Considering diagonal linear operators T_i in the conditions of Theorem 3, and the equality (for diagonal linear operators) between the cardinality of the spectrum and the degree of the minimal polynomial (Theorem 1), we obtain, from the previous theorem, the following result:

Theorem 5 ([9]) *Let $A_1, \ldots, A_m$ be finite nonempty subsets of $\mathbb{F}$. For p large enough we have*

$$|s_k(A_1, \ldots, A_m)| \geq \left\lfloor \frac{|A_1| + \cdots + |A_m| - m}{k} \right\rfloor + 1.$$

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Great Moments in XXth Century Mathematics

In this issue we present the answers of two researchers, E. C. Zeeman and Thomas J. Laffey, to the question "If you had to mention one or two great moments in XXth century mathematics which one(s) would you pick?".

Christopher Zeeman

Of course one is tempted to mention famous theorems that have earned Field's Medals, but I would like to draw attention to a moment in the early 1960's that witnessed the rebirth of the whole subject of geometric topology.

Topology grew out of analysis at the beginning of the century, became geometric in the 1910's - 1920's, then turned algebraic in the 1930's - 1940's. By the 1950's algebraic topology reigned supreme while geometric topology was all but dead. But then in the 1960's four results ushered in its rebirth, and gave rise to a great resurgence of geometric topology, as well as spinning off new subjects like differential topology, dynamical systems and chaos. The four results were:

Smale's proof of the Poincaré Conjecture in dimensions greater than or equal to 5 (bypassing the lack of proof in dimensions 3 and 4).

Mazur's (and Morton Brown's) proof of the Schoenflies Conjecture in higher dimensions (under the hypothesis of local flatness, thereby bypassing the psychological obstruction of the Alexander Horned Sphere).

Milnor's proof of the different differential structures on the 7-sphere (thus lauching differential topology).

My own unknotting of spheres in 5 dimensions (leading to the piecewiselinear unknotting of n-spheres in (n+k) - space, for all $k \geq 3$).

Thomas J. Laffey

I have chosen two results in Algebra which have had a profound effect.

I was still an undergraduate when my teacher, Professor Seán Tobin, announced to the class one day that a paper had just appeared by Feit and Thompson proving that all finite groups of odd order are solvable. I knew that Burnside had conjectured this but only later came to appreciate the amount of innovative techniques and arguments that had to be created to prove it—Richard Brauer in his talk in 1970 on the occasion of the award of a Fields Medal to John Thompson said in relation to the Burnside conjecture that before the Feit-Thompson paper: "Nobody did anything about it, simply because nobody had any idea how to get started". The paper contains the initial version of many of the tools used in the classification of the finite simple groups as well as techniques used in studying p-groups, p-nilpotence of finite groups etc. The fact that a paper of such extreme length and complexity successfully resolved the conjecture also encouraged mathematicians to attack other outstanding conjectures.

The development of (particularly non-commutative) ring theory and the theory of algebras has been a highlight of twentieth century Mathematics. Jacobson's Density Theorem revolutionised this area. The resulting emphasis on matrix-type rings led to the theory of PI-algebras, central identities etc. and had a major impact on representation theory and functional analysis.

After completing a doctorate in Sussex University under the supervision of Walter Ledermann, Thomas Laffey joined the Mathematics Department of University College Dublin in 1968 and has remained there since. He served two terms as head of department (1986-90 and 1996-99).

His principal research interests are in algebra, particularly in finite group theory and algebraic linear algebra.

He was the founding editor of the Newsletter (now Bulletin) of the Irish Mathematical Society and is currently one of the two editors of the Mathematical Proceedings of the Royal Irish Academy and a member of the editorial board of three other journals.