Great Moments in XXth Century Mathematics

by Roger W. Carter

I have chosen for my great moment the appearance in 1976 of the paper by Deligne and Lusztig 'Representations of reductive groups over finite fields'.

This paper provided the breakthrough in understanding the characters of the irreducible representations of the finite groups of Lie type.

These groups are the analogues over finite fields of the simple Lie groups over the complex or real numbers. The classical linear, symplectic and orthogonal groups over finite fields had been studied at the beginning of the 20th century by L. E. Dickson, and Dickson also published papers on some of the exceptional groups over finite fields. However it was not until 1955 that Chevalley discovered a systematic way of constructing the finite analogues of the simple Lie groups.

The uniform description of these finite groups of Lie type, and their twisted versions due to Steinberg, Tits, Suzuki and Ree, posed the challenge of describing in a uniform manner the irreducible complex characters of such groups. The first step had been taken by J. A. Green in 1955 in a remarkable paper in which he determined the irreducible characters of the general linear groups $GL(n, q)$. This paper was in many ways ahead of its time. It was clear that an analogous representation theory should exist for all the finite groups of Lie type. Indeed I. G. Macdonald made some famous conjectures about how such representations should behave. But these remained only conjectures until the appearance of the paper by Deligne and Lusztig. By considering the action of the group on a certain algebraic variety over a finite field and taking the l-adic cohomology groups, Deligne and Lusztig obtained families of irreducible characters which both proved Macdonald's conjectures and also opened the way for the determination of all the irreducible characters.

This work was carried out by Lusztig in the years following the Deligne-Lusztig paper. Eventually the degrees of all irreducible representations were obtained, as well as much additional information about the character table. This beautiful and elaborate theory is without doubt one of the great achievements of 20th century mathematics.

Roger Carter obtained his Ph.D from Cambridge University in 1959. After spending a postdoctoral year as a Humboldt scholar at Tubingen University he spent the period 1960-5 as Lecturer in Mathematics at the University of Newcastle upon Tyne. He took up a position at Warwick University in 1965, the year in which Warwick admitted its first undergraduates, and has been a member of the Warwick department since then. He served as Chairman of the Mathematics Department for five years, and is now an Emeritus Professor.

Carter's research interests are in algebra, and include group theory, representation theory, algebraic groups and Lie theory. He is the author of a number of books on these subjects, as well as research papers. He has been involved in organising several large scale international symposia at Warwick, and one at the Isaac Newton Institute in Cambridge.