

Stewart Alexander Robertson was born in Dundee, Scotland, in 1933. He studied at the universities of St Andrews and Leeds, lectured at the University of Liverpool, and has held a Chair of Pure Mathematics in Southampton since 1970.

In his career he supervised some twenty Ph D theses and introduced several important geometrical concepts (*Transnormality*, *Exact Fillings*, *Parallel Immersions*, to mention a few).

His interest in the popularisation of Mathematics led to the participation in *The French Museum Competition*, organized by *The Mathematical Intelligencer*, in 1981, in which he was awarded third prize, and to an invitation to contribute to the London Mathematical Society's Popular Lecture series in 1994.

Professor Robertson has strong links with Portugal and in May 1996 the Maths Department of Coimbra University held a *Topology and Geometry Day* in his honour.

May be you would like to start by telling us when you first realized that you would become a mathematician. Was there an adequate atmosphere for maths at school, or a particularly influential teacher?

'When I was very young, maybe only five or six years old or thereabouts, I wanted to be a scientist, which I imagined involved wearing a white lab coat and making all kind of marvellous discoveries that would make the world a wonderful place to live in. The source of this was the illustrations in a serial publication by one of the national newspapers, dedicated to the proposition that science was the key to future happiness. The second world war, ending with the two atom bombs, rather changed that fantasy, and besides, one day in the chemistry laboratory at school, I got a mouthful of caustic soda in an accident with a pipette. So I came quickly to the conclusion, at the age of about thirteen, that perhaps theoretical pursuits were a lot safer. I retain, however, a strong liking for the design ideas that permeate the Art Deco movement, which was very much part of the childhood experience of my generation. I attended the local school in the town (Broughty Ferry) where I grew up, a fairly large coeducational comprehensive in the long-established Scottish tradition. My teachers were very dedicated, and all were well-qualified academically. I remember the day when we were introduced to the idea that mathematical theorems did not just have to be learned as facts, but could be proved in

the spirit of Euclid. This seemed to me very exciting, and I suppose that set me on course towards a mathematical career, although at that time I had no idea of what such a life could be. Like many of my generation, I was the first member of my family ever to attend a university. Of all the many people who have helped me, I think I owe most to D.E Rutherford, who was a great teacher at the University of St Andrews, with a wonderfully ironic sense of humour and sceptical outlook.'

You contributed to several areas in Geometry. Is there one you would like to single out, either because you obtained major results or the results were particularly pleasing?

'Every professional mathematician hopes that some of their work will survive as part of the mathematical corpus stored up in the textbooks and studied by future generations. I should like to think that I have made a small contribution to the theory of symmetry for convex bodies, as a foot soldier in the Euclidean army. Geometrical symmetry has been my main interest since I came across the Platonic solids in my early undergraduate days, and I now feel that I am beginning to understand these and associated strange objects quite well. This has given me a great deal of pleasure.'

You have had a long career in research. Was or is there a problem whose solution eluded you?

'We all have a private file of pet unsolved problems. The one I should most like to see someone resolve in my lifetime concerns a topic that developed from a study of paper folding. This led to the concept of isometric folding for Riemannian manifolds. In particular, an isometric folding of an ordinary two-dimensional sphere is a map of the sphere to itself that sends each piecewise geodesic path on the sphere to another of equal length. It is conjectured that the space of all such maps that are not actual isometries, with the 'obvious' topology, is pathwise connected. So any such isometric folding can be deformed, through isometric foldings, to the map sending  $(x, y, z)$  to  $(x, y, |z|)$ ,  $x^2 + y^2 + z^2 = 1$ . So far, all efforts to decide this one way or the other have run into the sand. Another favourite is the conjecture that the only nontrivial automorphism of the symmetry type structure in the space of convex bodies is the duality involution. I have not the faintest idea about how to tackle this problem.

There is a third problem that I first thought about

more than thirty years ago, and which has been taken up by various colleagues, who have succeeded in solving some special cases, even if the original problem remains tantalisingly out of reach. This concerns the set of focal points of a smooth (say  $C^\infty$ ) connected compact hypersurface  $M$  without boundary, embedded in euclidean  $(n + 1)$ -space  $E^{n+1}$ . Any such  $M$  partitions  $E^{n+1}$  into two connected regions with  $M$  as common boundary. One of these is unbounded, and we call this the *outside* of  $M$ . The other is bounded, and we call this the *inside* of  $M$ . Now there is a map  $\eta$  from  $M \times \mathbf{R}$  to  $E^{n+1}$  which assigns to each pair  $(x, t)$  the unique point  $\eta(x, t)$  of  $E^{n+1}$  that lies at (signed) distance  $t$  along the inward-pointing normal line to  $M$  at  $x$ . This map

conjecture in full generality, even for  $n = 2$ , remains unsettled, to the best of my knowledge.

I am not as confident as I once was that the conjecture is true.'

How do you see the development of Mathematics in this century? Which is the mathematical achievement you would like to point out as the most important one?

'I think that questions of this kind should be asked of much better mathematicians than myself. I don't really know enough about the general shape of the subject, nor do I have a good enough grasp of the detail



is smooth and the image of any singular point of  $\eta$  is called a **focal point** or *centre of principle curvature* of  $M$ . I suggested to my very first research student, Sheila Carter (now at the University of Leeds), that 'as an exercise' she should prove the conjecture that the intersection of the set of focal points of  $M$  with the inside of  $M$  is nonempty. It soon became clear that this was by no means a straightforward 'exercise', and it took a lot of effort to make any progress at all. It is known, for example, that the conjecture is true if the inclusion of  $M$  in the closure of its inside induces an epimorphism of fundamental groups. Likewise, the conjecture is true if the fundamental group of  $M$  is abelian. However, the

in many key areas of current research. From my colleagues, I have gained some sense of how the really big discoveries of the nineteenth century are still being worked out. For example, the interaction of hyperbolic geometry, group theory and topology looks like one of the most fruitful and exciting areas of current research, with the prospect of great things still to come. Others might draw attention to the cross-fertilisation between geometry and theoretical physics in recent years. This is of course not new to the extent that it continues a tradition going back at least as far as Pythagoras, but it is important that it does continue: topics in pure mathematics tend to degenerate,

in my opinion, if they lose contact with the world of physical experience; conversely, the progress of science can be impeded by failure to recognise known mathematical structures and patterns. For reasons already indicated, I feel incompetent to judge which are the most important results of this century, but one obvious candidate is Gödel's theorem. Likewise, a candidate for the most dramatic achievement must be Wiles' solution of the Fermat problem. At a more general level, there is a striking difference in the way that mathematics is written down by present day mathematicians, compared with their predecessors in earlier centuries. There is now a standard language, and apparently a much higher level of rigorous argument. Unfortunately, this does not help much in the avoidance of errors, and clarity of expression is no substitute for creativity and insight.'

Now that you are about to "ride off into the sunset", how would you like to be remembered by your former students? You have had quite a large number of research students over the years. Would you like to comment on the two-way process involved in the supervision of a PH D dissertation?

I hope that most of my research students will remember their years at Liverpool or Southampton with nostalgia for a time that may have seemed dominated by hard work but also included lots of laughs. Supervising research students is very stimulating because it forces you to focus on problems that you might otherwise leave aside, and it also creates a feeling of urgency to get things finished, since nearly all students are short of both time and money. It is also very rewarding to see young mathematicians developing self-belief as they begin to produce original work of their own. Good research students always produce good ideas, and it is a great feeling when a research project starts to succeed in a variety of directions at once as a kind of community effort. One of the longer term benefits for me is

that almost all of my students have obtained permanent university positions, and several have continued to work with me on joint research projects over many years. I shall miss that part of my professional life very much, even if I shall no longer be under continuous pressure to finish that proof, or produce those diagrams, by yesterday at the latest. I have been lucky to have research students not only from the UK, but also from various parts of the Middle East, Africa, and mainland Europe, including Portugal. This has given me an insight into other cultures and intellectual traditions, and a sense of belonging to a world-wide community, much of which is struggling to survive in difficult economic and social conditions. I believe that the learned societies have a big responsibility to provide leadership and to try to help mathematicians all over the world both directly and by advising policy makers. I am pleased to have had seen the progress of such work in close-up as an officer in the London Mathematical Society and in the now emerging European Mathematical Society.'

What do you think the future holds for you in terms of intellectual work? We know that you do not want to pursue further mathematical research.

Although I have no plans to start new research projects, I hope to finish a couple of expository pieces of work that might one day appear in book form. I should like very much to develop other interests that began in early childhood, and now find expression in the study of poetry, painting, drawing, photography, aesthetics and philosophy. One dream, which I shall probably fail to turn into reality, is to produce a study of the aesthetics of pure mathematics. Retirement brings escape from the hurly-burly of everyday university life, but it isn't really feasible to continue at professional level in technical mathematics without being a full-time member of the working community. Besides, I'll soon be too old and dodderly for that sort of thing, and may have reached that state already.