The competition *Symmetries of Lisbon's* calçada *on π Day in 2025*‡

by **José Francisco Rodrigues***

At the proposal and initiative of the International Mathematical Union (IMU), the UNESCO General Conference, at its 16th plenary meeting on 25 November 2019, proclaimed the 14th of March of each year as the *International Mathematics Day*.

Among the considerations underlying this decision is the recognition that "global awareness of, and enhanced education in, mathematical sciences are vital to addressing challenges in areas such as artificial intelligence, climate change, energy and sustainable development, and to improving the quality of life in both the developed and the developing worlds" and that "the applications of mathematical sciences are vital for advances in all types of engineering and computer science, while responding to the growing needs of automation and providing access to information via the Internet (the World Wide Web) for the wellbeing of society", recognising "the importance of creating conditions conducive to gender equality in mathematical sciences by promoting successful female role models in science, from Hypatia of Alexandria to Maryam Mirzakhani, not forgetting Emmy Noether, Sophie Germain or Mary Winston Jackson."

Thus, in the third month of each year the 14th day was chosen for International Mathematics Day, also known as *π Day*, since 3.14 is an approximation of that irrational number, already known to Archimedes, the ancient mathematician who demonstrated that the number π is, in any circle, the ratio of the perimeter to the diameter and the ratio of the area to the square of the radius.

In 2025, the theme associated with mathematics for the celebration of π Day was *Art and Creativity*, announced in nine languages (English, Arabic, Chinese, French, German, Korean, Portuguese, Spanish and Turkish — https://www.idm314.org/) and implemented in 922 locations across all continents. In Portugal alone, there were 58 initiatives, of which the one held at the Academy of Sciences was one of four registered in Lisbon.

On the 14th of March 2025, the Academy of Sciences celebrated this symbolic date for the first time in its Noble Hall with two hybrid events: a conference [S] promoted by the Young Scientists Seminar (SJC), where academic Maria Ivette Gomes gave a lecture entitled *Statistics, the science of data*, followed by two presentations by members from that seminar, Emmanuel Cruzeiro with his presentation *Mathematical challenges in quantum foundations* and João Cancela with his presentation entitled *Between the illusion of accuracy and the fear of quantifying: the role of mathematics in the social sciences*; and the [P] ceremony for the presentation of the awards for the *Competition Symmetries of Lisbon's* Calçada.

In January 2025, the Academy of Sciences launched a challenge to students in grades 9 to 12 (corresponding to ages 14 to 18, approximately) from schools across the country to create proposals for the five missing symmetry patterns in Lisbon's *calçada*,[1] with the aim of completing the set of 24 flat symmetries in its streets and squares. A prize of one thousand euros to each of the five winners was announced.

[1] *Calçada* is a Portuguese cobblestone pavement, generally for pedestrian use, made from natural, small stones, varying in shape. (https://calcadaportuguesa.org/en/o-que-e-a-calcada-portuguesa-2/)

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* Academy of Sciences of Lisbon

Figure 1. The symmetry of the Mar Largo pattern in a contemporary photograph of Lisbon [1][2]

Portuguese pavement is a form of public art and an integral part of the national cultural heritage, which has spread throughout the country and around the world, from Brazil to Macau, passing through Africa. Its symmetries began in Lisbon in 1849 with the *Mar Largo* (Wide Sea) of Rossio geometric pattern, an expression from *Os Lusíadas*, from the eighth verse of stanza 66 of canto IV, regarding the succession of D. João II by D. Manuel, who "Took on the conquest of the wide sea".

The *Mar Largo* pattern, which still exists in Rossio to this day, despite being interrupted between 1919 and 1975, has one symmetry by reflection, like a mirror, and two symmetries by rotation, with 180º turns around two points. It is one of the 17 possible symmetries of crystallographic patterns of the plane, identified by the Russian mineralogist Evgraf Fedorov and German mathematician Artur Schönflies in the late 19th century. Added to these symmetries are the seven symmetries of the friezes, which can be characterised by human footprints, respectively, with steps in normal gait, with feet together, with both feet landing alternately facing forwards and backwards, with feet sideways, with both feet sideways and rotating 180º, with a hop on one foot and with a hop on one foot rotating 180º.

The fact that there can only be exactly 24 flat symmetries, and no more, is a mathematical theorem. In 2017, the city of Lisbon completed all seven types of friezes, but so far only 12 patterns have been identified, meaning that five symmetries are still needed for it to become the first city in the world to materialise this mathematical theorem in its streets and squares.

The Competition *Symmetries of Lisbon's* Calçada aimed to reward the best five proposals for the five missing symmetry patterns, each proposal being a unique motif, consisting of a distinct decorative ornament, the repetition of which forms one of these five patterns, namely the patterns with the symmetries *333, 333, 632, 22X and 0, which in crystallographic notation are represented, respectively, by p3m1, p3, p6, p2gg and p1 and, for the purposes of the competition, labelled symmetry A, B, C, D and E.

The initiative received immediate support from Ciência Viva, the Directorate-General for Education, Lisbon City Council, the Portuguese Calçada Association, Lisbon Tourism and the Ludus associations, the Association of Mathematics Teachers and the Portuguese Mathematical Society.

The jury, chaired by José Francisco Rodrigues (Academy of Sciences of Lisbon), was composed of Rosalia Vargas (Ciência Viva — National Agency for Scientific

Figure 2. The Rossio Square in Lisboa with the *Calçada Mar Largo* in a postcard circa 1900.

Culture), Ana Silva Dias (Cultural Heritage Protection Division of Lisbon City Council), Ana Cannas da Silva (Department of Mathematics, ETH Zurich), António Prôa (Portuguese *Calçada* Association), Pedro Macias Marques (Directorate-General for Education) and Ana Margarida Rodrigues (Portuguese Mathematical Society).

Although the competition period was relatively short, in about six weeks, 127 proposals were received from students across the country, some with multiple proposals, distributed as follows: 35 for symmetry A; 15 for symmetry B; 26 for symmetry C; 22 for symmetry D and 29 for symmetry E.

Assessing the mathematical correctness, artistic quality and feasibility of implementation on the calçada of each of the 127 proposals, after two meetings, the jury decided on the following allocation of the five prizes and seven honourable mentions:

Symmetry Award A

Eduardo Manuel Fontoura Rebelo Rodrigues, from *Escola Profissional de Recuperação do Património de Sintra*

Symmetry Award B

Mila Luana de Gouveia Loureiro, from *Escola Básica e Secundária da Cidadela, Cascais*

Symmetry Award C

Vladyslav Kravets, from *Escola Profissional de Recuperação do Património de Sintra*

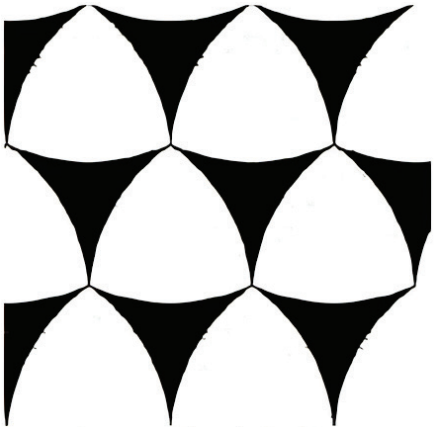
Symmetry Award D

Francisco Miguel Ledo Nazareth Pais Costa, from *Agrupamento de Escolas D. Filipa de Lencastre, in Lisbon*

Symmetry Award E

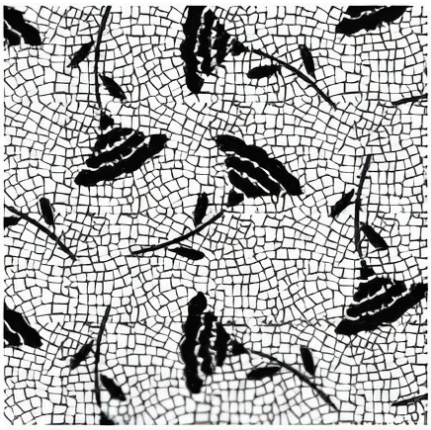
Sebastião José Pacheco Mendes, from *Escola Secundária de Alcácer do Sal*

A



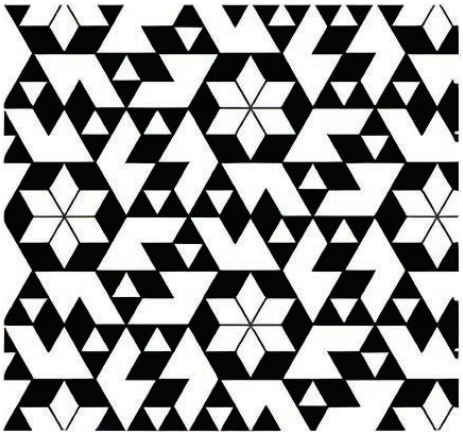
*** 3 3 3** por Eduardo Rodrigues
Escola Profissional de Recuperação do Património de Sintra
Sintra

B



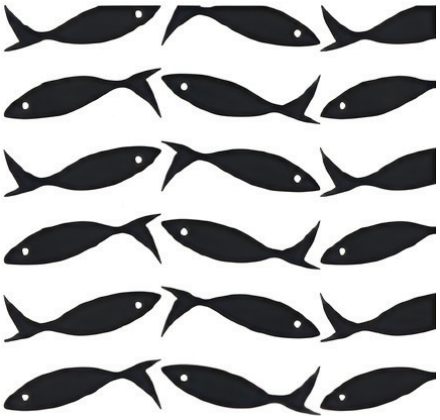
3 3 3 por Mila Loureiro
Escola Básica e Secundária da Cidadela
Cascais

C



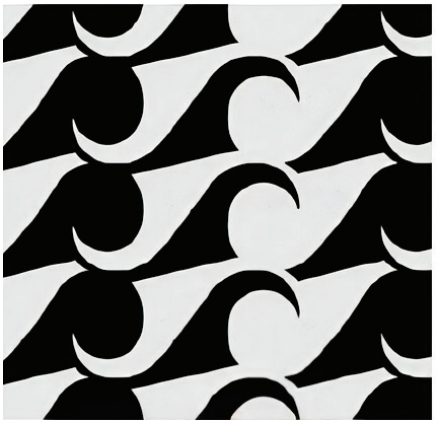
6 3 2 por Vladyslav Kravets
Escola Profissional de Recuperação do Património de Sintra
Sintra

D



2 2 X por Francisco Costa
Escola D. Filipa de Lencastre
Lisboa

E



0 por Sebastião Mendes
Escola Secundária de Alcácer do Sal
Alcácer do Sal

Honourable mention for symmetry A
Yixuan Wu, from *Escola Secundária D. Afonso Sanches, Vila do Conde*, and Vladyslav Kravets from *Escola Profissional de Recuperação do Património de Sintra*

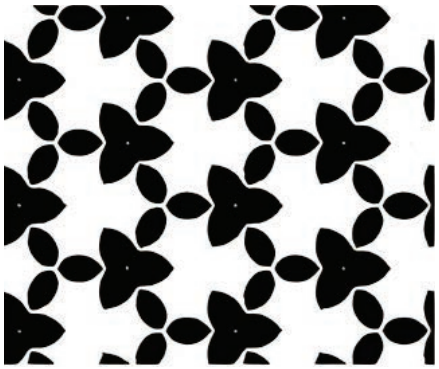
Honourable mention for symmetry B
Hugo Cabrita Henriques, from *Externato Marista de Lisboa*

Honourable mention for symmetry C
Mila Luana de Gouveia Loureiro, from *Escola Básica e Secundária da Cidadela, in Cascais*

Honourable mention for symmetry D
Patrícia Martins Pessoa, from *Escola Básica e Secundária Artur Gonçalves, in Torres Vedras*

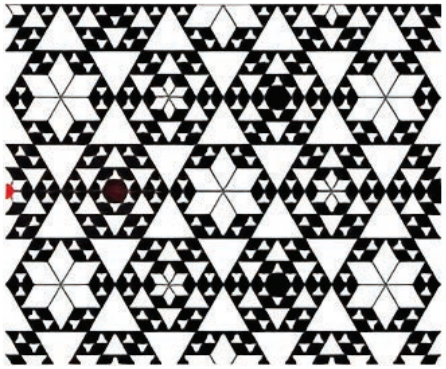
Honourable mention for symmetry E
Vasco José Catarino Neves, from *Escola Básica e Secundária da Anadia* and Hugo Cabrita Henriques from *Externato Marista de Lisboa*.

A



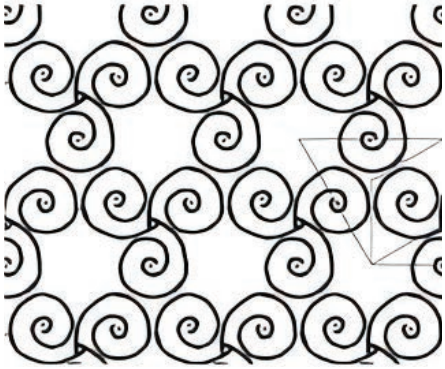
*** 3 3 3** por Yixuan Wu
Escola Secundária D. Afonso Sanches
Vila do Conde

A



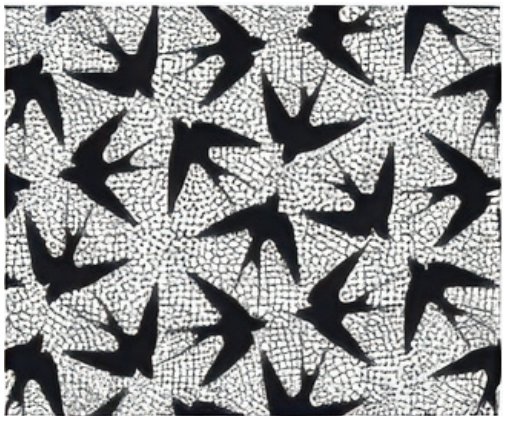
*** 3 3 3** por Vladyslav Kravets
Escola Profissional de Recuperação do Património de Sintra
Sintra

B



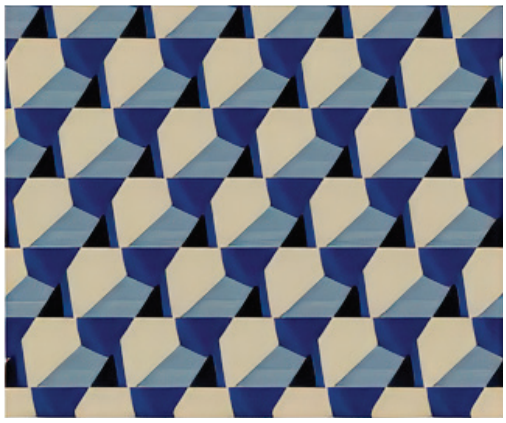
3 3 3 por Hugo Henriques
Extrenato Marista de Lisboa
Lisboa

C



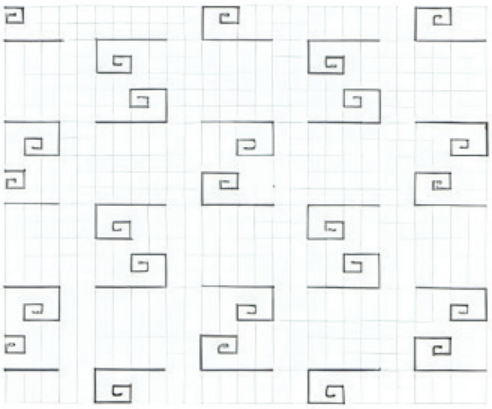
6 3 2 por Mila Loureiro
Escola Básica e Secundária da Cidadela
Cascais

E



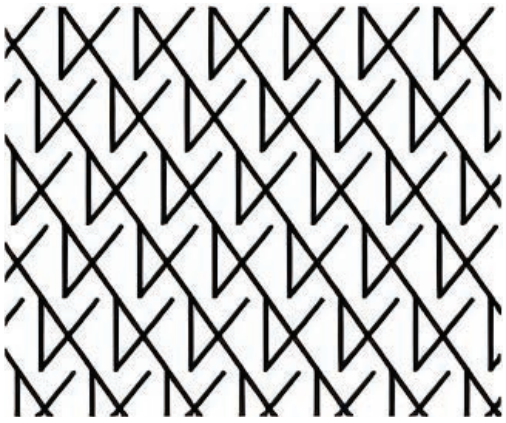
0 por Vasco Neves
Escola Básica e Secundária da Anadia
Anadia

D



2 2 X por Patrícia Martins Pessoa
Escola Básica e Secundária Artur Gonçalves
Torres Novas

E



0 por Hugo Henriques
Externato Marista de Lisboa
Lisboa

Figure 3. The winner of the C -Pattern Award, congratulated by the President of the Academy

This extraordinary collection of twelve proposals poses an enormous challenge for the city of Lisbon to complete the symmetries in its streets and squares and become the first city in the world to complete the 24 flat symmetries in its pavements. They also encourage the expansion to all regions of Portugal, where Portuguese *calçada* already exists or may come to exist, the dissemination and exploration of the algorithms of plane symmetry, whether through school activities or art competitions.

Bibliography

[1] Ana Cannas da Silva, Simetria Passo a Passo: Calçadas de Portugal, Edições CTT. 2016. https://people.math.ethz.ch/~acannas/Outreach/ctt simetria passo a passo apresentacao.pdf

[2] Ana Cannas da Silva, et al., As simetrias nas calçadas de Lisboa, Rev. Ciências Elem. V12(03) 025 (2024). doi: 10.24927/rce2024.025 https://rce.casadasciencias.org/rceapp/art/2024/025/

[3] Eduardo Veloso, Simetrias e transformações geométricas, Associação de Professores de Matemática, Lisboa, 2012. http://id.bnportugal.gov.pt/bib/bibnacional/1841208

[4] Paula Delgado e Paula Figueiredo, Calçada Portuguesa — A arte aos nossos pés. https://arquivomunicipal.lisboa.pt/publicacoes/documento-do-mes/detalhe/calcada-portuguesa

[S} https://www.youtube.com/watch?v=k4hGCUtBG1E

[P] https://www.youtube.com/watch?v=XgJggEU5jV4