

## Almeida Costa and Algebra in Portugal

In the 1940s, I was a Mathematics student in the Faculty of Science at Lisbon University, and at that time, what we learned of the so-called *Modern Algebra* consisted only of groups of permutations and their application to Galois Theory about equations solvable by radicals. There was nothing at all about abstract groups, rings or other algebraic structures.

While I was a student, I had the opportunity of attending an extra-curricular seminar organised by some of the most respected mathematicians of the time, who had recently returned from studies abroad and were trying to do something about the academic backwardness that they found in Portugal. One of the subjects studied in this seminar was Group Theory, and the bibliography contained, amongst half a dozen books by foreign authors, one by a Portuguese: *Elements of Group Theory* (Elementos da Teoria dos Grupos) by A. Almeida Costa (1942, 153 pp.). Thus it was that I first heard of the scientist who, in the same year, had published *Abelian Groups, Rings and Non-Commutative Ideals* (Grupos abelianos, anéis e ideais não comutativos) (173 pp.) and, immediately afterwards, in 1943, *Elements of Ring Theory* (Elementos da Teoria dos Anéis) (282 pp.)

How had Almeida Costa become so enthusiastic about these matters?

In 1928/9, he assumed responsibility for the courses in Astronomy in the Faculty of Science in Oporto, and when, in this same year, the National Education Committee (*Junta de Educação Nacional* - henceforth J.E.N.) was set up, he immediately applied to them for a monthly subsidy in order to prepare his doctorate dissertation. His proposal was received favourably, since he clearly revealed exceptional intelligence and diligence, and his area of study (certain elements of surface theory by vector calculus methods) was recognised to be of topical interest. However, the scholarship was not awarded that year because “the candidate had not yet revealed his talent for research through written work”.

The young Almeida Costa was not discouraged, and two years later, his first publications began to appear: *Notes on Vector Calculus* (Notas de Cálculo Vectorial) and *On the Dynamics of Holonomic Systems* (Sobre a dinâmica dos sistemas holónomos), which he presented with his application for a professorship.

In 1934, he once more requested a grant from the J.E.N.,

this time in order to travel abroad to pursue his studies into astronomy and geometry at the University of Paris, the latter under Élie Cartan (but with “the possibility of replacing, at his own responsibility, the study in geometry with physics, under the direction of Louis de Broglie”).



Almeida Costa

However, the Committee was very demanding in its concession of the grant, particularly as regarded the subject matter proposed and national acceptance of the candidate’s specialization. But Almeida Costa did not give up, and in this struggle, revealed his determination to fight for what he believed in, a quality that remained with him all his life. In successive applications, he revealed himself to be highly informed about what was happening in Europe, and in 1937 (three years after his first application), having personally explained to Prof. Max von Laue the aims of his scholarship (the study of theoretical physics, particularly quantum theory and theory of relativity), he finally received a grant to study at the Physikalischer Institut in Berlin, where he remained for 22 months.

There, he began to attend courses on Matrices and Group Theory, as well as *Theoretical Electricity* and *Thermodynamics*. When the Institute for High Culture

(which had replaced the J.E.N.) observed that he should not diversify his attentions so much, he replied, “I cannot forget that a background in mathematics is absolutely indispensable for the serious study of theoretical physics. Pages and pages of physics books are given over to matrix theory, series expansions of complete functions, orthogonal or not . . . group theory . . .”.

The influence of Hilbert was obvious in the teaching of mathematics and theoretical physics at that time, and the last studies done by Almeida Costa in Berlin demonstrated his familiarity with methods of physical mathematics according to Courant and Hilbert, and quantum groups according to Hermann Weyl, another of Hilbert’s disciples.

In conclusion: until September 1937, Almeida Costa had lived in the world of applied mathematics (essentially mechanics and astronomy) at the Faculty of Science in Oporto; the J.E.N. effectively rerouted him in the direction of theoretical physics, when he went to study in Berlin; and as a result of this, he dropped his interest in astronomy, and from the 1940s, moved into a completely different scientific world where he demonstrated zeal in transmitting to others what he had learned abroad.

In 1950, he became a Full Professor of Celestial Mechanics at the Faculty of Science in Oporto, and two years later, accepted the Chair in Algebra at the Faculty of Science in Lisbon, where he was able to lecture on subjects more in keeping with the choices he had made in the previous decade. This eventually made possible the modernization of the teaching of this subject.

It was thus in 1952 that I met him personally. Hardly had he arrived in Lisbon when he instigated a series of seminars on the topics that most interested him, seeking to transmit to the young academics his enthusi-

asm for “modern” algebra. During the academic year, there were regular expositions (at least once a week) on group theory, theory of rings and non-commutative ideals, and field theory. I became familiar with his massive work *Hypercomplex Systems and Representations* (Sistemas hipercomplexos e representações), and perhaps because of this, he himself suggested that I, still a teaching assistant, should be responsible for the course of Physical Mathematics, lecturing among other things group representations.

His enthusiasm for his subject was such that he even suggested that students who had passed his course with the minimum of 10 should re-sit the exam in the next session, since he could not be sure that they had attained sufficient mastery of the material! It was as if he felt, in his innermost self, the words of the famous algebraist Paul Halmos, “It saddens me that educated people don’t even know that my subject exists!”

He was elected correspondent member of the Lisbon Academy of Sciences in 1959, and full member in 1972, and five years later, became its president. His activities in algebra did not cease upon his retirement in 1973, for the following year, the third volume of his *Course in General Algebra* (Cours d’Algèbre Générale) appeared, and he continued to present papers to the Academy until 1978, the year of his death.

Almeida Costa worked until the end, and was the greatest specialist in Semiring Theory, and the true founder of the Portuguese school of algebra.

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