Pedro Nunes and Mercator: a Map From a Table of Rhumbs

by Pedro Freitas*

Pedro Nunes and the loxodrome

The 16th century was a period of great scientific and technological development in Europe. Portugal was no stranger to this general atmosphere, having developed new sailing techniques, which were necessary to navigate outside the Mediterranean, and below the Equator, where the North Star is not visible.

Along with more practical developments, there was also an interest in abstract physical and mathematical problems, inspired by concrete needs and questions. Among the people that were interested in these problems, stands the figure of Pedro Nunes (1502–1578), a remarkable mathematician and astronomer. Figure 1 is sculpture of Pedro Nunes in a monumento to the discoveries.

Pedro Nunes started his scientific studies in Salamanca, around 1517, where he got a degree in Medicine, in 1525 (this was the usual course of studies at the time for someone interested in a higher scientific education). He then returned to Portugal, and taught Moral Philosophy, Logic, and Metaphysics at the University of Lisbon (starting between 1529 and 1531). He was appointed Royal Cosmographer in 1529 and Chief Royal Cosmographer in 1547, a post that he held until his death.

Among his publications, we refer the *Tratado da sphera* (1537), which includes the *Treatise on certain doubts of navigation* and the *Treatise in defence of the nautical chart*, to which we will return, and *De crepusculis* (1542). In this second book, he studies, and solves, an important problem of his time: the determination of the duration of the twilight, depending on the latitude and the day of the year, and provides many other new and relevant observations, including the description of

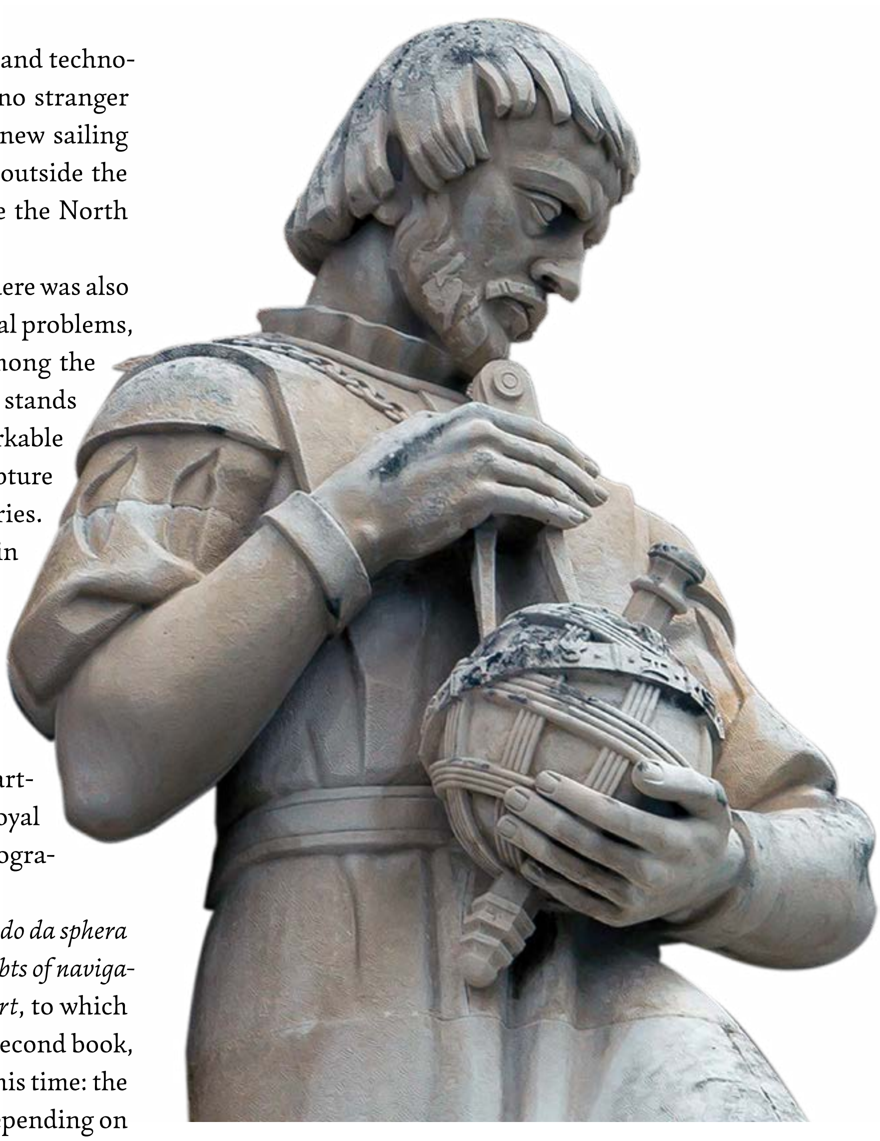

Figure 1.— A sculpture (by Leopoldo de Almeida) representing Pedro Nunes

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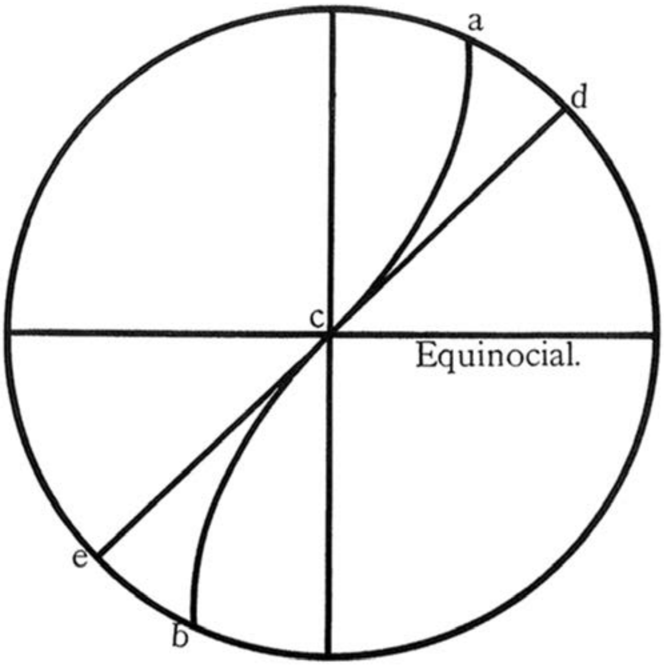

Figure 2.— A great circle (dce) and a rhumb line (acb)

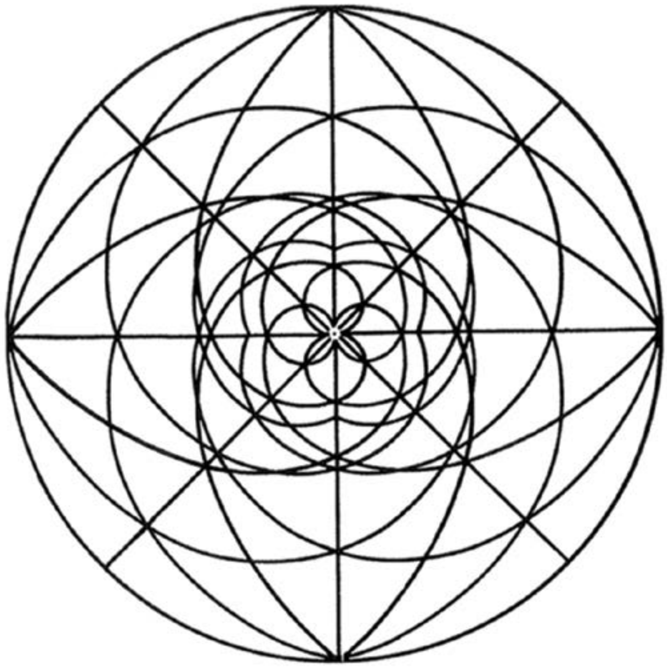

Figure 3.— Loxodromes seen from the pole in a figure by Pedro Nunes

a new instrument, the *nónio* (see [6] for a study of this book). In both works, Nunes gives a strictly mathematical treatment to practical problems. These books, especially *De crepusculis*, with its rigorous and extensive solution to an old problem, afforded Nunes a high intellectual standing, at an international level.

These and other works were later collected and expanded by Nunes in *Petri Nonii salaciensis opera*, his collected works, published in Basel, in Latin, in 1566 — a commented edition was recently organized, between 2002 and 2010, and published by Fundação Calouste Gulbenkian.

In the course of his career of Royal Cosmographer, Nunes came across problems relating to navigation, which were in great extent inspired by practical considerations. In [2], Pedro Nunes mentions some specific questions that the navigator Martim Afonso de Sousa asked him. One of them was about the correct way to navigate along a great circle, which is the path of least distance on a sphere (this is Nunes' reformulation of the original question). The knowledge of the north, given by the compass or the North Star, would allow keeping the ship at a course maintaining a given angle with meridians, and apparently there was a belief that this would ensure the ship would travel along a great circle. Pedro Nunes gave this problem quite some thought. Figure 2, taken from [2], and the accompanying text, show that he realized that this belief was misguided: the great circle is not the course the ship would take if this angle (called bearing) were kept constant.

Nunes distinguishes very clearly these two forms of navigation, noting that if one wants to navigate along a great circle, the bearing has to be constantly adjusted. The curve that the ship follows if the angle with meridians is kept constant came to be known as a rhumb line (this was the name used by Nunes), a loxodromic curve, or simply a loxodrome. The word rhumb refers to this constant angle with meridians, to be kept constant in order to navigate along the curve. The method Nunes suggests to correct angles in order to travel along a great circle, based on spherical trigonometry, turned out to be too difficult to implement on board, and apparently was never adopted by Portuguese sailors. In fact, Nunes suffered much criticism regarding the abstraction and complexity of his methods, in his time, even though it remains to be ascertained if such criticism was deserved or not.

At any rate, the treatment of this new curve,[1] the loxodrome, was extended to a theoretical level not seen before for non-conical curves. Figure 3, also taken from [2], and reproduced in the cover of the Proceedings of an International Conference held in Portugal in 2002, shows a few loxodromes as seen from the pole.

One of the results Pedro Nunes proved about this curve, only clarified and published a few years later in [3], is that it does not enter the pole (unlike what the figure above sug-

[1] Pedro Nunes makes a reference to Ptolomy's *Geography* when describing the loxodrome, but this is probably just a way to relate it to a famous name, as no reference of this curve has been found in Ptolomy.

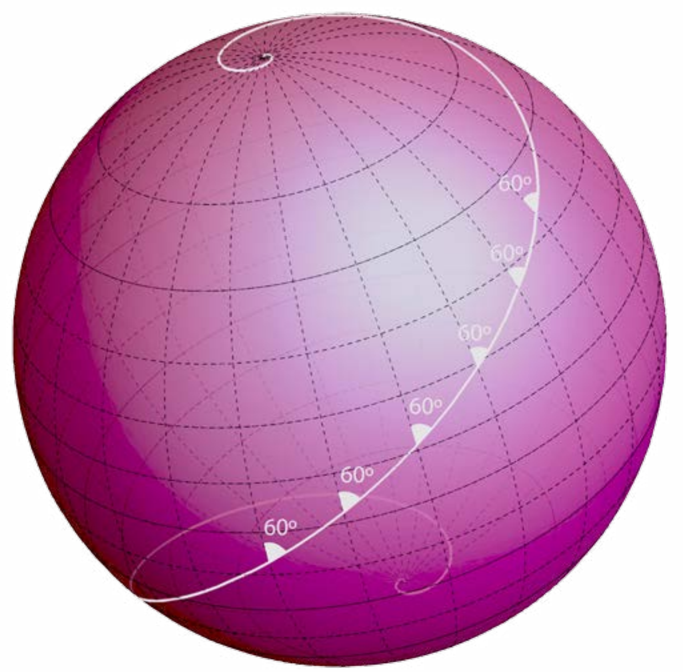

Figure 4.— A loxodrome making an angle of 60° with meridians

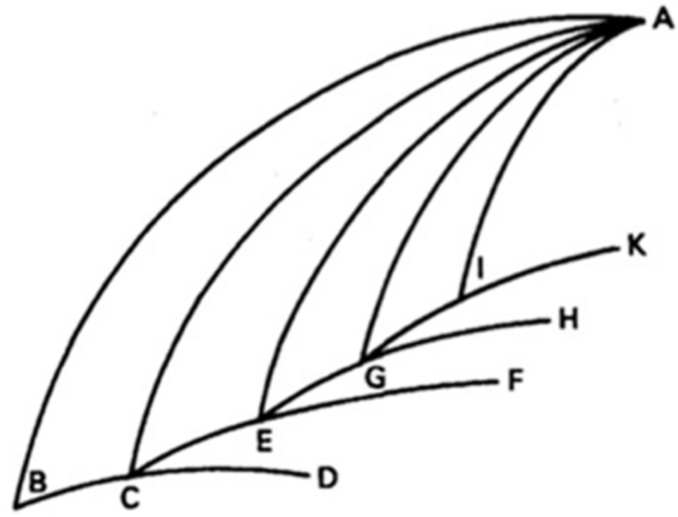

Figure 5.— Approximation of a rhumb by arcs of great circles

gests), yielding an *infinite line* on a sphere. Figure 4 shows a loxodrome on a sphere.

The equation for a loxodrome making an angle α with meridians is given by

$$\varphi = -\frac{\pi}{2} + 2\arctan e^{\lambda \operatorname{cotan} \alpha}$$

where φ is the latitude and λ is the longitude, and can be deduced from the definition of the curve using integral calculus (see [4] for a brief deduction of this formula and [5] for a more extensive study of this curve). The description of the curve in Nunes' time, however, was not done with a formula but with a table of pairs of longitudes and latitudes, along each rhumb. Usually, seven angles were chosen, making angles of 11,25° between them, evenly dividing the right angle between the equator and a meridian.

Nunes' method for constructing such a table is described in [3]. Once an angle was chosen, the rhumb is approximated by arcs of great circles. Figure 5 illustrates this.

In the figure, *A* represents the pole and the lines *BCD*, *CEF*, etc, are arcs of great circles. The lines originating at *A* are arcs of meridians. Nunes uses a theorem by Gebre about sines in spherical triangles to iteratively calculate the angles and lengths involved. After this description the method, Nunes includes an empty table, inviting the "laborious lads" to supply the calculations.

The Mercator chart

The expression *rhumb line* can also refer to a straight line, drawn on a navigation chart. When Pedro Nunes started working on these problems, there were no charts with the property that loxodromes would be represented as straight lines. In [2], Nunes actually refers to the need to create such a chart, which would make navigation problems much easier. The navigation could be charted along a certain rhumb, which would just be a straight line on a map, and correspond to a certain bearing, something that could be achieved with a compass.

The first chart with this property was published by Gerardus Mercator in 1569, and this cartographic projection came to be known as *Mercator projection*. Figure 6 (see next page) shows a modern map using this projection. The circles, called Tissot indicatrices, all have the same area on the globe.

The map shows that horizontal and vertical lines represent parallels and meridians, respectively, but parallels must be more spaced as the latitude increases (the scale factor at a given latitude is the secant of this latitude). This leads also to area deformation—for instance, Greenland looks as big as Africa, which is fourteen times larger—but it does have the property that loxodromes translate to straight lines.

The main problem of producing such a map is determining the rule for this increase in spacing of parallels. Mercator left nothing written about the method he used to calculate this spacing increment. He does refer that the rule for creating the map is that the proportion between lengths of meridians and of parallels should be the same in the map as in the globe. This implies angles are not distorted (in other

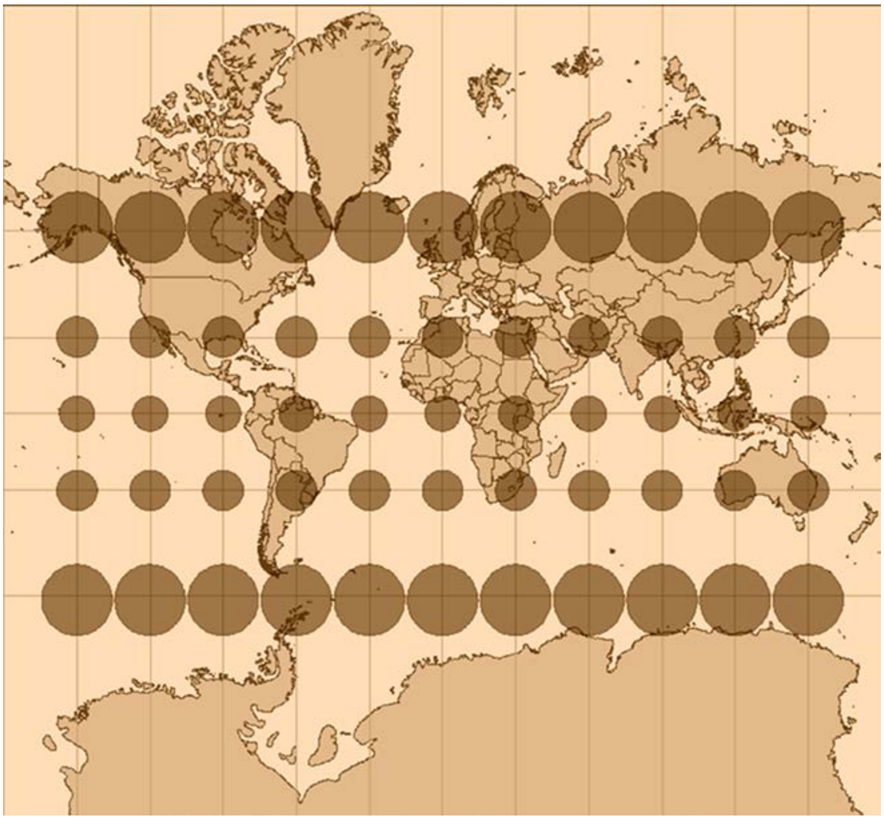

Figure 6.— A modern map using the Mercator projection (Stefan Kühn, Wikipedia).

words, it is a conformal projection). However, it does not provide a practical method for drawing the actual map.

This has been a problem of cartography for many years, several theories having been presented. There was a recent breakthrough, though, that originated in a detailed analysis of the errors in the original chart. We follow article [1] in describing this new approach.

First of all, one has to separate the calculation errors (inherent to the method) from errors due to the physical distortion of the sheet on which the map was printed (article [1] was the first one to distinguish these two types of errors). The key to detecting the physical distortion was a figure on the bottom right of the map, called *Organum directorium* (see Figure 7).

A graduated quarter circle with a mesh of meridians and parallels appears in this figure, along with the angles marked on the quarter circle. This means that we know what were the angles considered when making computations for the drawing of the parallels, and by comparing them to the actual angles and y-coordinates of the parallels on the map, we can ascertain what was the physical distortion the map suffered after printing, thus separating it from the errors due to calculations. As an illustration, we can see in the figure an angle α, along with a y-coordinate Y_α, which is used to draw a parallel.

Finally, a last question remains: after isolating the physical errors, what was the method used to draw the map? An answer that is simultaneously natural and ingenious consists of taking the property that the map should have and *turn it into the method* for drawing it. In other words: if the aim is that loxodromes should be straight lines, then consider one of these loxodromes, and make it a straight line! The (simplified) process is as follows: after drawing a graduated equator on a piece of paper, draw a straight line forming the given rhumb angle with the equator, and use a table of rhumbs to mark, on each longitude, the corresponding point on the rhumb line. This gives you y-coordinates for the parallels, which can then be drawn.

For this we need a table of rhumbs, a sequence of pairs of longitudes and latitudes, something that Pedro Nunes proposed. The authors of [1] tested a few tables available in 1569, and found that there was a remarkable match between the map and a table for the second rhumb (22,5°). This table used constant intervals of one degree of longitude, yielding differences in errors smaller than one fifth of a degree, when compared with the errors in Mercator's map.

Conclusion

Our story started about five hundred years ago, with Pedro Nunes' idea of a new curve on the globe, which would facilitate navigation, and his call for a map in which these curves would be straight lines. The map was created by Gerardus Mercator a few decades later, but the method of its making remained a mystery until our present days, only to be solved recently by Joaquim Gaspar and Henrique Leitão. And in this solution, Pedro Nunes happened to also have a side role, by launching the idea (along with a method) of constructing tables of rhumbs.

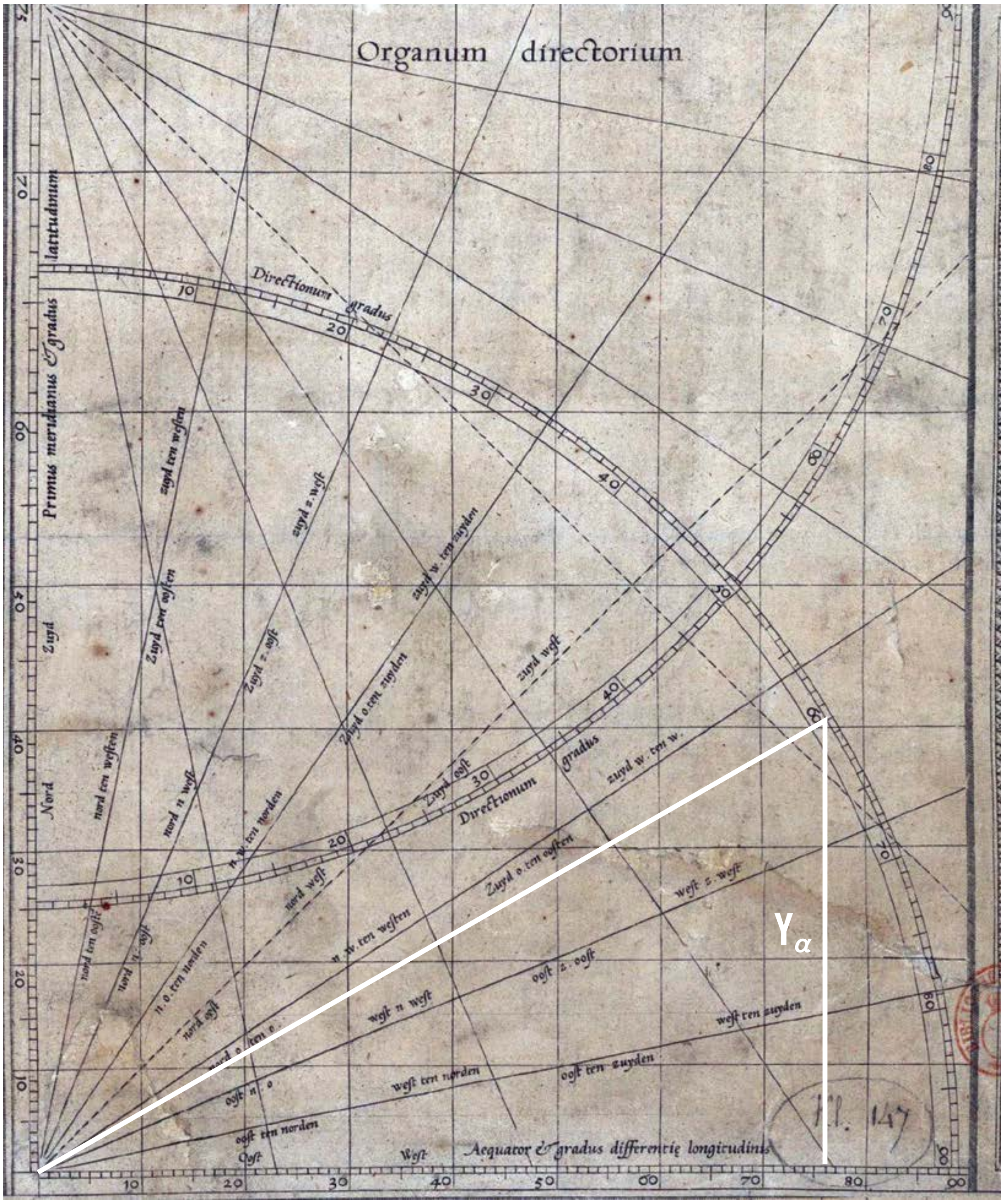

Figure 7.— Organum directorium from Mercator's 1569 map, taken from [1].

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For more information on Pedro Nunes' life and works, the reader can visit the site http://pedronunes.fc.ul.pt, *by Bruno Almeida and Henrique Leitão.*