## What's New in Mathematics

Royal Welcome to Abel Laureate Peter Lax. Norway's Crown Prince Regent awarded the 2005 Abel Prize to Peter D. Lax on May 24. The city of Oslo and the Norwegian Academy of Science and Letters prepared for several days of events that honored Lax, including the prize ceremony, lectures by and in honor of Lax, a banquet at the Akershus Castle, and some special events for local teachers and students.


HRH the Crown Prince Regent presented the Abel Prize 2005 to Peter D. Lax. Photo: Knut Falch/Scanpix (The Abel Prize/The Norwegian Academy of Science and Letters).

Lax received the US $\$ 980,000$ Abel Prize "for his groundbreaking contributions to the theory and application of partial differential equations and to the computation of their solutions." See the Abel Prize website (http://www.abelprisen.no/en/) for details about the international prize ceremony, all the festivities, and the 2005 Abel Symposium. In http://www.abelprisen.no/en/prisvinnere/2005/ documents/popular2005eng9.pdf you may find the article "Peter D. Lax, Elements from his contributions to mathematics", by Professor Helge Holden, where the work of this year's Abel Laureate is described.

The math beneath the Interplanetary Superhighway. The cover story for the April 162005 Science News was Erica Klarreich's "Navigating Celestial Currents," with subtitle: "Math leads spacecraft on joy rides through the solar system." The spacecraft in question was NASA's Genesis. [The joy ride ended in what NASA terms a "hard landing" in the Utah desert.

Fortunately much useful scientific information survived. More on the NASA website.] Klarreich's piece is about the way the mathematical analysis of the Solar System Gravitational Dynamical System (the sum of the gravitational fields of all the objects in the system) led to the discovery of extremely fuel-efficient orbits. Edward Belbruno, now at Princeton, pioneered this approach twenty years ago, and scored its first great success in 1991 when he rescued the Japanese Hiten spacecraft, stranded in Earth orbit without enough fuel, it seemed, to reach the moon. Belbruno showed how to exploit the chaotic nature of the SSGDS to calculate a long-duration, low-cost trajectory which would lead the spacecraft to its destination. Chaotic here does not mean disorderly, but refers to the enormous change in behavior that can be produced by a tiny change in initial conditions near an unstable critical point of the system. The Hiten rescue used the unstable critical points in the Earth-Moon system; there are three of them, known as the Lagrange points $L_{1}, L_{2}$ and $L_{3}$.


At $L_{1}$, the sum of Earth's gravity and the centrifugal force exactly balance the gravitational attraction of the Sun. At points on the (black) halo orbit the vector sum would pull a mass towards $L_{1}$; this can be balanced centrifugally by motion of the mass along the orbit.

The Genesis mission used the similarly defined and labelled points in the Earth-Sun system: with notation from the diagram above, at the $L_{1}$ point the Sun's gravity (red) is exactly balanced by the sum (blue) of the Earth's gravity and the centrifugal force produced by the yearly rotation of the Sun- $L_{1}$-Earth axis. (At $L_{2}$ and $L_{3}$ that centrifugal force balances the sum of the two gravities). The axis is crossed at $L_{1}$ by a twodimensional surface at each point of which the (green) resultant of the red and blue vectors is tangent to that
surface. A mass on that surface will fall towards $L_{1}$ unless it is orbiting rapidly enough, staying in the surface, to balance that attraction by centrifugal force. These are the "halo orbits" shown in Shane Ross's illustration below.


The Genesis mission parked in a halo orbit while it studied the solar wind, then looped around $L_{2}$ for a leisurely and inexpensive trip back to Earth. Image by Shane Ross (USC), used with permission.

But if the mass strays ever so slightly away from the surface, it will spiral either away from the Earth or away from the Sun. A tiny bit of fuel can send it on its way. We can think of "freeways" linking halo orbits around the three unstable equilibrium points. Want an inexpensive trip to Jupiter? Time your trajectory so that you're there when one of the Sun-Jupiter freeways crosses a Sun-Earth freeway; then a little nudge from your thrusters will do the trick. Klarreich's article is available online at www.sciencenews.org/articles/20050416/bob9.asp

Saunders Mac Lane, 1909-2005. Saunders Mac Lane, AMS President from 1973 to 1974, died recently. Mac Lane helped develop category theory and co-authored A Survey of Modern Algebra with Garrett Birkhoff. The MacTutor History of Mathematics Archive (www-groups.dcs.st-and.ac.uk/~history/ Mathematicians/MacLane.html) has Mac Lane's biography and list of publications. Read "Garrett Birkhoff and The Survey of Modern Algebra," (www.ams.org/notices/199711/comm-maclane.pdf) by Mac Lane in Notices of the AMS, December 1997. For the University of Chicago obituary for Saunders Mac Lane, which was released to the Associated Press, see www-news.uchicago.edu/ releases/05/050421.maclane.shtml.

Peter Lax in the New York Times. Peter Lax won the Abel Prize this year.


Peter D. Lax, Abel Laureate 2005 (Photo: New York University).

On that occasion, he was interviewed by Claudia Dreifus of the Times; the interview appeared in the Science section on March 29, 2005, with a photograph showing Lax in his NYU office in front of a blackboard bearing the prominent and talismanic chalk inscription: $\delta=\log 4 / \log 3$. Dreifus leads Lax through his early days: Budapest and Stuyvesant High School ("I didn't take any math courses at Stuyvesant. I knew more math than most of the teachers"). Lax was drafted in 1944 at the age of 18, and ended up at Los Alamos. "I arrived six weeks before the A-bomb test. ... Looking back, there were two issues: should we have dropped the A-bomb and should we have built a hydrogen bomb? Today the revisionist historians say that Japan was already beaten ... I disagree. ... I also think that Teller was right about the hydrogen bomb because the Russians were sure to develop it. And if they had been in possession of it, and the West not, they would have gone into Western Europe. What would have held them back? Teller was certainly wrong in the 1980's about Star Wars. ... The system doesn't work. It's a phantasmagoria." Dreifus asks what Von Neumann would think about the ubiquity of computers today. "I think he'd be surprised. ... But remember, he died in 1957 and did not live to see transistors replace vacuum tubes." Did he know John Nash? "I did, and had enormous respect for him. He solved three very difficult problems and then he turned to the Riemann hypothesis. ... By comparison, Fermat's is nothing." Does he believe high school and college math are poorly taught? "... In mathematics, nothing takes the place of real knowledge of the subject and enthusiasm for it."

2005 Wolf Prize. Gregory A. Margulis (Yale University) and Sergei P. Novikov (University of Maryland, College Park) have been named co-winners of the 2005 Wolf Prize in mathematics by the Wolf Foundation and will share $\$ 100,000$. The selection committee cited Margulis for his "monumental contributions to algebra" and Novikov for his contributions to algebraic topology, differential topology and mathematical physics. The prizes were awarded on May 22 in Jerusalem.

The math of craquelure. "Four Sided Domains in Hierarchical Space Dividing Patterns" is the title of an item published on February 9, 2005 in Physical Review Letters, and picked up in the "Research Highlights" section of the February 242005 Nature. The authors, Steffen Bohn, Stephane Douady and Yves Coudert (Rockefeller University and ENS, Paris) begin with the observation that, in the tilings formed by the cracks in ceramic glazes, the average number of sides of a tile is four. This seems unnatural, at first glance: Generically the edges of a tiling meet three by three. Euler's characteristic for a convex domain gives Vertices - Edges + Faces $=1$ or $V-E+F=1$. Since every edge joins two vertices, generically $2 E=3 V$; Euler's equation then gives $3 V-3 E+3 F=3$ and so $3 F-E=3$. When the number of faces is large we can write $3 F=E$ and since each edge is shared by two faces, this means that the faces must be, on average, six-edged. How the six edges become four sides in crackle finishes is clear from the picture below.


Craquelure in ceramics results from the differential shrinking of coats of glaze. The characteristic pattern is different from other naturally occurring tilings, which usually involve hexagons.

The authors explain the general mechanism at play: they define a hierarchical space-dividing pattern as one formed by "the successive divisions of domains and the absence of any further reorganization," and they show that "the average of four sides is the signature of this hierarchy." Another example is the organization of veins and sub-veins in the framework of a leaf (earlier work of theirs in this direction was referred to in the cover illustration of Science for February 6, 2004). Finally, they remark that the street network in a city where "growth resulted from self organization" is also of this type, and exhibit as evidence part of a 1760 map of Paris. Article available online at http://asterion.rockefeller.edu/steffen/.

Proof checking by computer assistants. Anyone who has ever been hoodwinked by a false proof of an
intricate statement will be grateful to know that computers have been trained to take over the job of checking arguments. This is explained by Dana Mackenzie in the March 42004 Science, in an article with the title "What in the Name of Euclid Is Going On Here?" Mackenzie evokes the following problematic "scenario that has repeated itself, with variations, several times in recent years: A high-profile problem is solved with an extraordinarily long and difficult megaproof, sometimes relying heavily on computer calculation and often leaving a miasma of doubt behind it." The remedy is now at hand: software packages ("proof assistants") which "go through every step of a carefully written argument and check that it follows from the axioms of mathematics." The best-known examples are Coq, HOL and Isabelle. Recently Coq was used by Georges Gonthier to check the proof of the Four-Color Theorem, the archetype of Mackenzie's scenario. It passed; Gonthier's paper is available online. Isabelle was put through its paces by Jeremy Avigad to check the proof of the Prime Number Theorem. HOL-light has been used by Thomas Hales to check the Jordan Curve Theorem, a warm-up perhaps for a verification of his work on the Kepler Conjecture. Mackenzie muses on the philosophical implications of these new developments. "Ever since Euclid, mathematical proofs have served a dual purpose: certifying that a statement is true, and explaining why it is true. Now these two epistemological functions may be divorced. In the future, the computer assistant may take care of the certification and leave the mathematician to look for an explanation that humans can understand."
"Noether's Novelty," by John Derbyshire. In National Review Online, 21 April 2005, Derbyshire recalls that German mathematician Emmy Noether died 70 years ago in April. He describes her as "the greatest female mathematician of the 20th century, and quite possibly of all time." The article places Noether in the context of her time, place and culture - both societal and mathematical. Although she produced "a brilliant paper resolving one of the knottier issues in General Relativity" praised by Einstein, and although David Hilbert fought on her behalf to have her appointed to the faculty at Göttingen during World War I, she had many uphill battles. She won the admiration of colleagues and students but was "ill-paid and un-tenured," and when the Nazis came to power in 1933 she lost her job. While her mathematician brother Fritz emigrated to Siberia, Emmy came to Bryn Mawr in Pennsylvania, where she died two years later. Derbyshire concludes the piece with a quote from her obituary written by Albert Einstein, published as a letter to the editor in The New York Times on May 5, 1935, in which Einstein classifies Noether as one of the "genuine artists,
investigators and thinkers" of the world.

Differential geometry and the Venus Flytrap. An international team led by Yoël Forterre has used "highspeed video imaging, non-invasive microscopy techniques and a simple theoretical model" to investigate how the Venus flytrap can snap shut rapidly enough to catch its prey. The authors, reporting in Nature for January 27, 2005, argue that "the macroscopic mechanism of closure is determined solely by leaf geometry." The image below shows clearly that both in the open state and the closed the leaf has positive gaussian curvature, and that it starts curved outward but ends curved inward.


The rapid closure of the Venus flytrap (Dionaea muscipula) leaf in about 100 ms is one of the fastest movements in the plant kingdom. Image from Nature (www.nature.com) 433 422, used with permission.

Choosing curvilinear $x$ and $y$ coordinates on each half of the leaf, with $x$ increasing in the direction of the spines, and $y$ increasing perpendicularly to the right, they observe that the change in the principal curvature $\kappa_{x}$ is the main actor in the phenomenon. "For a doubly-curved leaf ... bending and stretching modes of deformations are coupled, meaning that bending the leaf causes its mid-plane to be stretched. If the coupling is weak, the leaf can change its shape from open to closed by varying its gaussian curvature and stretch without a large energetic cost. In such a situation, the leaf deforms smoothly to accommodate the change in $\kappa_{x}$. If the coupling is strong, the leaf will not deform much (owing to the large energetic cost of stretching its mid-plane), until eventually the change in $\kappa_{x}$ becomes so large that the leaf snaps shut rapidly." The authors derive ("poroelastic shell dynamics") a mathematical model which accurately
mimics the detailed changes in leaf geometry. The Supplementary information posted on the Nature website (www.nature.com/nature/journal/v433/n7024/ suppinfo/nature03185.html) includes a $1 / 4$-speed video of the flytrap, tickled with a glass pipette as in the image above, snapping shut.

Wrong, Wrong and Wrong. Math Guides Are Recalled. That's the headline on an article by Susan Saulny in the March 252005 New York Times. The hapless New York math educators have done it again. This time, "City education officials were forced to recall test preparation materials for math exams late Wednesday after discovering that they were rife with errors, including basic arithmetic mistakes." Randi Weingarten, the head of the United Federation of Teachers, was reportedly outraged: "Tweed [the NYC Department of Education, located in the Tweed Courthouse] has no problem with excessively criticizing teachers for failing to meet its picayune mandates. But then it produces a test prep manual riddled with errors and misspellings. The hypocrisy is stunning." The Times printed two examples of questions with wrong answers and called on Alfred Posamentier, mathematician and dean of the City College School of Education, for the final word: "... in mathematics, where you have such an exact science, there is no room for error."

Amateur math in ancient Japan. Science magazine for March 18, 2005 ran a "News Focus" item by Dennis Normile, under the title 'Amateur' Proofs Blend Religion and Scholarship in Ancient Japan." Datelined Tokyo, the piece is prompted by an exhibition of Edo period sangaku (wooden tablets inscribed with geometric theorems) that opened at the Nagoya City Science Museum last May. During that period (1603-1868) "when Japan was isolated from the rest of the world, a unique brand of mathematics flourished in the country's shrines and temples. Amateur mathematicians crafted geometric theorems on elegant wooden tablets ... and offered them to the gods." The exhibition is due largely to the efforts of Hidetoshi Fukagawa, a high school math teacher who stumbled upon sangaku while "looking for material to enliven his classes," and has spent decades tracking them down and deciphering their contents. Some of the theorems stated (notably Soddy's Hexlet - see Bob Allanson's animation) were published on a sangaku many years (in this case, 114 years) before their discovery in the West. And this was all the work of "amateurs." As Fukagawa puts it: "There was no academia as we know it. So samurai, farmers and merchants all felt free to study mathematics." The tablets contain theorems but, in fact, no proofs. Fukagawa again: "Ostensibly, the tablets were left as gifts to the gods. In
reality, people were showing off and challenging others to work out the proof."

Algorithmic complexity in evolution. The idea of algorithmic complexity goes back, in some sense, to Leibniz (see Greg Chaitin's home page at www.umcs.maine.edu/~chaitin/). The general concept is suggested by Chaitin's definition: The (algorithmic) complexity of a sequence of 0 's and 1's is the length of the shortest computer program that will generate the sequence. An international team, led by Ricardo Azevedo (University of Houston), has recently applied this concept to the study of the development of multicellular organisms. Their work appears as "The simplicity of metazoan cell lineages" (Nature, January 13, 2005). "Lineage" refers to the fact that all the cells in an organism descend from a single cell, the fertilized egg. But a typical metazoan has a large variety of different kinds of cells (brain, skin, bone, etc.). So in the family tree, traced back from a single cell in the complete organism, there must be one or more nodes where a mother cell divided into two dissimilar daughters. Algorithmically, each of these nodes corresponds to a division and differentiation rule. In the study, part of the tree (a lineage) is reduced by identifying functionally similar nodes. The number of reduced nodes divided by the original total is the algorithmic complexity of the lineage.


A schematic (non-biological) lineage illustrating the reduction process. Here nodes R4 and R5 are collapsed to RR4 in the reduced lineage: the algorithmic complexity of the original lineage is $4 / 5=80 \%$.

The team computed the complexity for lineages in four different multicellular organisms: three species of freeliving nematodes (microscopic groundworms) and a sea squirt. The numbers worked out to $35 \%, 38 \%, 33 \%$ and $32 \%$ in the four cases. In a first analysis, the team "compared each real lineage to lineages with the same cell number and distribution of terminal cell fates but generated by random bifurcation." They found that real lineages were $26-45 \%$ simpler than the corresponding random lineages. Conclusion: evolution selects for simpler lineages. (Tentative explanation: "the specification of simpler cell lineages might require less genetic information, and thus be more efficient.") In a second analysis, they "used evolutionary simulations to search
for lineages that had the same terminal cell number and fate distribution as the actual lineages but were simpler." They found that after 20,000 to 50,000 generations they "could evolve lineages that were $10-18 \%$ simpler than the ancestral, real lineages." One explanation is "developmental constraints imposed by the spatial organization of cells in the embryo." They added these constraints to their simulations and conclude that "the metazoan lineages studied here are almost as simple as the simplest evolvable under strong constraints on the spatial positions of cells."

Six degrees of self-similarity. The "six degrees of separation" phenomenon (so named when the network is acquaintance among people today) is often observed in complex networks. The "six" becomes the average diameter of the network. A Letter in the January 272005 Nature shows that many naturally occurring complex networks are also self-similar. The authors (Chaoming Sung and Hernán Makse (CCNY), Shlomo Havlin (Bar-Ilan)) focus on "connectivity between groups of interconnected nodes on different length scales," which they study by a renormalization procedure (see caption below).


The authors' network renormalization, applied to a schematic network with 8 nodes. For each box length $I_{B}$ the network is tiled with boxes in which all the nodes are $<I_{B} u b>$ steps away from each other. Then the boxes are replaced by nodes, which inherit connections, and the renormalization into boxes is repeated, with the same length criterion. The procedure terminates when the network has been collapsed to a single node. The total number of boxes required is $N_{B}\left(I_{B}\right)$. Finally $\log N_{B}\left(I_{B}\right)$ is plotted against $\log I_{B}$. If the points fall on a line, the network is said to be self-similar, with fractal (box) dimension $d_{B}$ equal to minus the slope of that line. Image from Nature 433 392, used with permission.

They apply this analysis to the following networks: hyperlinkages among the 325,729 web pages of a subset of the World Wide Web; 392,340 actors (linked if
they have been cast together in at least one film); and various networks from molecular and cellular biology. All of these networks turn out to be self-similar: $d_{B}$ for the WWW is 4.1 , for the actors 6.3 . Here is the author's illustration of renormalization carried out on their WWW sub-network, with $I_{B}=3$ :


Box size 3 renormalization of a 325,729-page subnetwork of the WWW. Image from Nature 433 392, used with permission.

More Origami Mathematics. Each Tuesday the "Science Times" features a "Scientist at Work." For February 15, 2005 the scientist was Erik Demaine of MIT. Margaret Wertheim wrote the piece; she tells us that Demaine "is the leading theoretician in the emerging field of origami mathematics, the formal study of what can be done with a folded sheet of paper." Some pretty amazing things can be done. For example, Demaine, working in 1998 with his father Martin Demaine and with Anna Lubiw, showed that any polygonal shape can be made out of a single piece of paper by folding it flat appropriately and making one complete straight cut. The Times shows a swan; Demaine's Fold-and-Cut webpage (theory.lcs.mit.edu/~edemaine/foldcut/) gives angelfish, butterfly, jack-o'lantern and more. Wertheim leads us through some of Demaine's other interests: linkages ("A linkage is a set of line elements hinged together like the classic carpenter's rule.") which are related to protein folding ("... molecular biologists would like to be able to predict from the chemical structure of a protein what shape it would fold into") and graph theory ("known to be fiendishly difficult, but Dr. Demaine is confident he can make headway once he immerses himself in its arcane lore.")


A "cube" constructed by Demaine, Demaine and Lubiw out of hypads, origami hyperbolic paraboloids. Image courtesy Erik Demaine, used with permission.

There is more in the article and much, much more on Demaine's website at theory.csail.mit.edu/ $\sim$ edemaine/. For earlier media math origami see this bulletin for December, 2004.

Relativity. Incompleteness. Uncertainty. Thus runs the first paragraph of Eric Rothstein's February 142005 "Connections" column (every other Monday, in the New York Times). The piece is a meditation on Einstein, Gödel and Heisenberg, occasioned by the publication of Rebecca Goldstein's new book "Incompleteness: The Proof and Paradox of Kurt Gödel" (Atlas Books; Norton). Rothstein contrasts Heisenberg, whose "allegiance to an absolute state, Nazi Germany, remained unquestioned even as his belief in absolute knowledge was quashed," with Einstein and Gödel who "fled the politically absolute, but believed in its scientific possibility." Most of the column is saved for Gödel's Incompleteness Theorem. "Before ..., it was believed that not only was everything proven by mathematics true, but also that within its conceptual universe everything true could be proven. Gödel shattered that dream. He showed that there were true statements in certain mathematical system s that could not be proven. And he did this with astonishing sleight of hand, producing a mathematical assertion that was both true and unprovable." Rothstein, following Rebecca Goldstein, gives Gödel's result a positive twist: "But what if the theorem is interpreted to reveal something positive: not proving a limitation but disclosing a possibility? ... In this, Gödel was elevating the nature of the world, rather than celebrating powers of the mind. There were indeed timeless truths. The mind would discover them not by following the futile methodologies of formal systems, but by taking astonishing leaps, making unusual connections, revealing hidden meanings."

Crocheted Manifold. As Daniel Engber reported in the Chronicle of Higher Education for January 21, 2005, a team at the University of Bristol has used yarn and a crochet hook to build a model of the Lorenz manifold.


A close-up view of the crocheted Lorenz manifold. The origin, the center of the bulls-eye pattern on the right, is just hidden from sight. The wire looping through the origin is the strong stable manifold of the system. The manifold's vertical axis of symmetry can be seen as a diagonal across the upper half of this image. Photo: University of Bristol, used with permission.

This is the 2-dimensional stable manifold of the origin in the Lorenz system

$$
\left\{\begin{array}{l}
x^{\prime}=\sigma(y-x) \\
y^{\prime}=\rho x-y-x z \\
z^{\prime}=x y-\beta z
\end{array}\right.
$$

with the classic choice of parameters $\sigma=10, \rho=28$, and $\beta=8 / 3$. According to Engber, Hinke Osinga and Bernd Krauskopf realized that the computer program they had devised for generating the Lorenz manifold could be adapted to produce a set of crocheting instructions.


Osinga and Krauskopf with their model of the Lorenz manifold. Photo: University of Bristol, used with permission.
"Each computed point on the manifold translates to a type of crochet stitch. A mere 85 hours
and 25,511 stitches later, the project was finished." Osinga and Krauskopf's work appeared in the fall issue of The Mathematical Intelligencer; their preprint is available as a PDF file online (http://www.enm.bris.ac.uk/anm/preprints/2004r 03.html). The crocheted Lorenz manifold struck the fancy of the international media, including the
BBC: Mathematicians crochet chaos
news.bbc.co.uk/1/hi/education/4099615.stm,
CBC Radio: Crocheting Chaos
www.cbc.ca/aih/STEAM/2004/crocheted_chaos_2004 1216.html,
the Austrian ORF:
science.orf.at/science/news/131349,
Channel One in Russia:
www.1tv.ru/owa/win/ort6_main.main?p_news_title_ id=73197\&p_news_razdel_id=9.
"Blood, math and gore." It could work." That's the end of Alessandra Stanley's review of the new TV series "Numb3rs," in the January 212005 New York Times. The plot line involves "Don, a decent, workaholic F.B.I. agent who turns to his math genius younger brother, Charlie" for help in tracking down a serial rapist. As Stanley tells it, "Charlie looks at a water sprinkler and has an Archimedean moment: he realizes that the same principle that allows him to track the path of drops to determine their point of origin could be applied to the distribution of crime scenes on a map." (She quotes one character as saying: "If this works, we'll have a whole new system of investigating criminal cases.")

More about "Numb3rs". A more academic view was taken by NPR's "Math Guy" Keith Devlin, interviewed by Scott Simon on "Weekend Edition - Saturday" for January 22, 2004. Scott: "There's a scene where the mathematician brother is writing out a formula on the board. Firstly he seems to be listening to head-banging rock music and in addition to that he seems to be in the grip of a fever. Is that commonly what happens when mathematicians write out formulas?" Keith: "... Most people's impression of a mathematician, if that impression is of an elderly guy in a tweed suit and worn down shoes, they'd better walk around a university like Stanford or Cal Tech or MIT and just take a look. In fact when David Krumholtz was preparing for this role, he hung around Cal Tech for a while and just watched what he saw." Scott plays a clip in which Charlie consults a fellow mathematician who tells him: "Charlie, when you're
working on human problems, there's going to be pain and disappointment." Keith: "This reflects one of the most interesting changes in the whole history of mathematics. ... Over the last few hundred years increasingly we've found that we can take this mathematics which was originally developed to study the physical world and apply it to the world of people, and by using computer graphics superimposed on action you can show people that mathematics, this abstract stuff, really applies to the real world and, in the case of a crime series, with positive outcomes for society." Scott: "Do you expect that this series could do for mathematics what 'The Simpsons' did for cartoons?" Keith: "I would hope it does succeed because the one thing they're trying to do is make mathematics look cool. I know it's cool, all my friends know it's cool. We do have an image problem, and I think a TV series like this can help get over it." The interview is available online at WWW.npr.org/templates/story/story.php?storyId =4462642.

Terror Network Theory. On December 11, 2004, Jonathan Farley was interviewed on Air America's "So What Else Is News" by the program host, resident whizkid Marty Kaplan. Farley, currently a Visiting Scholar at Harvard, turns out to be a mathematician with a mission. Inspired by the real and hypothetical mathematical derring-do evoked in "A Beautiful Mind," he has found an application of lattice theory to the war on terror. His problem is the structure of terror cells and what it takes to disrupt them. A current approach, he tells us, is to view a terrorist cell as a graph, "a picture where you've got a bunch of nodes or dots which represent the individuals, and then lines which connect individuals if they have some sort of communications link, or if they lived in the same flat in Hamburg at some time ..." Graph-theoretically, a cell is disrupted if the graph is disconnected. Farley noticed that with that kind of analysis "you're missing a key mathematical component of the terrorist network, namely its hierarchy. And that's where I come in, because my branch of mathematics, called lattice theory, deals with hierarchy and properties of order." Kaplan proposes a concrete example: suppose a cell has 15 people, "and the government has picked off 4 of them. To what degree can the government feel as though they have shut that cell down?" Farley explains that for a precise estimate you would need to know the structure of the cell, but he shows how, for a 15 -node binary tree, hierarchically ranked from top to bottom, the graph-calculation and the lattice-calculation give very different answers. "If you've captured 4 guys you're pretty sure you've disrupted the cell, under the old way of thinking. But when you take the lattice-theoretic perspective, you see that actually you only have a $33 \%$ chance of disrupting
the cell in that case."


Removal of four nodes at random has a $93 \%$ chance of disconnecting a 15 -node binary tree, but only a $33 \%$ chance of breaking all top-to-bottom chains of command.

He elaborates: "If 4 people have been captured at random, it might still be possible for terrorist plans to be passed on from the leader down to one of the people at the bottom, one of the eight foot soldiers, in which case you might have another September 11, you might have a shoe-bombing ..." And finally: "Mathematics won't help you catch the terrorists, but it will help you analyze how good a job you've done in the past." Farley's work has also been covered by Ivars Peterson in Science News Online (www.sciencenews.org/articles/20040110/ mathtrek.asp, January 10, 2004).

Ant Geometry. "Pheromone trails are used by many ants to guide foragers between nest and food. But how does a forager that has become displaced from a trail know which way to go on rejoining the trail?" Richard Feynman (in "Surely you're joking ...") considered this problem and speculated that a direction might be written into the pheromone trail (e.g. A-B-space-A-B-space). In fact, the ants use information encoded in the geometry of the (plentiful) bifurcations along the trail. This has been conclusively shown by Duncan Jackson, Mike Holcombe and Francis Ratnieks, a computer science-biology team at the University of Sheffield; work reported in the December 162004 Na ture.


Exits on the Pheromone Highway branch on average $53^{\circ}$ from the "away" direction, so each intersection reads like an arrow pointing home. Image © Duncan Jackson, used with permission.

The Sheffield team worked with colonies of Pharaoh's ant (Monomorium pharaonis). In one of their reported experiments, they allowed individual ants to walk along experimental, straight trails: "any reorientations occurring were as likely to be correct as incorrect in relation to the polarity of the trail when it was originally formed." In contrast, when meeting a trail bifurcation, " $43 \%$ of fed ants [who would presumably be heading home] made U-turns upon meeting the bifurcation point when walking in the 'wrong' direction: that is, away from the nest. Conversely only $8 \%$ of fed ants walking the 'correct' way made ... corrections that led them to heading incorrectly away from the nest." On the other hand, of the unfed ants, who presumably would be heading out for food, " $47 \%$ made course corrections at the bifurcation point when moving the 'wrong' way" while "only $8 \%$ walking the 'correct' way made incorrect course changes." The authors add: "Note that although only $45 \%$ of the ants corrected their orientation at a single bifurcation, real networks contain many bifurcations and many opportunities for course correction." [Interpreted as Markov chains, these numbers say that in the steady state $84 \%$ of the fed ants and $85.4 \%$ of the unfed will be moving in the appropriate direction.]

The Magic of Math, in Queens. On November 24, 2004 the New York Times ran "From Internet Arm Wrestling to the Magic of Math," Edward Rothstein's review of the new wing of the New York Hall of Science, in Queens. Rothstein glances at the high-tech baubles of the new installations, but saves most of his admiration for the Mathematica exhibition, which the Hall of Science recently acquired from the California Science Center. Mathematica was created for IBM in 1961 by the celebrated design team of Charles and Ray Eames. Rothstein remembers seeing it as a child: "I still recall wired structures rising out of soapy liquid, their swirling surfaces demonstrating solutions of mathematical problems; the cubic array of bulbs that translated simple multiplication into three-dimensional patterns of light; the suspended Möbius strip - a surface with only one side and one edge - on which a train continuously ran." And he ponders the difference between this exhibition, assembled at the apex of the post-sputnik wave of enthusiasm for science, and the flashier but shallower productions of today. "Mathematica samples varied branches of mathematics, not blanching from explaining functions or projective geometry; contemporary ex-
hibitions set their sights lower, restricting each display's focus. Mathematica knows you won't fully understand it all ... . Contemporary displays are more concerned that you grasp a single concept. They are play stations in a science lesson."

Father of fractals. That's the title of Jim Giles' News Feature in the November 182004 Nature. Benoit Mandelbrot is the Father; the article is illustrated with a large and spooky image of part of "the set that bears his name." Giles gives us a capsule intellectual history of Mandelbrot, taking him from his 1963 paper on selfsimilarity in graphs of cotton prices, through his years as an "academic wanderer" and the 1982 publication of The Fractal Geometry of Nature, when "the worlds of math and physics took notice." After a brief and completely non-technical digression on fractals, we come to the main point of the paper: Mandelbrot's attitude. Apparently, he has not been very nice. "As so often happens in academia, questions of precedence were central." We hear reports of aggressivity and misbehavior at conferences. Then Giles focuses on Mandelbrot and Vilfredo Pareto (1848-1923), who had published "similar studies on power laws in economics" many years before. Giles claims that in the most recent reprinting of Mandelbrot's 1963 paper on cotton, "many references to Pareto have been removed." And that one paper by a third author, Mandelbrot and the stable paretian hypothesis, appears in the same collection with a new title: Mandelbrot on price variation. Are we supposed to be horrified? In fact Giles ends up fairly conciliatory: "Even researchers who have been the subject of his attacks praise his contributions to maths." [A deeper analysis would examine the divisions in post-war French society, politics and science (even mathematics!), and how they played themselves out in Mandelbrot's career.]

Prime Number Record Extended. The Great Internet Mersenne Prime Search (GIMPS) has discovered the largest known prime number. The number, $2^{25,964,951}-1$, has almost eight million digits and is the 42 nd known Mersenne prime. (Mersenne primes are prime numbers of the form $2^{p}-1$.) The number was discovered on the computer of Dr. Martin Nowack, a German eye surgeon, through GIMPS, a distributed computing project (www.mersenne.org/prime.htm).

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