebraic combinatorics and algebraic graph theory. The first explicit mathematical paper in the theory of graph spectra was published in 1957 by L. Collatz and U. Sinogowitz [1]. However, we may say that the theory of graph spectra began not after 1931,

when E. Hückel used graph spectra in an implicit way in quantum theoretical treatment of the chemistry of benzenoid hydrocarbons [3]. According to the list of publications referred in [4], between 1931 and 1957, a few papers dealing with graph spectra theory implicit techniques applied to chemistry and molecular physics as well as some papers relating matrix theory and graphs, with applications in mathematical physics, economics

and geometry, were published. After 1957 an increasing number of papers on graph spectra has been published (at least 60 papers during the sixties, 158 during the seventies, 297 during the eighties, 425 during the nineties and, so far, more than 400 during the current decade).

The first comprehensive book on the topic, “*Spectra of Graphs - Theory and Applications*” by D. Cvetković, M. Doob and H. Sachs, appeared in 1979 [2], becoming a land mark and a priority reading for everyone interested in this field. In the last two decades several spectral techniques for treating graph theory problems have been developed: e.g. the use of graph eigenspaces, the star complement technique and many others. There are connections of the theory of graph spectra with other parts of combinatorics and also with algebra and geometry. It is very much used in theoretical chemistry but also has some relevance to other applied fields, e.g. physics, electrical engineering and computer science.

The Aveiro Workshop on Graph Spectra was the first international meeting organized as a forum for researchers on graph spectra and related topics. This three-day meeting took place in the Mathematics Department of University of Aveiro, between 10 and 12 of April 2006 (the web page is still active at ceoc.mat.ua.pt/conf/graph2006). This workshop deserved the attention of the Portuguese Mathematical Community, promoting the contact of some of his members (coming from several parts of the country) with the state of the art and with most of the recent advances in the topic. Indeed, 26 among the 60 participants, coming from Aveiro (10), Bragança (1), Coimbra (2), Covilhã (2), Lisbon (8), Porto (2), Setúbal (1), were in contact with international experts on graph spectra and their presentations. The foreign participants, including renowned specialist, came from Brazil (8), Canada (2), Germany (1), Italy (4), Malta (1), Poland (2), Serbia and Montenegro (3), Spain (4), The Netherlands (2), UK (4) and USA (3). The plenary presentations ranged through the following: *Spectral radius of tournaments and bipartite graphs* (Richard Brualdi, Univ. of Wisconsin - Madison, USA); *Graph spectra and graph isomorphism* (Chris Godsil, Univ. of Waterloo, Canada); *Star complement in finite graphs* (Peter Rowlinson, Univ. of Stirling, UK); *The Laplacian and Cheeger inequalities for direct graphs* (Fan Chung Graham, Univ. of California, San Diego, USA); *Old and new results on algebraic connectivity of graphs* (Nair Abreu, UFRJ, Rio de Janeiro, Brazil); *Constructing graphs with integral Laplacian spectra* (Steve Kirkland,

Univ. of Regina, Canada); *Spectral characterization of distance-regular graphs* (Edwin van Dam, Tilburg Univ., The Netherlands); *Generalized adjacency matrices* (Willem Haemers, Tilburg Univ., The Netherlands); *Signless Laplacians of finite graphs* (Dragos Cvetković, Univ. of Belgrade, Serbia and Montenegro). The scientific program was complemented with 18 contributed talks with many recent results (part of them delivered by leader researchers on the theory of graph spectra) and with a problems session (coordinated by Dragan Stevanović from University of Nis, Serbia and Montenegro). Taking into account the high level of most of the presentations, their authors were invited to submit the papers to be referred for publication in a special issue of *Linear Algebra and Its Applications*, to be edited by Dragos Cvetković, Willem Haemers and Peter Rowlinson.

For the organization of this workshop was crucial the main support of the Chairman of Scientific Committee, Dragos Cvetković and the very important advice also received, during the preparation of the workshop, from the other colleagues of Scientific Committee: Nair Abreu, Richard Brualdi, Chris Godsil, Willem Hamers and Perter Rowlinson. Finally, I can not forget my colleagues from the Organizing and Local Committees: Raul Cordovil, Leal Duarte, Carlos Luz, Guedes de Oliveira, Agostinho Agra, Paula Carvalho, Rosa Amélia, Paula Rama and Cristina Requejo from whom, since the very beginning, all the commitment and requested collaboration was received.

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Domingos Moreira Cardoso (Organizing Committee's Chairman)

CIM SCIENTIFIC COUNCIL MEETING 2006

Scientific Report

The 2006 Meeting of the CIM Scientific Council took place in *Hotel Quinta das Lágrimas* in Coimbra on February 11, with the following timetable:

10:30-16:30 Meeting of the Scientific Council (including a work lunch).

17:00-18:00 Lecture by Mário Martinez (State Univ. of Campinas, Brazil):

Lower-sum order-value optimization.

18:30-19:30 Lecture by Enrique Zuazua (Univ. Autónoma de Madrid, Spain):

Propagation, dispersion, control and numerical approximation of waves.

20:00 Dinner.

The Scientific Council Meeting took place in a pleasant environment and was attended by thirteen of its fifteen Members. The CIM Activity Plan for 2007 was

approved. Due to the outstanding scientific quality of its participants, the Scientific Council Meeting was the ideal opportunity for a broad discussion with the President about the strategies and future activities of the Centre.

The Seminars were attended by 50 participants, which is exactly the maximum number expected by CIM in their website announcement. This public session of the Scientific Council Meeting was highly participated and stimulating and many and interesting questions were put by the audience to the lecturers. A dinner was served to all the participants in this last event, which had the opportunity to socialize with the members of the scientific council. The Rector of the University of Coimbra also accepted the invitation of the Direction of CIM to participate in this dinner.

Joaquim João Júdice (President of the Executive Board of CIM)

Poisson vs. Symplectic Geometry

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Abstract

We survey the relationship between symplectic and Poisson geometries, emphasizing the construction of the symplectic groupoid associated with a Poisson manifold.

1 Introduction

Poisson and symplectic geometries are usually referred to as the geometries underlying classical mechanics. Let us recall briefly why this is so. In every introductory course in mechanics one learns Hamilton's equations describing the motion of a mechanical system with Hamiltonian H :

$$\begin{cases} \dot{q}_i = \frac{\partial H}{\partial p_i} \\ \dot{p}_i = -\frac{\partial H}{\partial q_i} \end{cases} \quad (i = 1, \dots, n) \quad (1.1)$$

where (q_1, \dots, q_n) are the position and (p_1, \dots, p_n) are the momenta, which together form coordinates on the phase space of the system.

Symplectic geometry originates from the following interpretation of equations (1.1). Introduce the closed 2-form:

$$\omega := \sum_{i=1}^n dp_i \wedge dq_i. \quad (1.2)$$

Since this 2-form is non-degenerate, every smooth function H determines a vector field X_H by the requirement:

$$i_{X_H} \omega = dH.$$

Now (1.1) is just the equation for the integral curves of this vector field. More generally, one defines a **symplectic manifold** to be a manifold M equipped with a symplectic form, i.e., a closed, non-degenerate, 2-form ω . Then every smooth function $H : M \rightarrow \mathbb{R}$ determines a **Hamiltonian vector field** X_H by exactly the same procedure. Darboux's theorem (see [9]) states

that, around any point, there exist local coordinates (q_i, p_i) such that ω takes the form (1.2), so locally we recover the standard formulation.

A slightly different interpretation of equations (1.1) leads to Poisson geometry. One defines a bilinear, skew-symmetric bracket of functions on the phase space by setting for any pair of functions F and G :

$$\{F, G\} := \sum_{i=1}^n \left(\frac{\partial F}{\partial q_i} \frac{\partial G}{\partial p_i} - \frac{\partial F}{\partial p_i} \frac{\partial G}{\partial q_i} \right) \quad (1.3)$$

and observes that Hamilton's equations can be written in the form:

$$\begin{cases} \dot{q}_i = \{q_i, H\} \\ \dot{p}_i = \{p_i, H\} \end{cases} \quad (i = 1, \dots, n).$$

More geometrically, any smooth function H determines a **Hamiltonian vector field** X_H by:

$$X_H(\cdot) = \{\cdot, H\},$$

and Hamilton's equations are just the equations for the integral curves of this vector field. Another justification for the introduction of the Poisson bracket is the study of first integrals of the system: if F and G are two first integrals, then their Poisson bracket $\{F, G\}$ is also a first integral. This is because, for any triple of functions F, G and H , we have the Jacobi identity:

$$\{F, \{G, H\}\} + \{G, \{H, F\}\} + \{H, \{F, G\}\} = 0.$$

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All this motivates defining a **Poisson manifold** to be a manifold M equipped with a **Poisson bracket** $\{ , \}$, i.e., a Lie bracket on the algebra of smooth functions $C^\infty(M)$ which satisfies the Leibniz identity:

$$\{F, GH\} = \{F, G\}H + G\{F, G\}.$$

Then any smooth function $H : M \rightarrow \mathbb{R}$ determines a **Hamiltonian vector field** X_H by the procedure above. But now, contrary to the symplectic situation, locally we may not recover anymore the standard form (1.3) of the Poisson bracket. In fact, we have the following theorem, which maybe consider as the first significant result in Poisson geometry:

Theorem 1.1 (Weinstein [10]). *Let $(M, \{ , \})$ be a Poisson manifold. For every $x_0 \in M$ there exists coordinates $(q_1, \dots, q_n, p_1, \dots, p_n, y_1, \dots, y_l)$ centered at x_0 such that:*

$$\{F, G\} = \sum_{i=1}^n \left(\frac{\partial F}{\partial q_i} \frac{\partial G}{\partial p_i} - \frac{\partial F}{\partial p_i} \frac{\partial G}{\partial q_i} \right) + \sum_{j,k=1}^l \pi_{jk}(y) \frac{\partial F}{\partial y_j} \frac{\partial G}{\partial y_k},$$

where $\pi_{jk}(y) = -\pi_{kj}(y)$ are certain functions of the y_j 's alone which vanish at 0.

Therefore, contrary to symplectic geometry, where there are no local invariants, in Poisson geometry it is important to understand the local structure as well.

More generally, what can be said about the relationship between symplectic and Poisson geometry? In one direction we have that every symplectic manifold is a Poisson manifold: to every symplectic form ω on a manifold M , one associates a Poisson bracket by setting:

$$\{F, G\} := \omega(X_F, X_G).$$

Conversely, the Darboux-Weinstein theorem above implies that a Poisson manifold is (singular) foliated by symplectic (immersed) submanifolds.

However, there is a much more subtle and deeper relation between Poisson and symplectic geometry: to every Poisson manifold one can associate a canonical symplectic object. Moreover, its properties encode both the local and global behavior of a Poisson manifold. In the remainder of this paper we will explain how this object arises naturally, and we will discuss briefly its relevance in the study of both local and global properties of a Poisson manifold.

2 Contravariant geometry

Let $(M, \{ , \})$ be a Poisson manifold. What kind of paths should one consider in M which take into account the Poisson geometry? Because M is foliated into symplectic submanifolds, paths in M should preserve this

foliation. However, this is a singular foliation and to take care of this we must give some ‘‘internal geometry’’ to the paths. More precisely, let $\pi : T^*M \rightarrow M$ be the cotangent bundle and consider the bundle map defined by:

$$\# : T^*M \rightarrow TM, \quad dH \mapsto X_H.$$

It is easy to check that the symplectic leaves of M are in fact the integral leaves of the distribution $\text{Im}\# \subset TM$.

Definition 2.1. A **cotangent path** is a path $a : [0, 1] \rightarrow T^*M$ such that:

$$\#a(t) = \frac{d}{dt}\pi(a(t)).$$

The space of cotangent paths will be denoted by $P_\Pi(M)$.

Note that the base path $\gamma(t) = \pi(a(t))$ of a cotangent path lies in a symplectic leaf. These kind of paths were introduced first by A. Weinstein, and they show up in virtual every global construction in Poisson geometry. For example, in [7] one studies connections in Poisson geometry and shows that parallel transport is defined along cotangent paths.

There is a *general principle* in Poisson geometry that every construction in standard (covariant) geometry can be dualized to a (contravariant) construction in Poisson geometry. The cotangent paths we have introduced is just one instance of this principle. Another instance is Poisson cohomology. Recall that de Rham cohomology of a manifold is the cohomology of the complex of differential forms $(\Omega^\bullet(M), d)$, where the differential of a r -form is given by the usual formula:

$$\begin{aligned} d\omega(X_0, \dots, X_r) = & \sum_{k=0}^r (-1)^{k+1} X_k(Q(X_0, \dots, \widehat{X}_k, \dots, X_r)) + \\ & \sum_{k<l} (-1)^{k+l+1} \omega([X_k, X_l], X_0, \dots, \widehat{X}_k, \dots, \widehat{X}_l, \dots, X_r), \end{aligned} \quad (2.1)$$

where $X_0, \dots, X_r \in \mathfrak{X}(M)$ are vector fields, $[,]$ denotes the usual Lie bracket of vector fields, and the hat over a factor means omitting that factor. Following the general principle above, in Poisson geometry one considers the dual objects to differential forms, i.e., the multivector fields $\mathfrak{X}^r(M)$, and defines a contravariant exterior differential $d_\Pi : \mathfrak{X}^r(M) \rightarrow \mathfrak{X}^{r+1}(M)$ by:

$$\begin{aligned} d_\Pi Q(\alpha_0, \dots, \alpha_r) = & \sum_{k=0}^r (-1)^{k+1} \# \alpha_k(Q(\alpha_0, \dots, \widehat{\alpha}_k, \dots, \alpha_r)) + \\ & \sum_{k<l} (-1)^{k+l+1} Q([\alpha_k, \alpha_l]_\Pi, \alpha_0, \dots, \widehat{\alpha}_k, \dots, \widehat{\alpha}_l, \dots, \alpha_r), \end{aligned} \quad (2.2)$$

where $\alpha_0, \dots, \alpha_r \in \Omega^1(M)$. Here $[,]$ is a Lie bracket on 1-forms induced from the Poisson bracket, which on exact 1-forms is given by

$$[dF, dG]_{\Pi} = d\{F, G\},$$

and extends to any pair of 1-forms by requiring that

$$[\alpha, F\beta]_{\Pi} = F[\alpha, \beta]_{\Pi} + \#\alpha(F)\beta.$$

It is easy to see that d_{Π} is indeed a differential: $d_{\Pi}^2 = 0$. Hence it defines the **Poisson cohomology** $H_{\Pi}^{\bullet}(M)$ of the Poisson manifold. It is not hard to see that for a symplectic manifold the Poisson cohomology is isomorphic to the usual de Rham cohomology. However, in general, the Poisson cohomology is quite hard to compute.

Observe that the defining equation of a Hamiltonian vector field can be written in the form $X_H = d_{\Pi}H$. It follows that $d_{\Pi}X_H = 0$ for any Hamiltonian vector field. More generally, any vector field X such that $d_{\Pi}X = 0$ is called a **Poisson vector field**. It is easy to check that X is a Poisson vector field iff its flow preserves Poisson brackets. The first Poisson cohomology group is just the quotient of the Poisson vector fields by the Hamiltonian vector fields.

In geometry one integrates 1-forms over curves. Dually, in Poisson geometry one integrates vector fields over cotangent paths: if $X \in \mathfrak{X}(M)$ is a vector field and $a \in P_{\Pi}(M)$ is a cotangent path with base path γ , then one defines:

$$\int_a X := \int_0^1 \langle X(\gamma(t)), a(t) \rangle dt.$$

The usual integral of *closed* 1-forms is invariant under homotopy and depends only on the end-points of the curve provided the form is exact. In Poisson geometry there is also a notion of **cotangent homotopy** between cotangent paths (the precise definition can be found in [6]), and we have:

Proposition 2.2 ([6]). *The integral of a Poisson vector field is invariant under cotangent homotopies. For a Hamiltonian vector field the integral depends only on the end-points of the cotangent path.*

In ordinary topology one defines the fundamental group $\pi_1(M, x_0)$ of a pointed space (M, x_0) to be the loops based at x_0 modulo homotopies, where the group multiplication arises from concatenation of paths. If M is connected, changing the base point leads to isomorphic fundamental groups. If one considers (not necessarily closed) paths modulo homotopy then we do not get a group anymore because we cannot always multiply two paths. We get instead a groupoid $\Pi_1(M) \rightrightarrows M$: there are source and target maps

$$\mathbf{s}([\gamma]) = \gamma(0), \quad \mathbf{t}([\gamma]) = \gamma(1),$$

and the product $[\gamma] \cdot [\tau]$ is defined provided $\mathbf{s}([\gamma]) = \mathbf{t}([\tau])$.

In Poisson geometry we consider the analogous **Poisson fundamental groupoid** $\Sigma(M) \rightrightarrows M$ formed by cotangent paths modulo cotangent homotopies:

$$\Sigma(M) := P_{\Pi}(M) / \sim$$

Of course we can consider only cotangent paths whose base paths are loops based at x_0 , and these form the **isotropy group**

$$\Sigma(M, x_0) = \mathbf{s}^{-1}(x_0) \cap \mathbf{t}^{-1}(x_0),$$

which should be thought of as the *Poisson fundamental group* based at x_0 . The Poisson fundamental groups at different base points are isomorphic provided the base points lie in the *same* symplectic leaf of M (otherwise, they may be non-isomorphic). Also, contrary to the fundamental group of a space, Poisson fundamental groups are usually non-discrete topological groups. This is because they contain information about the local behavior at x_0 of our Poisson structure. More generally, the groupoid $\Sigma(M)$ provides both local and global information about the Poisson structure. But before we turn into that we need to study its geometry.

3 Symplectic Groupoids

The Poisson fundamental groupoid $\Sigma(M)$ is a topological groupoid since it is a quotient of the Banach manifold $P_{\Pi}(M)$. Moreover, $\Sigma(M)$ has at most one smooth structure compatible with the quotient topology for which the projection $P_{\Pi}(M) \rightarrow \Sigma(M)$ is submersion. Whenever this smooth structure exists $\Sigma(M)$ becomes a Lie groupoid (i.e., the groupoid structure is compatible with the smooth structure) and we say that M is an **integrable** Poisson manifold. The obstructions to integrability were determined recently in [6, 5], solving a long standing problem in Poisson geometry (and Lie groupoid theory). Let us explain briefly how they arise.

First of all, for each $x \in M$ there exists attached to (M, π) a certain Lie algebra \mathfrak{g}_x . As a vector space, we have $\mathfrak{g}_x := \text{Ker } \#_x \subset T_x^*M$ and the Lie bracket is the restriction of the Lie bracket on 1-forms. We call \mathfrak{g}_x **isotropy Lie algebra** at x . Now, if $\Sigma(M)$ is smooth, each isotropy Lie group $\Sigma(M, x)$ is a Lie group with Lie algebra \mathfrak{g}_x which, in general, is neither connected nor simply connected. If we denote by G_x the 1-connected Lie group with Lie algebra \mathfrak{g}_x , the connected component of the identity $\Sigma(M, x)^0$ is isomorphic to G_x / \mathcal{N}_x where $\mathcal{N}_x \subset G_x$ is a certain normal discrete subgroup called the **monodromy group** at x . One can show that the monodromy group can also be described as the image

of a certain homomorphism $\partial : \pi_2(S, x) \rightarrow G_x$, where S denotes the symplectic leaf through x . Moreover, this description is still valid in the non-integrable case, but now the monodromy groups $\mathcal{N}_x = \text{im}(\partial)$ need not be discrete subgroups anymore. In fact, the main theorem of [5] implies:

Theorem 3.1. *A Poisson manifold is integrable iff the monodromy groups $\mathcal{N}_x \subset G_x$ are uniformly discrete as $x \in M$ varies.*

From now on we will assume that (M, π) is an integrable Poisson manifold so that $\Sigma(M)$ is a Lie groupoid. We will show now that $\Sigma(M)$ is a symplectic manifold and that the symplectic form ω is *compatible* with the groupoid multiplication, i.e., that

$$m^*\omega = \pi_1^*\omega + \pi_2^*\omega \quad (3.1)$$

where $m : \mathcal{G}^{(2)} \rightarrow \mathcal{G}$ is the multiplication in \mathcal{G} , defined on the space $\mathcal{G}^{(2)} \subset \mathcal{G} \times \mathcal{G}$ of composable arrows, and $\pi_1, \pi_2 : \mathcal{G}^{(2)} \rightarrow \mathcal{G}$ are the (restrictions of the) projections to the first and second factors.

In order to understand how the symplectic form appear we recall an alternative construction of $\Sigma(M)$ due to Cattaneo and Felder [1], and which is related with the Poisson-sigma model of string theory. Let us denote by $P(T^*M)$ the set of all paths in the cotangent bundle, so that $P_\Pi \subset P(T^*M)$. Since $P(T^*M) \simeq T^*P(M)$ is the cotangent bundle of the manifold of paths in M , it carries a natural symplectic form ω_{can} . Now the results in [1] (see the explanations in [6]) show that there exists a Lie algebra action

$$P_0\Omega^1(M) \rightarrow \mathfrak{X}(P(T^*M))$$

where $P_0\Omega^1(M)$ denotes the Lie algebra of time-dependent 1-forms α_t satisfying $\alpha_0 = \alpha_1 = 0$, with Lie bracket $[\cdot, \cdot]_\Pi$. The cotangent paths $P_\Pi \subset P(T^*M)$ form an invariant submanifold and two cotangent paths lie in the same orbit iff they are cotangent homotopic.

Now observe that the space of cotangent paths is precisely the level set $J^{-1}(0)$ of the map $J : P(T^*M) \rightarrow P_0\Omega^1(M)^*$ given by:

$$\langle J(a), \eta \rangle = \int_0^1 \left\langle \frac{d}{dt} \pi(a(t)) - \pi^\sharp a(t), \eta(t), \gamma(t) \right\rangle dt.$$

We have the following result due to Cattaneo and Felder [1]:

Theorem 3.2. *The Lie algebra action of $P_0\Omega^1(M)$ on $P(T^*M)$ is Hamiltonian, with equivariant moment map $J : P(T^*M) \rightarrow P_0\Omega^1(M)^*$.*

Hence the groupoid $\Sigma(M)$ can be described alternatively as a Marsden-Weinstein reduction:

$$\Sigma(M) = P(T^*M) // P_0\Omega(M). \quad (3.2)$$

We deduce:

Corollary 3.3. *If $\Sigma(M)$ is smooth, then it admits a symplectic form which turns $\Sigma(M)$ into a symplectic groupoid.*

Proof. We only need to check the compatibility of the symplectic form with the product. First note that we have the following explicit formula for the symplectic form ω_{can} in $P(T^*M)$:

$$\omega_{\text{can}}(U_1, U_2)_a = \int_0^1 \bar{\omega}_{\text{can}}(U_1(t), U_2(t)) dt,$$

for all $U_1, U_2 \in T_a P(T^*M)$, where $\bar{\omega}_{\text{can}}$ is the canonical symplectic form on T^*M . The additivity of the integral shows that that condition (3.1) holds at the level of $P(T^*M)$, hence it must hold also on the reduced symplectic space $\Sigma(M)$. \square

4 Local vs. Global properties

The symplectic groupoid $\Sigma(M)$ encodes both local and global properties of a Poisson manifold, which otherwise would be difficult or impossible to understand. We will illustrate these with two examples.

Let $(M, \{ \cdot, \cdot \})$ be a Poisson manifold and assume that the Poisson bracket vanishes at a point x_0 . This means that in the local form given by Theorem 1.1 there is only the second term (no (p, q) coordinates), so for x close to x_0 we have:

$$\{y_i, y_j\}(x) = c_{ij}^k y_k + \dots$$

where the dots represent higher order terms. Here the c_{ij}^k are just the structure constants of the isotropy Lie algebra $\mathfrak{g}_{x_0} = T_{x_0}^*M$ in the basis $\{d_{x_0}y_1, \dots, d_{x_0}y_m\}$. The *linearization problem* asks for new local coordinates where the higher order terms vanish (see [8] for a survey of this problem). We have the following deep theorem:

Theorem 4.1 (Conn [2, 3]). *If the isotropy Lie algebra is semisimple of compact type then there exists linearizing coordinates.*

The original proof due to Conn is a hard analysis proof based on the Nash-Moser method. He constructs successive coordinate systems which give better approximations to the linearizing coordinates and which do converge to the linearizing coordinates. Using the symplectic groupoid $\Sigma(M)$, Crainic and the author gave a soft geometric proof of this result, along the following lines:

- The hypothesis of the theorem is equivalent to the isotropy Lie group $\Sigma(M, x_0)$ being a compact, 1-connected Lie group.

- By Reeb stability, the s -fibers are compact and \mathbb{Q} -homological 2-connected. Hence the groupoid $\Sigma(M)$ is a proper 2-groupoid with 2-connected fibers.
- By the vanishing cohomology theorem of Crainic [4], the differentiable groupoid cohomology of $\Sigma(M)$ vanishes. By the Van Est Theorem (see [4]), it follows that the 2nd Poisson cohomology group $H_{\Pi}^2(M)$ also vanishes.
- The vanishing of $H_{\Pi}^2(M)$ allows one to apply a contravariant version of Moser's Path Method (see [9]) to obtain the linearizing coordinates.

Notice how this proof makes clear the relevance of the assumption, while in the original proof the assumption is hidden in the analysis (it is used to build certain norms necessary for the Nash-Moser method to work).

The previous example was about local properties of a Poisson manifold. Let us give a different example where both global and local properties are present. We claim that the following result holds:

Theorem 4.2. *If a Poisson manifold $(M, \{ \cdot, \cdot \})$ integrates to a compact symplectic groupoid $\Sigma(M)$ then the Poisson bracket cannot vanish at any point.*

Proof. Let us assume that the Poisson bracket vanishes at some point x_0 . Just like we observed in the proof above, it follows that $\Sigma(M)$ is a compact groupoid with 2-connected fibers, and from that we conclude that $H_{\Pi}^2(M)$ vanishes. Now the Poisson bracket always defines a class $[\Pi] \in H_{\Pi}^2(M)$, and hence this class must be trivial. At the level of the groupoid, this means that the cohomology class $[\omega] \in H^2(\Sigma(M))$ of the symplectic form in $\Sigma(M)$ is trivial. But this is not possible since, by assumption, $\Sigma(M)$ is a compact manifold. \square

As examples of Poisson manifolds (M, π) with $\Sigma(M)$ compact, we can take compact symplectic manifolds with finite fundamental group. I don't know of any other examples, and either they do not exist or they will provide an extremely interesting class of Poisson manifolds. So I believe it is important to solve the following:

Open Problem. Are there (non-symplectic) Poisson manifolds which integrate to a compact symplectic groupoid $\Sigma(M)$?

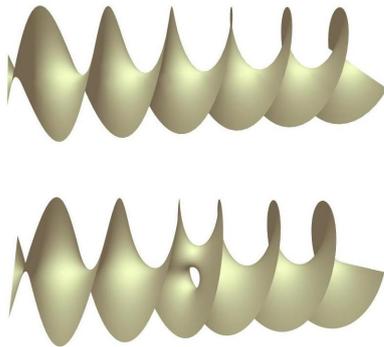
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“A Helix with a Handle”. That’s the title of Fenella Saunders’ piece in the May-June 2006 *American Scientist*. The subject is what its authors – Mathias Weber (Indiana), David Hoffman (Stanford) and Michael Wolf (Rice) – describe as “the first properly embedded minimal surface with infinite total curvature and finite topology to be found since 1776, when Meusnier showed that the helicoid was a minimal surface.” (Their paper, which appeared in the November 15, 2005 *PNAS*, is available online – www.indiana.edu/~minimal/research/helicoid.pdf).

In fact the new surface is closely related to Meusnier’s; it is properly described as a helicoid with a handle, or a “genus-one helicoid”, and is asymptotic to the helicoid at infinity.

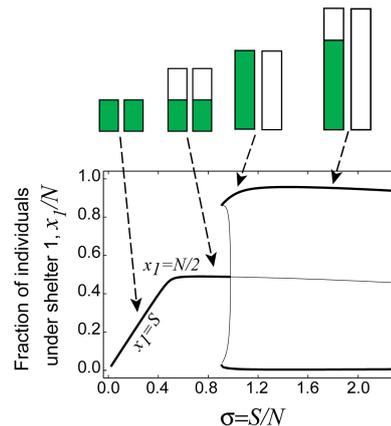


The helicoid and the genus-one helicoid. This picture shows a segment of a cylindrical core through each of the surfaces, which actually extend to infinity in every direction. Image courtesy of Indiana University.

Saunders tries to start her readers off gently: “Dip a loop of wire into a soapy solution, and the film that covers the loop will be what mathematicians call a minimal surface.” But soon we hear: “At any point, a minimal surface is maximally curved in one direction and minimally curved in the opposite direction, but the amount of curvature in each direction is exactly the same.” The readers may have better luck with the project’s interesting history. “Over a decade ago, Hoffman, with Fusheng Wei ... and Hermann Karcher ... , had created computer simulations of such handled helicoids, but an airtight demonstration of minimal surfacehood eluded them.” They knew what it looked like, but they

could not prove that it really was an embedded minimal surface. Saunders quotes Hoffman: “I think the information about how to solve this problem was lurking in the pictures all the time, but we just had to think about it for a long time and have the theory catch up with the evidence we had.”

Dynamics of Roach Congregation. “Group-living animals are often faced with choosing between one or more alternative resource sites.” Thus begins the abstract of a paper published April 11, 2006 in the *Proceedings of the National Academy of Sciences* (**105** 5835-5840). The authors, a French-Belgian team led by Jean-Marc Amé and José Halloy, report on “an experimental and theoretical study of groups of cockroaches (*Blattella germanica*) tested in a circular arena ... with identical shelters.” When the number of shelters is two, the phenomenon can be described by the graph below, giving the occupancy of shelter 1 as a function of shelter size. Until the size of a shelter is enough for the whole population, the roaches split between the two shelters. But as soon as there is room for everyone in each of the shelters, then the roaches all occupy one and not the other.



Occupancy of two shelters as a function of shelter size S for a fixed number N of individuals. When S/N is less than .5, both shelters are filled; for S/N between .5 and 1, the animals split evenly between the shelters; if S/N is 1 or more, all the animals congregate in one of the shelters. Adapted from *PNAS* **105** 5835-5840, from which the equations below are taken. Image courtesy of José Halloy ULB.

This behavior is predicted by a mathematical model. First, the researchers determined from experiment that the probability Q_i of an individual leaving shelter i varies inversely with the crowdedness (the ratio of the number x_i of animals in the shelter to the shelter size S):

$$Q_i = \frac{\theta}{1 + \rho\left(\frac{x_i}{S}\right)^n}$$

where θ , ρ and n are experimentally derived parameters. On the other hand, the probability R_i for an exploring cockroach to join shelter i decreases linearly with the crowdedness:

$$R_i = \mu\left(1 - \frac{x_i}{S}\right)$$

where μ is experimentally derived. These two laws can be combined into a system of differential equations:

$$\frac{dx_i}{dt} = \mu x_e \left(1 - \frac{x_i}{S}\right) - \frac{\theta x_i}{1 + \rho\left(\frac{x_i}{S}\right)^n}$$

(here x_e is the number of unattached individuals) subject to the constraint

$$x_e + x_1 + x_2 + \dots + x_p = N$$

if there are p shelters and a total of N individuals. The authors solved numerically for the steady states, which for $p = 2$ appear schematically in the graph above. Larger numbers of shelters give more complex bifurcation. The authors explain why this collective behavior gives the optimal outcome for each individual roach, and speculate that “the collective decision-making process studied here should have its equivalent in many gregarious animals” This work was picked up in the April 6 2006 *Nature* Research Highlights.

Mathematical Incompleteness in the Scientific American. The March 2006 *Scientific American* features a report by Gregory Chaitin, entitled “The Limits of Reason,” describing his own work on the incompleteness of mathematics. “Unlike Gödel’s approach, mine is based on measuring information and showing that some mathematical facts cannot be compressed into a theory because they are too complicated” and that therefore “... a theory of everything for all of mathematics cannot exist.” Chaitin outlines his theory, including the irreducible number Omega: the first N digits of Omega cannot be computed using a program significantly shorter than N bits long. He sketches the argument that computing the first N binary digits of Omega would solve the halting problem for all programs of length up to N ; so the uncomputability of Omega follows from Turing’s proof of the unsolvability of the halting problem. It follows from its definition that “an infinite number of bits of Omega constitute mathematical

facts ... that cannot be derived from any principles simpler than the string of bits itself. Mathematics therefore has infinite complexity, whereas any individual theory of everything would only have finite complexity and could not capture all the richness of the full world of mathematical truth.” Chaitin then spends some time pondering the scientific and philosophical consequences of his work. “Irreducible principles – axioms – have always been part of mathematics. Omega just shows that a lot more of them are out there than we suspected.” “If Hilbert had been right, ... there would be a static, closed theory of everything for all of mathematics, and this would be like a dictatorship. ... I much prefer an open system. I do not like rigid, authoritarian ways of thinking.” “Extensive computer calculations can be extremely persuasive, but do they render proof unnecessary? Yes and no. In fact, they provide a different kind of evidence. In important situations, I would argue that both kinds of evidence are required, as proofs may be flawed, and conversely computer searches may have the bad luck to stop just before encountering a counterexample that disproves the conjectured result.”

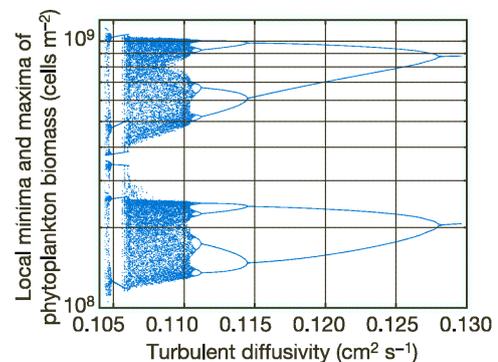
Fractals finger suspect Pollocks. Alison Abbott reports in the February 9 2006 *Nature* on a new mathematical development in the saga of the 32 small “possible Pollocks” recently discovered on Long Island. Large poured works by Jackson Pollock bring prices in the tens of millions of dollars; if these paintings are authentic they are very valuable. But their authenticity, accepted by some experts, has been challenged by others. Enter the physicist Richard Taylor. Taylor had published in 1999 (*Nature* **399** 422) his group’s discovery that Pollock’s poured works showed (as Abbott explains it) “two distinct sets of fractal patterns. One was on a scale larger than 5 cm; the other showed up on scales between 1 mm and 5 cm.” and furthermore “that the fractal dimension of Pollock’s works ... increased through the years as the artist refined his technique.” In a later experiment, he analyzed “14 Pollock paintings, 37 imitations created by students at the University of Oregon, and 46 paintings of unknown origin.” Abbott quotes Taylor: “The only shared thing in Pollock’s very different poured paintings is a fractal composition that was systematic through the years.” The non-Pollocks, when they had fractal structure, had different fractal characteristics. So it was natural for the Krasner-Pollock foundation to send six of the putative 32 for Taylor to examine. His diagnosis: “I found significant deviation from Pollock’s characteristics.” The foundation’s final judgment has not yet been promulgated. The *Nature* piece has several echoes in the *New York Times*. It gets picked up as a news item by Randy Kennedy (“Computer Analysis Suggests Paintings Are Not Pollocks”) on February 9. Their art critic Michael Kimmelman weighs in with “A Drip by Any Other Name” on February 12: “... the curious truth is that

while a few drips and splashes can imitate Pollock's touch ... it is nearly impossible to replicate ... the full-scale complex rhythms and overlapping patterns, the all-over, depthless, balletic and irregular space he created." And then on the February 19 Op-Ed page, Professor Don Foster (English, Vassar, "Mind Over Splatter") brings us an academic perspective. He starts out serious: "At the heart of the controversy lie critical questions about artistic meaning and value that have vexed literary scholars no less than art historians." But he leaves us with: "Meanwhile, Jackson Pollock may be chuckling in his grave: if the object of Abstract Expressionist work is to embody the rebellious, the anarchic, the highly idiosyncratic – if we embrace Pollock's work for its anti-figurative aesthetic – may faux-Pollock not be quintessential Pollock? May not a Pollock forgery that passes for authentic be the best Pollock of all?"

The differential geometry of quantum computation. "Quantum computers have the potential to solve efficiently some problems that are considered intractable on conventional classical computers." This is the start of "Quantum Computation as Geometry," a report in the February 4 2006 issue of *Science*. The authors are four members of the School of Physical Sciences, University of Queensland; a team led by Michael Nielsen. They continue: "Despite this great promise, as yet there is no general method for constructing good quantum algorithms, and very little is known about the potential power (or limitations) of quantum computers." What they propose in this report is an alternative approach to understanding the difficulty of an n -qubit computation, i.e. the complexity of the quantum algorithm that would be needed to carry it out. Such a computation corresponds to a unitary operator U (a $2^n \times 2^n$ matrix with complex entries). The authors' definition of difficulty is the length $d(I, U)$ of the shortest path from the identity matrix to U , where length is measured in a metric which penalizes all computational moves which require gates with more than two inputs. They show that this distance is "essentially equivalent to the number of gates required to synthesize U ." "Our result allows the tools of Riemannian geometry to be applied to understand quantum computation. In particular we can use a powerful tool – the calculus of variations – to find the geodesics of the space." They remark that thinking of an algorithm as a geodesic "is in contrast with the usual case in circuit design, either classical or quantum, where being given part of an optimal circuit does not obviously assist in the design of the rest of the circuit." Finally they show how "to construct explicitly a quantum circuit containing a number of [one and two-qubit] gates that is polynomial in $d(I, U)$ and which approximates U closely."

Chaos in the deep. "Reduced mixing generates oscil-

lations and chaos in the oceanic deep chlorophyll maximum" appeared in the January 19 2006 *Nature*. The authors, an Amsterdam-Honolulu collaboration led by Jef Huisman and Nga N. Pham Thi, investigated the stability of deep chlorophyll maxima (DCMs) – layers of high concentration of phytoplankton who flourish where there are sufficient nutrients welling up from the bottom and sufficient light filtering down from the top. The point of the article: "we extend recent phytoplankton models to show that the phytoplankton populations of DCMs can show sustained fluctuations." The authors set up a mathematical model, a reaction-advection-diffusion equation for the phytoplankton population density P coupled to a partial differential equation for the nutrient availability N . A common parameter in both equations is the "turbulent diffusivity" κ , the coefficient of the second-derivative terms. If κ is sufficiently large, "nutrients in the top layer are gradually depleted by the phytoplankton. The nutricline moves downwards, tracked by the phytoplankton population, until the population settles at a stable equilibrium at which the downward flux of consumed nutrients equals the upward flux of new nutrients." To investigate the behavior for lower κ , the authors ran "numerous simulations using a wide range of turbulent diffusivities." "The model simulations predict that the DCM becomes unstable when turbulent diffusivity is in the lower end of the realistic range. By a cascade of period doublings, reduced turbulent mixing can even generate chaos in the DCM."



The numerical solution of the coupled $P - N$ differential equations shows bifurcation and eventually chaos as the mixing parameter is decreased. This is a close-up picture of the evolution of the local maxima and minima of the phytoplankton population as a function of turbulent diffusivity, near the low end of the realistic range $0.1 < \kappa < 1$. Image from *Nature* **439** 324, used with permission.

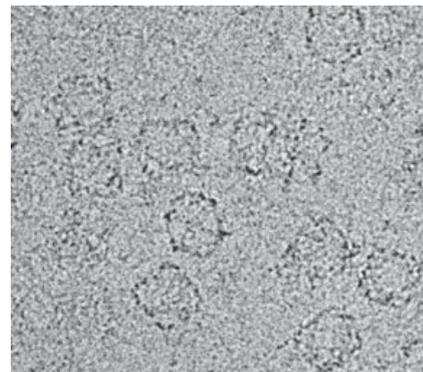
Their explanation for the periodic behavior: if κ is low, the phytoplankton sink faster than the nutrients are welling up; without sufficient light their numbers decline. This lets more nutrients through up to more luminous layers, and "fuels the next peak in the DCM."

An ominous note: “Climate models predict that global warming will reduce vertical mixing in the oceans.”

Plant growth and the Golden Ratio, re-evaluated. The *Botanical Journal of the Linnaean Society* ran in its January 2006 issue an article by Todd Cooke (Maryland) with the title: “Do Fibonacci numbers reveal the involvement of geometrical imperatives or biological interactions in phyllotaxis?” From the abstract: “This paper reviews the fundamental properties of number sequences, and discusses the underappreciated limitations of the Fibonacci sequence for describing phyllotactic patterns.” Apparently golden-ratio giddiness has spread to botany, and this paper aims to be a corrective. Prof. Cooke’s main point is that although “it is inescapable that the spiral phyllotaxes of vegetative shoots are overwhelmingly characterized by low Fibonacci numbers,” the common belief that “such spiral arrangements are attributable to the leaf primordia being positioned in optimal packing” must be questioned and ultimately rejected. The argument, as I understand it, runs as follows. Suppose consecutive primordia are arranged at an exactly constant angular difference. If that difference is *exactly* the golden angle, here given as 137.5° , then one does indeed achieve optimal packing. But “even slight variation from the Fibonacci angle disrupt[s] optimal packing.” E.g. constant angle 137.45° or constant angle 137.92° don’t work. “It is difficult, if not impossible, to imagine any biological system being capable of organizing itself with such discriminating accuracy as a direct response to a hypothetical geometrical imperative for optimal packing. It seems more likely that the spiral phyllotaxes observed ... are the outcome of some biological process, the consequence of which is that such structures tend to approach optimal packing.” There are two points here. The mathematical one is shaky. The golden ratio is (supremely) irrational, and the evidence for its occurrence in the likeliest interval between consecutive primordia (*viz.*, the appearance of numbers of spirals corresponding to its rational approximators $2/3$, $3/5$, $5/8$, etc.) is excellent. On the other hand the question whether optimal packing is an “imperative” or a “consequence” does not seem to me to be one that science can answer. The end of this article addresses the identification of the biological process governing phyllotaxis. Cooke refers to the 1992 *Physics Review Letters* paper (68, 2089-2010) by Stéphane Douady and Yves Couder, where they “managed to create spiral phyllotaxis on a lab bench” working with mutually repelling ferrofluid drops floating on silicon oil in a varying magnetic field. Presumably something analogous is happening at the growing tip of a plant. “The ... mechanism ... appears to involve the interaction of mathematical rules, generating process, and overall geometry. In particular, it seems quite plausible that the mathematical rules for phyllotaxis arise from local inhibitory interac-

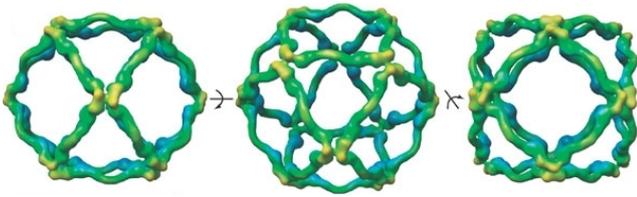
tions among existing primordia. These interactions are apparently mediated by the expression of specific genes whose products regulate growth hormones” This work was picked up in the “Research Highlights” of the February 9 *Nature*.

Cuboctahedral vesicles in eukaryotic cells. A eukaryotic cell is a complex, three-dimensional organism. Just as our food is ingested in one place and moved to another for processing, with the nutrients then ferried about the body by the bloodstream, so in a cell’s internal economy a critical role is played by transportation. The agents of intracellular transport are vesicles: molecular cages that enclose their cargo and move it from A to B. A paper in the January 12 2006 *Nature* explores the structure of one type of vesicle: those whose skin is made from the coat protein complex II, or COPII. The authors (a Scripps Research Institute team of 8, led by Scott Stag) explain that the structural part of COPII consists of a lattice formed by the protein complex Sec13/31.



Part of a micrograph of Sec13/31 cages preserved in vitreous ice. The cages are approximately 600\AA (0.06 microns) in diameter; their images show the planar projection of their cuboctahedral structure. Image from *Nature* 439 235, used with permission.

Using electron cryo-microscopy, they determined that the most elementary cages formed by Sec13/31 have the structure of a cuboctahedron, but they suggest that in order to enclose larger cargoes, the same units could organize into the small rhombicuboctahedron, the icosidodecahedron or the small rhombicosidodecahedron. These semi-regular solids all share with the cuboctahedron (and the octahedron) the property that four edges meet at each vertex. The condition corresponds to the asymmetry in the molecular realization of the Sec13/31 complex: the two ends are different, so it cannot assemble into a network with odd-ordered vertices.



The three axes of symmetry of the Sec13/31 cage. Each edge is a Sec13/31 protein complex. The color (blue-green-yellow) encodes distance from the cage center. Note the asymmetry in the edges. Cage diameter approximately 600Å. Image from *Nature* **439** 236, used with permission.

Deal or No Deal? The *New York Times* ran a “Critic’s Notebook” column by Virginia Heffernan on December 24, 2005. The subject was the popular game show “Deal or No Deal,” and the title was “A Game Show for the Probability Theorist in Us All.” Here’s how the game works (you can test it out on the NBC website (www.nbc.com/Deal_or_No_Deal/game) – click on “Start game!” once it has uploaded).

- Twenty-six known amounts of money, ranging from one cent to one million dollars, are (symbolically) randomly placed in 26 numbered, sealed briefcases. The contestant chooses a briefcase. The unknown sum in the briefcase is the contestant’s.
- In the first round of play, the contestant chooses 6 of the remaining 25 briefcases to open. Then the “banker” offers to buy the contestant’s briefcase for a sum based on its expected value, given the information now at hand, but tweaked sometimes to make the game more interesting. The contestant can accept (“Deal”) or opt to continue play (“No Deal”).
- If the game continues, 5 more briefcases are opened in the second round, another offer is made, and accepted or refused. If the contestant continues to refuse the banker’s offers, subsequent rounds open 4, 3, 2, 1, 1, 1, 1 briefcases until only two are left.
- The banker makes one last offer; the contestant accepts that offer or takes whatever money is in the initially chosen briefcase.

The psychology is what makes the game fun. As Heffernan explains: “So far, no game theorist from the Institute for Advanced Study has appeared to try his

hand at ‘Deal or No Deal’ and play as a cool-headed rationalist. Instead the players on the American show are, like most game-show contestants, hysterics.” In fact the three scientists at Erasmus University who conducted an exhaustive analysis of the decisions made by contestants in the Dutch version of the game (jackpot 5 million Euros) remark that “For analyzing risky choice, ‘Deal or No Deal’ has a number of favorable design features. The stakes are very high: ... the game show can send contestants home multimillionaires – or practically empty-handed. Unlike other game shows, ‘Deal or No Deal’ involves only simple stop-go decisions that require minimal skill or strategy. Also, the probability distribution is simple and known with near-certainty. Because of these features, ‘Deal or No Deal’ seems well-suited for analyzing real-life decisions involving real and large risky stakes.” Their report is available online (papers.ssrn.com/sol3/papers.cfm?abstract_id=636508).

Mathematical patterns in asthma attacks. A mathematically powered breakthrough in the study of the incidence of asthma attacks, with potentially important therapeutic implications, was reported in the December 1 2005 *Nature*. Urs Frey (University Hospital of Berne) works in pediatric respiratory medicine; Béla Suki (Boston University) is a physicist who “analyses complex nonlinear systems, such as the factors that contribute to avalanches” (quote from an “Authors” sketch at the beginning of the issue). With their collaborators, they analyzed the records of a “previously published, randomised, placebo-controlled, double-blind crossover study” following 80 asthmatic subjects for 3 six-month treatment periods. In that study, the PEF (peak expiratory flow) of each subject was measured twice daily; the subject was also assigned a daily asthma symptom score. The team’s strategy was to “examine whether the statistical and correlation properties of the time series of PEF recordings can be used to predict the risk of subsequent exaggeration of airway instability.” They can. To disentangle the effects the authors created a “nonlinear stochastic model of the PEF fluctuations” (“a cascade connection of a linear dynamic system followed by a second order nonlinear system with no memory. ...”) They were able to tune this model to match the statistical characteristics of the experimental data, and then use it to measure the impact of the characteristics separately. One startling conclusion from their analysis is that short-acting bronchodilators, such as the popular drug albuterol, can actually aggravate medium-term risk of an asthmatic attack.

AN INTERVIEW WITH MARCELO VIANA

First, we congratulate you for being awarded, recently, the first ever Ramanujan Prize, which “distinguishes young mathematicians for conducting outstanding research in developing countries”. What does this prize mean to you ?

It was a great pleasure and honor to be awarded the first Ramanujan Prize, of course. The prize is a great recognition of the quality of what I, and people close to me, have been doing. I do perceive it as a distinction granted to the individual, but I am aware that it also pays tribute to the Mathematics that is being done in Brazil and, more broadly, in Latin America.



Marcelo Viana after receiving the Ramanujan Prize for 2005 (ICTP, Trieste, Italy, December 15, 2005).

Photo: ICTP Photo Archives, Massimo Silvano.

How is it to pursue a top mathematics career in a developing country ?

It is really very exciting. Economical, social, and educational conditions are much less favorable than in developed countries. That is frustrating sometimes, but it also means that our work must have a broader scope than in other parts of the world. It is not only about proving theorems and teaching graduate courses, we must be involved in improving education at all levels,

spreading good mathematical activity across the country, helping bring mathematical “literacy” to everyone. That is what it makes it so exciting: there are many things to be done, many opportunities to make a real impact.

Your main workplace has been the IMPA in Rio de Janeiro. You are now its Deputy Director. Can you describe what makes this institute a very special place ?

IMPA is a small institute, with a very well defined mission, which is to create mathematics, to train high level researchers, and to disseminate mathematical knowledge at all levels. Since it was founded, in 1952, IMPA has developed a tradition of scientific excellency that, I believe, is now part of its essence. Being small means there is a very personal contact between faculty students and staff, which adds to the great scientific ambience, as do the many visitors coming around the year. And, being located in one of the nicest places I have ever seen, between the Botanical Garden and what is said to be the largest urban forest in the world, can only help, right ?

What has been the impact of the activities of IMPA for Mathematics in Brazil and, more generally, in Latin America ?

For one thing, IMPA is the main graduate school in Mathematics in Latin America. It has granted more than 250 doctoral degrees and nearly 500 master degrees so far. Most of these students came from Latin American countries, including all the regions of Brazil, and what they have been doing for the development of Mathematics in the continent can not be overestimated. Most university departments in Brazil include former students from IMPA, as do many institutions in Argentina, Chile, Colombia, Cuba, Ecuador, Mexico, Peru, Uruguay, Venezuela. More recently, we have been attracting students from Bolivia and Paraguay as well.

Incidentally, Portugal is among the countries with the largest number of researchers with a doctoral degree from IMPA: I remember 10, at least.

Another important aspect are our collaboration agreements with several universities in Brazil. Through these agreements, IMPA and its faculty support the graduate teaching, help organize meetings and Summer Schools, and interact scientifically with researchers in those institutions. In the Latin American scenario we have strong

ties with the most important centers, both directly and through UMALCA, the Latin American Mathematical Union. Researchers from IMPA have been very active in organizing Schools, as a means to attract the best students of the region to a career in Mathematics, and other scientific events in various countries in the region. For instance, I have just come back from Santiago de Chile, where I helped organize the first International Congress on the Applications of Mathematics, held jointly by UMALCA, the European Mathematical Society - EMS, and the Society for Industrial and Applied Mathematics - SIAM.

Do institutes like IMPA influence the way mathematicians do research?

To some extent, yes. To begin with, we keep a vigorous visiting program. Around the year and, specially, during our Summer Program in January-February, we host a large number of researchers, both from Brazil and from abroad. This gives them a chance to get away from their daily obligations and benefit from the working conditions at IMPA, which are great.

There is also a certain style of doing Mathematics. A Dutch colleague once told me that interacting with colleagues from IMPA he discovered that “research is fun!”. I have heard similar comments from other colleagues, and this is something I try to convey to my own students. Successfully, I have reasons to believe.

There is yet another point. Recently, a famous American mathematician wrote that a place like IMPA is “a beacon to the mathematics community” in the developing world. I do believe that letting our talented youth know that personal and professional realization is possible in our countries through Mathematics may be one of the most beneficial influences.

Let us now talk a little about you. When did you first become interested in Mathematics? What was it about Mathematics that attracted you?

It happened gradually in school. We moved quite a lot when I was a kid, because my mother was a teacher and she was appointed in various places by the government. Eventually we settled down in Póvoa de Varzim, which is where I went to high school, in the *Liceu Eça de Queirós*. I was quite successful in most topics, but I had a clear preference for exact sciences. Among them, Mathematics was the “neatest”, the most “proper”. So I was more and more inclined to it and when the time came to make a definite decision, that was easy.

Were any people or events particularly influential in your choice of Mathematics as a career? Were there teachers who made a particular difference?

I always had very good Mathematics teachers, who added to the general feeling that everything about the subject was quality stuff. In that sense they all were influential in my picking Mathematics as my first choice when I applied for college.

At later stages, Prof. Arala Chaves, from the University of Porto, and Prof. Jacob Palis, from IMPA, played key parts.

You were born in Brazil but moved with your parents to Portugal almost immediately after. Can you tell us something about your youth in Portugal? How were you as a student?

We moved around the North of Portugal for a while, then we settled down in Póvoa de Varzim, which is where my parents are from. As I said, I was generally quite successful at school. This is not always a good thing, because somehow it puts you a little bit apart from the others. At those ages, school is the foremost ground for social interaction and I was eager to establish friendships.

Did you find that your undergraduate education in Portugal prepared you well academically?

Yes, definitely. The education I got was quite solid and I did not experience any particular difficulties when I moved to graduate school. I believe it would have been the same if I had joined any other graduate school, in any part of the world. But I did find out later that part of my education was somehow out of touch with “reality”. Many advanced topics I was taught were not as central as they had been, and could have been replaced. Also, I came to miss a more “experimental” approach to mathematical knowledge. These things have improved substantially over the last two decades, because there is nowadays much more research activity in Portugal. But I still notice in some of my Portuguese students a tendency to view mathematical issues as finished objects, a difficulty at realizing that (re)formulating questions is an important part of the game.

After graduating from the University of Porto, you got your PhD under Jacob Palis at IMPA, with a thesis entitled “Strange attractors in higher dimensions”. How did you first become interested in Dynamical Systems? What fascinates you in the area?

That I chose Dynamical Systems as my research topic was primarily due to Prof. Manuel Arala Chaves, from the University of Porto. He introduced me and my colleagues to the subject in the last year of the Licenciatura, at the *Faculdade de Ciências do Porto*. He directed me to visit the *École Normale Supérieure* in Paris, in 1984, and it was also at his advice that I attended a scientific meeting at the University of Coimbra in 1985, that was a crucial event in my career.

At that meeting in Coimbra I met Prof. Jacob Palis, who would become my doctoral advisor. He convinced me to come to IMPA, and is largely responsible for my remaining here after two decades (rather than the two or three years I hoped to get away with, when I came).

After your PhD you decided to leave definitely Portugal. Why? Did you notice big differences when you arrived at IMPA?

I did come back to Portugal for a while, but somehow I did not feel happy. A good part of it was due to personal reasons, to some important changes that were taking place in my personal life. Somehow, it was difficult to separate things. It is true that I already felt very well at IMPA, professionally. But, all in all, personal issues ended up being crucial.

Mathematical research in Portugal is mostly based at the universities. Do you have any suggestions to improve the working environment at universities?

Science is done by individuals or groups of individuals, and even more so in the case of Mathematics. Institutions are often more of a nuisance, if I may say so. In the last couple of decades, Portugal has moved a good deal in the direction of acknowledging the role of individuals and groups, for instance in the way funding is being distributed, and I have no doubt that was fundamental for the excellent evolution experienced in this period. I believe there is still room to proceed in this direction. A fine scientific ambiance is often the work of one or more people with strong leadership. As many other institutions with a past and tradition, the Portuguese universities give limited room for the action of such leaders.

I also gather from conversations that the teaching and administrative load has been rising, at least in some institutions. Such duties are certainly part of a scientist's job description, as are other academic activities beyond the teaching plus writing papers binomial. However, creative thinking does demand a lot of time and tranquility. A few years ago, during a meeting held in Lisbon by the Minister for Science and Technology, I suggested the creation of a mechanism for "teaching buy-outs" aimed, specially, at the most talented young researchers.

A common trace of your work has been the search for very general theorems and global theories suitable for the majority of dynamical systems. Is this the kind of approach to mathematics that attracts you (generalization instead of specialization ...)?

In my experience, Mathematics is done from examples. I really do not understand those areas (and there are a few honorable ones!) where experts are not able to

indicate simple interesting examples to motivate their statements. One of my favorite advices to the students is "do not try to solve the whole thesis problem at the same time: look for an interesting particular case, and focus on it for a while." My own work has been much driven by examples, the Hénon map, the Lorenz attractor, DA diffeomorphisms, and so on. But I also believe the ultimate goal is to reach statements with the "right" level of generality. For instance, the Role Theorem was first discovered for polynomials; the statement we now find in Calculus books is better, not because it is more general, but because the result has nothing to do with the algebraic structure, it is really only about differentiability.



Marcelo Viana delivering his lecture "Lorenz strange attractors" at the Math Colloquium of the University of Coimbra. (February 14, 2006)

In an article in the book "Mathematics Unlimited — 2001 and Beyond" (MU2001) you give an overview of the challenges that Dynamical Systems face in this century. What are some of the most significant unsolved problems in the area?

In the 1960's people believed that it would be possible to give a global description of how "most" dynamical systems behave. Steven Smale, made a concrete proposal, inspired by his previous work in Morse theory: he believed that globally most dynamical systems look very much like gradient flows. It was rapidly clear that this was not really so: Smale himself gave the first counter-example! I think the most exciting questions are: Can such a global picture be obtained, and how? In the mid 1990's, Palis proposed a new program to this effect, and there has been a lot of progress. In that article, I mentioned some important specific challenges in this direction.

One intriguing issue, which has been blocking progress for quite some time, is the so-called closing lemma. In

some cases, one can prove that if a dynamical system has a trajectory that is almost closed then, by modifying the system a little, one can cause the orbit to actually close. There has been a great deal of effort to try to extend the validity of this statement, which would have tremendous implications. However, after four decades, the results are really meager. At this point, I think we should change the strategy, and move on developing the field *without* the closing lemma. There has been some progress along these lines, specially in low dimensions.

In MU2001 you say that even Hilbert, in his 1900 famous speech, could not foresee the birth and extraordinary development of Dynamical Systems (in spite of two of his problems — 16th and 21st — being related to it). Do you have any explanation for this ?

Hilbert's list is mostly about specific mathematical problems. One exception is the sixth problem, where he asks for the axiomatization of Physics, and we have to acknowledge that was premature, at the very best. Hilbert had a remarkable insight and, together with Poincaré, he may have been the last "universal" mathematician. But the kind of forces that led to the development of Dynamical Systems, very often arising from the experimental sciences, were not particularly close to his heart (they were much closer to Poincaré's).

How would you describe Dynamical Systems to a layman ?

I usually describe it as the mathematical discipline that studies systems that evolve in time, in order to understand and predict this evolution. This definition is a bit "imperialistic", in the sense that it includes almost anything, but a few examples help make it more clear: the motion of planets, the evolution of ecological environments, the spreading of epidemics, fluids in motion. In all these, and many other systems, one would like to understand how and why they evolve, and how external factors may affect that evolution. Dynamical Systems provides the conceptual tools to model and predict their behavior.

You are in Coimbra to participate at the Annual Scientific Council meeting of CIM. What do you think about CIM and its role in the Portuguese mathematical community? and about Portuguese mathematics in general ?

I think it is very important for the country to have an institution such as CIM. Limited as its budget is, the Centro has been having a very positive effect at large, that goes much beyond the concrete activities it supports. CIM also serves as a forum and meeting ground at the national level, which I find very important as the Portuguese mathematical community has grown bigger

and more sophisticated. I believe its influence and importance will steadily grow in the future.

You are also going to present a talk in Coimbra for the students of the Delfos programme. What advice do you give to young students who are coming to like mathematics and to show real talent ?

Just do it. I often try to convey to young people the pleasures of personal and professional realization one can attain through Mathematics, or Science in general, and how so very few professional activities can boast the same.



Marcelo Viana delivering his *Delfos Lecture* at the University of Coimbra (February 12, 2006).

How can we encourage young people to take up mathematics, especially in the schools ?

This is something we are very concerned in Brazil as well. There are many mechanisms for attracting young talents to Mathematics. But they all involve having researchers and educators exposing students to Mathematics in a way that displays its beauty and, at the same time, gives the students the chance to exhibit their own capacity to face challenges. We all know this is often not the way Mathematics is presented in the lecture room, unfortunately.

You are a superb lecturer and seem to enjoy very much explaining ideas to others which suggest that you like teaching. What makes teaching fun for you? What is your way of teaching ?

Teaching is a great way to interact with people. I enjoy conducting the audience to the understanding of the subject, withholding information till the right moment, disclosing the secrets when time is ripe. We mathematicians are trained to be rigorous, to do our best not to

say anything that is wrong. In my opinion, that is a mistake. Mathematical knowledge is ultimately stored in an orderly, rigorous fashion. But that is not the way it is discovered and so it is not the way it ought to be taught.

Tell us about this mysterious link “Bola da Vez” on your website ?

The expression can not really be translated and I am not even sure how much sense it makes in Portugal. But, this being Brazil, you may guess it has to do with football. It all started with an exchange of teasing remarks between two of my students. As a result, there is now an actual football that is passed along from one student to another. Whenever a student finishes, (s)he signs and dates one of the hexagons, and gives the ball to the colleague who’s expected to finish next. This “honor” tends to be more appreciated when the thesis defense is already in sight, of course. At some point I thought it would be good to have a replica of that ball on my website, with the students photos.

What are your mathematical plans for the near future, what areas and problems have your attention at the moment ?

I have several unfinished projects, about equilibrium states, partially hyperbolic systems, linear cocycles, dynamics of flows. My various coauthors kindly, and regularly, remind me how upset they are we are taking so long... So, the first priority will be to finish these projects. Then I would like to go deeper into the study of Teichmüller flows, on which I have been working over the last couple of years. I would like to really get into conservative Dynamical Systems, where one studies systems that preserve a volume form. And I keep going back to problems that I was unable to solve the first time.

What things interest you other than Mathematics ?

My family, of course. Including my five delicious nephews and nieces. I also like reading and listening to music. I tend to be conservative, and listen to the things I already know and like, but I have been making some discoveries, specially about ancient and medieval music. I really like History. Not so much recent History, but I have read about almost anything prior to the XXth century. Oh yes! I try to do some physical exercise, but my heart is not really into it.

Interview conducted by Maria Manuel Clementino and Jorge Picado (University of Coimbra)

Marcelo Viana is a Professor of Mathematics and Deputy Director of the Institute of Pure and Applied Mathematics (IMPA), Rio de Janeiro, Brazil.

Born in Rio de Janeiro on March 4, 1962, of Portuguese parents, he was educated in Portugal, for where he came at only the age of three months. He received his B.S. from the University of Porto in 1984, where he held a position, and got his Ph.D. from the IMPA in 1990, with a thesis entitled “Strange attractors in higher dimensions”, written under the direction of Jacob Palis. He was awarded a Guggenheim fellowship to work at UCLA and Princeton in 1993, and since then he has been a visitor to main centers around the world. Marcelo Viana is a well-known mathematician worldwide in the fields of dynamical systems where he has made important contributions. His research focuses on dynamical systems, ergodic theory, and bifurcation theory. He has published more than 50 research articles in international journals (including *Acta Mathematica*, *Annals of Mathematics*, *Inventiones Mathematicae* and *Publications Mathématiques de l’IHES*) and has already supervised seventeen Ph.D. thesis.

Marcelo Viana has both Portuguese and Brazilian nationalities. Although he works in Brazil, he keeps strong links with mathematics in Portugal, by participating in research evaluation panels, conferences and seminars, and by supervising several Portuguese Ph.D. students. He is also a member of the Scientific Council of CIM.

Invited to give talks at two consecutive International Congresses of Mathematicians, Zurich-94 and Berlin-98 (the former as Section Speaker and the latter as Plenary Lecturer) he is considered a superb lecturer. Besides, he was also invited to give a Plenary Lecture at the International Congress on Mathematical Physics, (Paris, 1994). He is on the editorial board of seven mathematical journals: *Ergodic Theory & Dynamical Systems*, *Dynamical Systems: An International Journal*, *Portugaliae Mathematica*, *Discrete and Continuous Dynamical Systems*, *Journal of Stochastics and Dynamics*, *Nonlinear Differential Equations and Applications*, *Dynamics of Partial Differential Equations*.

Among his several prizes and distinctions are The Mathematical Union for Latin America and the Caribbean Award in Mathematics, 2000, the Great Cross of Scientific Merit, granted by the President of Brazil in 2000, Member of the Third World Academy of Sciences (elected in 2000), Third World Academy of Sciences Award in Mathematics, 1998, Member of the Brazilian Academy of Sciences (elected in 1997).

Last December Marcelo Viana was the first recipient of the Ramanujan Prize, which honors a researcher, younger than 45 years old, who has conducted outstanding research in a developing country. The prize is funded by the Niels Henrik Abel Memorial Fund and it is awarded by the Abdus Salam International Centre for Theoretical Physics in conjunction with the International Mathematical Union.

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