# What's New in Mathematics

## A new kind of Science?

"By relying on mathematical equations to describe the world, scientists for centuries have grossly limited their powers of explanation, asserts Stephen Wolfram" is the start of Richard Monastersky's piece *(Chronicle of Higher Education*, May 17, 2002) on the publication of Wolfram's long-awaited opus, "A New Kind of Science". The book is described by Jim Giles (*Nature*, May 16, 2002) as "a call for researchers to turn away from calculus and other conventional mathematical tools ... ." What is to replace calculus? Since John Conway's "Game of Life" (with roots in von Neumann's work in the 1940s, but first brought to wide attention by Martin Gardner in the October 1970 *Scientific American*) we have all known that a cellular automaton can start from a couple of simple rules and generate patterns of amazing complexity. Wolfram's fundamental innovation, as best reported by Edward Rothstein (*New York Times* "Arts and Ideas" section, May 11, 2002) is to posit that such automata are actually at work behind the complex systems (turbulence, consciousness, the local structure of space-time) that currently baffle scientific inquiry. "Not only can complex designs and processes arise from the simplest of rules, but ... simple rules actually lie behind the most sophisticated processes in the universe." And the corollary: some complex processes cannot be handled by scientific laws in the way we know them. "All we can do in such cases is discover the simple rules that give birth to the complexity. ... Everything else can be found only by 'experiment': the process must run its course."

## New/old math probes the Big Bang.

"A reconstruction of the initial conditions of the universe by optimal mass transportation" is the title of an article in the May 16, 2002, *Nature* by an international team mostly based at the CNRS Observatoire de la Côte d'Azur in Nice. "We show that, with a suitable hypothesis, the knowledge of both the present non-uniform distribution of mass and of its primordial quasi-uniform distribution uniquely determines" a map from present positions to the respective initial ones. The mathematics they use, which they call the Monge-Ampère-Kantorovich (MAK) method, goes back in part to Monge's solution of how best to move a pile of dirt from one location to another: you construct a "cost" function and minimize it. They have tested the MAK reconstruction on "data obtained by a cosmological $N$-body simulation with 1283 particles," and exhibit the results. Caution: they note that "when working with the catalogues of several hundred thousand galaxies that are expected within a few years, a direct application of the assignment algorithm in its present state would require unreasonable computational resources."

## The number line is real.

The number line is real. Psychologically speaking. That's the conclusion reached by a team of psychologists (Marco Zorzi, Konstantinos Priftis, Carlo Umiltà) at the University of Padua. In "Neglect disrupts the mental number line" (*Nature*, May 9, 2002) they examine right-brain-damaged patients with persistent left neglect: these patients "show a spatial deficit for left-side stimuli. ... When asked to mark the midpoint of a line, they miss the midpoint and place it to the right. The misplacement increases as a function of line length, with a crossover effect (leftward displacement) for very short lines". The team showed that exactly the same systematic errors occurred in mental operations when the patients were asked to *name* the midpoint of an integral segment $[a, b]$ given its endpoints a and b. The errors occur in the same direction whether the endpoints were given in increasing or in decreasing order, e.g. 1-9 or 9-1, leading them to observe that "the number line is canonically orientated in a left-to-right manner". They conclude: "Although most people focus on symbolic aspects of numbers, ... thinking of numbers in spatial terms (as has been reported by great mathematicians) may be more efficient because it is grounded in the actual neural representation of numbers". The reference is to Hadamard's "The Mathematician's Mind" (Princeton, 1996) which describes his own use of mental imagery but in coordinate-free terms: "a confused mass, ..., a point rather remote from the confused mass, ..., a second point a little beyond the first, ..." etc. (his visualization of Euclid's infinity-of-primes theorem). He also quotes Einstein: "The words or the language, as they are written or spoken, do not seem to play any role in my mechanism of thought."

## Stephen Wolfram and *The New York Times*.

George Johnson reviewed "A New Kind of Science" in the New York Times Book Review, June 9, 2002. Johnson begins with the book as a physical object: "1,263 pages ... and 583,313 words," intimidating perhaps but with marvelous pictures. "Certainly no one has worked so hard to produce such a beautiful book." He then contrasts Wolfram's publishing style (everything, all at once) with "the normal thing," i.e. regularly posted unreadable papers in "fashionable zines" like Physical Review Letters or Physica D. Johnson presents a cogent digest of Wolfram's main tenet: "the algorithm is the pure, elemental expression of nature; the equation is an artifice." And several examples. "One idea after another comes spewing from the automata in Wolfram's brain." The publication of Wolfram's treatise was also covered in the Science Times for June 11. In "Did This Man Just Rewrite Science?" Dennis Overbye relays opinions from several scientists who have worked the same turf. Here is Edward Fredkin, a BU physicist and longtime proponent of viewing nature as a computer: "For me this is a great event. Wolfram is the first significant person to believe in this stuff. I've been very lonely." Fredkin goes on: "An equation is just a thing you write down on a piece of paper. $E = mc2$ can't keep you warm." But programs are different. "Put them in the computer and they run." George Johnson is at bat again in "What's So New in a Newfangled Science?" (The Week in Review, June 16). "Interesting ideas rarely spring up in isolation" is the theme of this article, making up for Johnson's neglect of that topic in the Book Review. He surveys some of the current work on the algorithmic universe, including MIT's Seth Lloyd, the author of 'Lloyd's hypothesis' (Everything that's worth understanding about a complex system can be understood in terms of how it processes information), and BU's Fredkin. He concludes: "That is how an idea progresses. But sometimes it takes a bombshell to bring it to center stage." and in fact, as Johnson tells us at the start of the piece, " 'A New Kind of Science' was holding its own last week atop Amazon's best-seller chart, along with 'Divine Secrets of the Ya-Ya Sisterhood' and 'The Nanny Diaries.' "

## A tough math problem in Internet routing.

A tough math problem in Internet routing is described in "Guessing secrets: applying mathematics to the efficient delivery of Internet content" by Ivars Peterson in the April 6, 2002, *Science News*. Internet route optimizers need to determine the geographical source of a webpage request in order to connect that "client" with the nearest server holding the webpage. The request comes via an intermediate computer called a nameserver, but only the nameserver's address is immediately available. The client's address must be ascertained by a kind of "20 questions" game with the nameserver. E.g. "is the first digit a '1'?" The problem becomes interesting when, as is often the case, the client has two or more addresses, because then the nameserver still gives a yes-or-no answer. Peterson presents an worst-case example with three addresses and an honest but inscrutable answering algorithm that makes it impossible to guess any digit of any of the addresses. In general, when the information is available, how should one ask the questions to obtain it most efficiently? The matter, which is related to "list decoding" of ambiguous messages, is treated by Tom Leighton, Ron Graham and Fan Chung in the Electronic Journal of Combinatorics.

## Primes in the *Times*.

Sara Robinson again, in the August 8, 2002, *New York Times*: "New Method Said to Solve Key Problem in Math." The problem is "to tell quickly and definitively whether a number is prime," a problem that has "challenged many of the best minds in the field for decades." *Quickly* here means *in polynomial time*. The new method is an algorithm devised by Manindra Agrawal, Neeraj Kayal and Nitin Saxena of the Indian Institute of Technology in Kanpur. Robinson explains that the discovery has little immediate commercial significance, since the probabilistic algorithms currently in use are faster and accurate enough for practical purposes. But the theoreticians have loved it ever since it was announced (by e-mail) on Sunday, August 4th. It is simple and elegant enough so that Carl Pomerance of Bell Labs, who got the news Monday morning, was able to explain it to his colleagues in an "impromptu seminar" that very afternoon; he commented to Robinson: "This algorithm is beautiful." The "AKS" paper is available online (http://www.cse.iitk.ac.in/news/primality.pdf) in PDF format. It bears as epigraph a quotation from Gauss (1801): "The problem of distinguishing prime numbers from composite numbers and of resolving the latter into their prime factors is known to one of the most important and useful in arithmetic. ... Further, the dignity of the science itself seems to require that every possible means be explored for the solution of a problem so elegant and so celebrated." The story was also reported by the Associated Press ("Prime Riddle Solved"); the Times story was picked up in *The Hindu* on August 9 ("New algorithm by three Indians").

### Solving an Escher puzzle.

"The Print Shop" is one of Maurits Escher's more paradoxical creations. In the lower left-hand corner we see, through a window, a man looking at a print on the wall of a print shop. But the top of the print swells out of the shop and as we follow it clockwise through the picture it leads us back to the outside of the shop where we started, so the shop itself is in the print. This is a continuous version of the "picture within itself" that we see, in the US, on Land O'Lakes Butter boxes and in Holland on packages of Droste chocolate. The center of the print has "a large, circular patch that Escher left blank. His signature is scrawled across it." So Sara Robinson describes it in the July 30, 2002, New York Times, where she tells how Hendrik Lenstra, a mathematics professor at Berkeley and at Leiden, solved the riddle of what goes in the center. The key turned out to be the revelation, by a friend who had watched Escher at work, that the artist had kept the distortions conformal (i.e. angle-preserving, like the Mercator projection). Lenstra was able to exploit this feature to give a complete mathematical analysis of the print, and to fill in the patch. The solution has been beautifully presented on the web-page Escher and the Droste effect (http://escherdroste.math.leidenuniv.nl/) on the Leiden website. The page shows the original print and an amazing animation of the solution. Do not miss this.

### Post-mortem on The Geometry Center.

The analysis is carried out by Jeffrey Mervis in Science for July 26, 2002: "The Geometry Center, 1991-1998. RIP." The Geometry Center was created at the University of Minnesota as one of the first NSF-funded Science and Technology Centers. "From the start, the Geometry Center faced long odds. Even its mission was controversial." The mission was "to attempt to introduce computer graphics and visualization into pure mathematics and geometry," Mervis was told by David Dobkin, who chaired the center's governing board. "It wanted to change the field, but people weren't ready for that." Another problem was the budget: $2 million a year from NSF funds otherwise typically doled out in $25,000 parcels to single investigators. "We were immediately a target for people who said we didn't deserve all that money," said Richard McGehee, who directed the Center during its final years. There is no lack of suspects, and Mervis glances at several others. But he gives the final word to Don Lewis, head of the NSF mathematics division at the time: "I didn't see any progress, so I pulled the plug." The Geometry Center which, as McGehee remarks, "had one of the first 100 Web sites", lives on virtually (http://www.geom.umn.edu/) at the U of M.

### 143-Year-old Problem.

143-Year-old Problem Still Has Mathematicians Guessing – the headline stretches almost across the top of a page in the July 2, 2002, *Science Times*. And right in the middle is a picture of the man himself, with the caption "In 1859, Bernhard Riemann made a hypothesis on prime numbers that hasn't been proved or refuted." The occasion is a meeting at NYU earlier this year, where "more than a hundred of the world's leading mathematicians" gathered to "swap hunches, warn of dead ends and get new ideas that could ultimately lead to a solution" of the Riemann Hypothesis. Bruce Schechter wrote this article, a beautiful piece of mathematical reporting. It blends ancient history (Hardy, Gauss, Riemann) with modern history (Hugh Montgomery, Peter Sarnak, Andrew Wiles) and enough authentic background about prime numbers, complex numbers and the zeta function to keep the exposition honest. Of course after this wonderful buildup the news is disappointing, if not surprising: "Mathematicians at the conference agreed that there was no ... clear evidence of a trail head" from which to set off in pursuit of the still elusive hypothesis. Even more tantalizing, the Riemann Hypothesis now appears as the door to a universe of undiscovered mathematics. As Montgomery puts it: "It should be the first fundamental theorem."

### Perfect Graphs.

Perfect Graphs and the "Strong Perfect Graph Conjecture" are the topic of a News Focus piece by Dana Mackenzie in the July 5, 2002, *Science*. As Mackenzie explains it the definition involves two invariants of a graph. The first, $\omega$, is the size of the biggest clique (set of nodes each of which is one step away from all the others). The second, $\chi$, is the number of colors it takes to color the nodes so that no two adjacent nodes are the same color. So $\chi$ is always bigger than $\omega$; if the numbers are equal, the graph is *perfect*. Mackenzie: "A perfect graph is like a perfect chocolate cake: It might be easy to describe, but it's hard to produce a recipe." A conjecture due to Claude Berge (CNRS, Paris) has been around since 1960: every imperfect graph contains either an "odd hole" or an "odd anti-hole." This is the Strong Perfect Graph Conjecture (SPGC). The odd hole is "a ring of an odd number (at least 5) of nodes, each linked to its two neighbors but not to any other node in the ring." The odd anti-hole is "the reverse: Each node is connected to every other node in the ring except its neighbors." The news is that a proof of the SPGC has been announced by Paul Seymour (Princeton), G. Neil Robertson (OSU) and Robin

Thomas (Georgia Tech). The proof is worth $10,000 (put up by fellow "perfect-graph aficionado" Gerard Cornuejols) and "the early betting is that they will collect the prize."

### Neurons do Math.

Neurons do Math, in the brains of monkeys and frogs, at least. This is the message of Single brain cells count, a Nature Science Update for September 6, 2002. The update, by John Whitfield, describes two recent sets of experiments. Monkeys: A. Nieder, D.J. Freedman and E.K. Miller (*Science*, **297** 1708-1711 (2002)) "showed groups of dots to macaques, and recorded the output from individual neurons in the monkeys' prefrontal cortex. ... The neurons ignore the dots' size, shape and arrangement and hone in on their number. Each cell's response peaks at its preferred number and tails off on either side." Frogs: C.J. Edwards, T.B. Alder and G.J. Rose (*Nature Neuroscience* **5** 934-936, available online) sampled neurons in the brains of female frogs (*Hyla regilla*) to understand how they distinguished between the aggressive calls and the advertisement calls of males of their species. The only difference between the two calls is their speed. "Female frogs' male-detector neurons fire only after they hear five or more rapid pulses, Rose and his colleagues find. If the pulses are too close or too far apart, the counter resets to zero - as if the nerve cells measure the spaces between pulses, rather than the sounds themselves."

### A mathematical phase transition.

Phase transitions occur in physical systems, often at a certain "critical temperature" (e.g. ice to and from water at zero degrees C). In "Analytic and Algorithmic Solution of Random Satisfiability Problems" (*Science*, August 2, 2002), Marc Mézard, Giorgio Parisi and Riccardo Zecchina (Orsay) bring methods from statistical mechanics to study a phase transition which occurs in a purely mathematical context: the probability that a randomly generated $k$-SAT problem has at least one satisfying ("SAT") assignment. The $k$ means that each constraint involves exactly $k$ variables, so $(A+B+c)(a+D+e)(b+E+C)(d+a+b) = 1$ is a 3-SAT problem with four constraints in the five Boolean variables $A$, $B$, $C$, $D$, $E$, with $a = \neg A$, etc. The $+$ is the logical "or": $x+y=1$ unless $x=y=0$, and multiplication is the logical "and": $xy = 0$ unless $x=y=1$. In this example $A=0$, $B=1$, $C=1$, $D=0$, $E=1$ is a "satisfying assignment." The role of temperature is played by the ratio $\alpha$ of the number of logical constraints to the number of variables. Clearly when there are many more variables than constraints the probability of a satisfying assignment is high, and vice-versa. David Mitchell, Bart Selman and Hector Levesque showed experimentally about 10 years ago that the transition from high to low occurs abruptly at a critical value $\alpha_c$ near 4.3 for $k=3$ and in addition that the computing time necessary to settle the problem peaks dramatically near $\alpha_c$. Mézard and his colleagues pin down $\alpha_c$ to 4.256 and locate another transition point $\alpha_d = 3.921$ such that between $\alpha_d$ and $\alpha_c$ "the space of configurations breaks up into many states, and there exists a nontrivial complexity" thus partly explaining the computation peak observed by Mitchell *et al*. They remark "From the strict mathematical point of view, the phase diagram we propose should be considered as a conjecture," an invitation for mathematicians to get involved in this aspect of mathematics.

### The next big thing.

The *Chronicle of Higher Education* (September 30, 2002; Section B, page 4) invited experts in Geography, Math, Information Technologies and Criticism to tell us "What will be the next big thing?" in their fields. The mathematics respondent was John Ewing of the AMS. "The next big thing in mathematics? Biology. ... The mathematics involved in studying the genome and the folding of proteins is deep, elegant, and beautiful ... a spectacular new area of research that is certain to grow enormously in the next 10 years." Ewing goes on: "During the coming decades, scientists and mathematicians will come to see the false distinctions between pure and applied mathematics. ... More and more, mathematicians will see their subject as underlying all science and social science – not as a humble servant but as an essential companion."

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