Interview

DGS II 2013

CIM thanks the participants **Elvio Accinelli** [UASLP, Mexico], **Alberto Álvarez-López** [UNED, Spain], **Michel Benaim** [Université de Neuchâtel, Switzerland], **Mário Bessa** [Universidade da Beira interior], **Fabio Chalub** [Universidade Nova de Lisboa, Portugal], **Ana Dias** [Universidade do Porto, Portugal], **Orlando Gomes** [ISCAL/IPL, Portugal], **Clara Grácio** [Universidade de Évora, Portugal], **Filipe Martins** [LIAAD INESC TEC, Portugal], **Bruno Oliveira** [Universidade do Porto, Portugal], **Joana Pais**, Universidade de Lisboa, Portugal], **Alexandre Rodrigues** [Universidade do Porto, Portugal], **Luís Filipe Silva** [CIBIO Universidade dos Açores, Portugal], **Luís Silva** [ISEL Lisboa, Portugal], and **Paulo Vasconcelos** [Universidade do Porto, Portugal] of the International Conference and Advanced School Planet Earth, Dynamics, Games and Science II [DGS II], Portugal, 28 August to 6 September 2013, for sharing with us their ideas and points of view in this interview.

The questions presented here are based on several interviews, in particular, the interviews published in previous CIM bulletins. CIM thanks Renato Soeiro and Alberto Pinto for organizing this interview.

On the DGS II meeting

What is your general impression of the meeting?

Elvio Accinelli: These kinds of meetings are of great interest for making progress in different areas of applied mathematics, and they create networks on research topics of common interest.

Alberto Álvarez-López: I can talk about the DGS meetings II and III, held in Lisbon and in Porto, respectively. I found them very interesting. I met people who work in areas similar to mine, and I could hear some colleagues' opinions about my own work. In addition, I enjoyed them very much for their social aspects.

Michel Benaim: Very good. It was very friendly and gave me the opportunity to meet and discuss with researchers having different backgrounds and mathematical cultures.

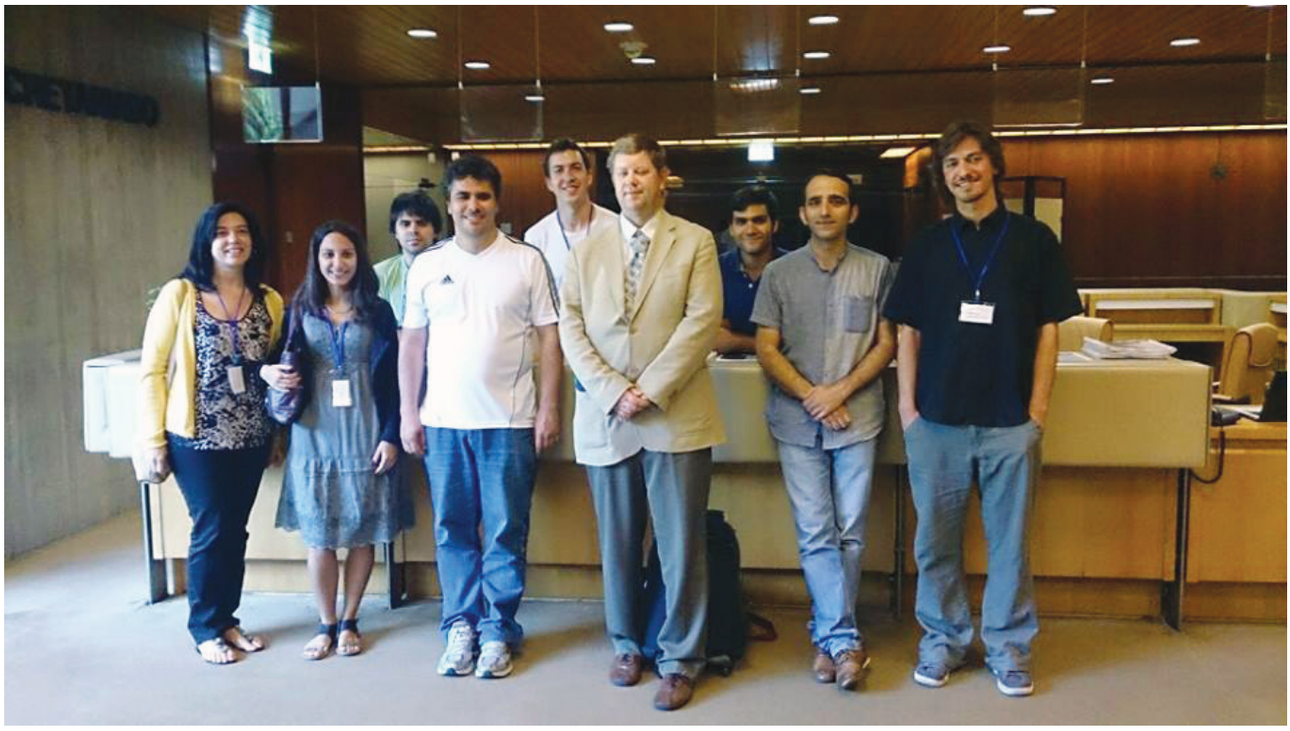
Mário Bessa: It was a good opportunity to meet several mathematicians working in related areas and develop some connections. I think that the Portuguese mathematical community should be more involved in this event.

Ana Dias: I found the meeting very interesting.

Orlando Gomes: The International Conference on Dynamics, Games and Science is, in my view, an extremely useful forum to discuss ideas and progress in research in a variety of fields concerning applied mathematics. In the events in which I have been present I have learned a lot about subjects on multiple areas ranging from evolutionary games to chaotic dynamics, stochastic optimization, and network analysis, just to cite a few.

Clara Grácio: I think this congress was an enjoyable opportunity to fulfill the objectives that I described in other questions as important for students and researchers who attended this event. I participated in the meeting held in September at the Calouste Gulbenkian Foundation. This meeting allowed us to talk to other colleagues, presenting our works in progress and discussing possibilities for continuing and improving that work, as well as future projects.

Filipe Martins: I think the Dynamics, Games and Science II conference was an amazing meeting featuring a wide range of topics and keynote speakers. My general opinion is that it was very well



organized and featured many brilliant presentations. I think these kinds of conferences are very important. For me, as a student, it was a huge boost in terms of encouragement to pursue a Ph.D., as I was finishing my Master's thesis at the time.

Bruno Oliveira: It was an excellent meeting where I had the opportunity to exchange ideas with many colleagues and learn from them.

Joana Pais: Very well organized. Amazing capacity of the organizers to put together an extremely interesting program, with a substantial group of well-known researchers. Very interesting talks, even though, in my opinion, they covered topics that were probably too diverse.

Filipe Silva: The importance of these types of meetings is the possibility of joining researchers who use mathematical tools in very different contexts, contributing to the transferability of knowledge between the different fields.

Luís Silva: The meeting was very interesting, bringing together an outstanding group of researchers, both domestic and foreign, in a fantastic and inspiring place.

Paulo Vasconcelos: The overall quality of the papers presented was great. The location was attractive, and the group lunches were full of life.

Something you would like to highlight?

Elvio Accinelli: These kinds of meetings are of great interest for making progress in different areas of applied mathematics and creating networks on research topics of common interest.

Alberto Álvarez-López: People from different "countries" of the world of mathematical applications could meet there, from pure mathematicians to biologists, economists, and engineers: a very interesting mixture. In addition, I would like to highlight the format of the sessions: short talks related to each other, which is perfect for cultivating interplay among senior and junior scholars. In fact, these events were a good opportunity for young researchers: they could show their own work and also listen to very relevant opinions from senior colleagues.

Ana Dias: The quality of the talks, the variety of the themes addressed at the talks, and the event

location, the Calouste Gulbenkian Foundation, all made these meetings very pleasant to experience.

Orlando Gomes: I believe that the strong feature of the Dynamic, Games and Science meetings is their interdisciplinary nature. With the use of mathematical methods as a unifying force, conferences offer a large variety of studies in a large variety of fields. Applications to economics and finance coexist with studies in themes relating to biology, ecology, or physics.

Luís Silva: I would like to highlight the quality of the plenary talks.

Paulo Vasconcelos: The intensive preparation by the organizing committee resulted in a smooth learning experience for the participants in a very pleasant setting.

How important do you think that events like this are for students and researchers?

Elvio Accinelli: Students in the process of completing their theses can find places to develop their research and to finish their work.

Mário Bessa: These types of meetings are quite important both for students and researchers because we have the chance of contact with related fields of expertise, thus gaining a deeper perspective on the application of our theoretical models in several different contexts.

Ana Dias: Very important not only for learning about new mathematical studies, but also for interchanging ideas, sometimes between mathematicians with different backgrounds.

Orlando Gomes: These events are a very good opportunity to share ideas, to learn, and to create networks among researchers. They are, of course, particularly important for young researchers who are starting a career by allowing them to present their work and establish the contacts they need to progress in their research effort. Graduate and undergraduate students have the opportunity at these events to have their first contact with the world of science.

Clara Grácio: In my opinion the scientific meetings are an excellent opportunity for researchers to present their work to their colleagues in order to receive feedback at an early stage of their research and are therefore an integral part of the process of science. These presentations also serve as informal reviews by peers, which may help researchers to develop, clarify, and improve their work and will no doubt help in the final phase of writing and submission to final publication. Also, and very important, the meetings allow researchers to hear about what others are studying, to develop relations with related disciplines by talking to colleagues from different institutions around the world, and to learn about new tools and research techniques that can be relevant to their work, other programs, and projects in common. These are truly scientific meetings arising in an academic environment where the questions and answers are natural, objective, honest, and fearless, and where the only goals are help, cooperation, and the development and dissemination of knowledge.

Bruno Oliveira: Events like this give researchers an opportunity to report their results to the scientific community. More importantly, in my opinion, they also open channels of communication between researchers, which enhances the work we develop. Regarding students, I think that these events give them a wonderful way of obtaining state-of-the-art knowledge from experts in these subjects.

Joana Pais: Very important. Research dissemination and networking are essential.

Filipe Silva: This might broaden their views, and make them see their daily research with different eyes.

Luís Silva: These kinds of events are extremely important both for students and for researchers. For the students they provide an excellent opportunity to make contact with senior researchers, learn about the most current issues, and even help them to decide about their future topics of research. These events allow the researchers to publicize their work and to exchange ideas with their colleagues.

Paulo Vasconcelos: The advanced school is an important meeting point for students with high level researchers, which can be a rare opportunity. Researchers enjoy the outstanding opportunity to publish proceedings within a prestigious and exigent editorial brand as well as participate in a book series devoted to applied mathematics.

How do you see the impact of this meeting on your field and outside of your field?

Michel Benaim: This type of meeting allows people with different backgrounds (game theory, dynamical systems, probability) but common interests (in the

present case "dynamics in games") to meet and is a good opportunity for cross-fertilization of ideas.

Fabio Chalub: Most of the meetings in the field of mathematics are "technique-based"; i.e., a number of professionals who have mastered the same techniques get together and discuss problems where these techniques were applied. This was a different kind of meeting in the sense that we had a large number of problems but no predefined mathematical technique. All areas of mathematics were represented in the conference, and the researchers could see where their expertise and abilities were required. This can forge a new generation of students who are more "problem-oriented" and who necessarily will learn more subjects, as opposed to the precocious specialization we see today.

Ana Dias: A good impact due, also, to the fact that some of the works will be published in a Springer book, which is a very good way of reaching readers from other fields.

Orlando Gomes: There are not many international quality scientific conferences or series of conferences in Portugal. This is a good example of a well-organized series of conferences that, I believe, has a good impact in promoting applied mathematics. As I see it, it is an interdisciplinary meeting with repercussions that go beyond mathematics; for instance, it is also an important event in my own research field, i.e., economics.

Joana Pais: I believe that, even though the impact on the field may be substantial, the outside impact is limited. This is not an exclusive feature of this particular event, but it is common to most (if not all) of the events of this nature. Clearly, it is a difficult exercise to translate the language of science into a language that the general public can understand. In fact, while there is no ambiguity in mathematics, when we translate mathematical language into words, our messages are probably not perceived the way we meant it. Still, disseminating scientific knowledge in the public sphere, particularly in the domain of social sciences, is important. It makes us think about why we believe that our research is necessary and useful.

Luís Silva: I think that this meeting may have a significant impact in the field, especially because this subject is relatively recent, and a meeting with this dimension of topics is not very common. In the particular case of Portugal, I think it presented a lot of subjects to several people.

What would you say is, generally, the impact of these events on specific areas, as they relate to and on the interplay between different areas or fields of knowledge?

Elvio Accinelli: These events are of great importance for creating networks between groups of different countries; consequently, they have a great impact on the work area as they allow one to learn about progress elsewhere.

Ana Dias: Good impact.

Orlando Gomes: This type of meeting is, as stated in previous answers, a way to promote the cross-fertilization of knowledge in various fields where game theory and dynamic processes matter. It is an extremely helpful event for all those who want to develop competence and explore new territories in applied science. New research projects, of an interdisciplinary nature, will certainly arise from the contacts researchers make in these conferences.

Bruno Oliveira: Of benefit to both students and researchers was the fact that this meeting covered a broad area of subjects in mathematics, in particular dynamical systems and game theory, and an even broader area of applications in the sciences, presenting research in several distinct topics of, for instance, economics and biology. This diversity can build bridges between different problems, allowing the attendees to further improve their work.

On your research:
Did you always want to be a mathematician?

Alberto Álvarez-López: Well, when I was a child, besides math I also liked language (I mean grammar and so on). But to tell you the truth, I always wanted to be a mathematician. Anyway, upon finishing my Bachelor's degree in Mathematics, I landed a position in a faculty of economics. Through the years I have discovered a wonderful field in which to apply mathematics that is very rich and interesting by itself!

Fabio Chalub: In fact, I graduated and received my Master's degree in Physics. During this time, I followed as many disciplines in mathematics as I could, and I got the impression that the most fundamental results in physics could only be entirely appreciated with a deep understanding of the mathematics behind them. In the end, I decided to do my Ph.D. in Mathematics involving the work on the border between math and physics.

Ana Dias: Looking back, my answer is yes.

Orlando Gomes: I am an economist, with research interests related to the mathematical modeling of economic phenomena. Economic processes have always fascinated me, and I believe that mathematics is necessarily the language through which economic events can be rigorously addressed and explained. My interest in modeling socio-economic events goes back to my undergraduate studies in economics (more than 20 years ago).

Clara Grácio: As we know, mathematics can be, sometimes, frustrating indeed, but it is in this struggle where the challenge itself lies. You experience a sense of accomplishment, even contentment, when you discover the missing piece of the puzzle, and mentally exclaim: That was it! Also, when you can establish unsuspected relationships between different areas of mathematics and/or other sciences, the coherence, connection, and immensity of mathematics emerge. I always liked the interconnection between the various areas of knowledge, from language or history to physics or biology, the wealth that allows us to move forward. And in order to advance in this way, mathematics is essential and indispensable. To the question of whether I always wanted to study mathematics, the answer is that mathematics has always been the first choice as long as the studies allow the monitoring of other areas.

Filipe Martins: I only thought of being a mathematician very recently. I decided to study mathematics as an undergraduate just two months before the start of the academic year. It was a pretty quick decision. I was trying to choose between mathematics and physics. The decision was taken completely by impulse, in 5 minutes.

Bruno Oliveira: It's a yes and no answer. Ever since I was young I had a fondness for mathematics. Later I gained an interest in physics, informatics and astronomy (from watching the TV series Cosmos by Carl Sagan). So, mathematics was always there, but linked to other sciences.

Alexandre Rodrigues: No, I did not always want to be a mathematician. In fact, I do not consider that I am a mathematician. I prefer to say that I am a researcher in mathematics. After completing my undergraduate degree I was convinced that I would like to be a high school teacher, but my desired career direction became clear while I was pursuing my M.Sc. degree. Even during that stage I considered exploring a different subject and switching to physics.

Filipe Silva: No, I always wanted to be a biologist, considering that life is probably the most complex and evolved form of matter/energy in the universe. However, during my research and teaching activities, I became progressively aware of the importance of using mathematical and statistical tools in biology, and in science in general.

Luís Silva: No. During most of my time in secondary school I was convinced that I wanted to be a psychologist.

Paulo Vasconcelos: Not always . . . but almost always!

How did you start working in this area? What was the motivation? Could you tell us about your mathematical beginnings and subsequent career development?

Elvio Accinelli: Motivated by social problems, I felt a vocation for economics. In the last year of primary school my teacher made me see that mathematics could be an excellent tool for thought. Later, when I was in prison as a political prisoner, I met José Luis Massera, who greatly influenced my thinking. Some years later, in the IMPA I had the opportunity to learn mathematical economics. Since then I feel real pleasure working in this area.

Michel Benaim: I have always worked at the interface of probability and dynamics. My interest in game theory started in Nefeli Cafe, a coffee shop located in Berkeley, near the math department, 20 years ago. At this time I was working with Moe Hirsch on some applications of topological dynamics for investigating the long-term behavior of certain stochastic processes called "stochastic approximations." A friend of mine, Paolo Ghirardato, at this time a Ph.D. student in economics, suggested that I look at a preprint by Drew Fudenberg and David Kreps on "stochastic fictitious play." It turned out that the techniques I was developing with Moe Hirsch proved to be very useful for analyzing stochastic fictitious play and more generally leaning processes in game theory.

Mário Bessa: After I finished my Bachelor's in Mathematics at the University of Porto, a colleague of mine asked me if I would like to go to some informal conversations about mathematics, taking place once a week, with Professor Jorge Rocha at the University of Porto. Since Jorge Rocha is a dynamicist, I started

learning about this area, and immediately I began to enjoy dynamics. Then, I finished my Master's thesis in dynamical systems with Jorge Rocha and I went to IMPA for a Ph.D. program with a thesis also in dynamical systems, supervised by Marcelo Viana. Finally, I returned to Portugal where I completed six years of a post-doc program and taught at the Polytechnic Institute of Coimbra. Now, I am an associate professor at the University of Beira Interior.

Fabio Chalub: During my Ph.D. study, I followed a course in the mathematical models used in ecology and, immediately after that, some colleagues and I started a discussion group in math-biology. I became fascinated with the topic and decided to work on it during my post-doc, in Vienna. I studied models for cell motility and had some relevant results during that time. I also enjoyed the fact that the math-biology group in Vienna is very well established, and I could learn new topics. At a meeting in Vienna, I met Jose Francisco Rodrigues, from the Lisbon University, and he told me about a post-doc position in Lisbon and his particular interest in starting a group in mathematical biology. I went to Lisbon intending to stay 2 years, but after a few months my wife and I were seeking opportunities for a longer stay. This was 12 years ago! In 2005, I got a position at Universidade Nova de Lisboa, and since then I have been there, first as an assistant professor, then as an associate professor, and now as an "investigador FCT" researcher.

Ana Dias: Professor Isabel Labouriau introduced me to the area of dynamical systems for my Master's thesis.I would say that the contact with Professor Isabel Labouriau in the Applied Mathematics Department and the job I got at the University of Porto were the main starting points in my becoming a mathematician. Any trip to any place for work has a story, and when we return we bring memories. For sure my period at Warwick University during my Ph.D. study was the most important period of my research, because during that time I found out what I really liked to work on, and my supervisor, Professor Ian Stewart, had a fundamental role in that discovery.

Orlando Gomes: My work in theoretical economic research started with my Master's course (1995--1996). The possibility of approaching economic processes through the use of mathematical tools, namely dynamic systems (linear and nonlinear, deterministic and stochastic, in discrete and in continuous time), fascinated me, and I have pursued studies in this area ever since. The first models that I approached related economic growth processes. Economic growth was the theme of my Master's thesis and of my Ph.D. thesis (which I completed in 2002). Later, I diversified my studies to areas that involve business cycles, monetary policy, international trade, individual decision-making, social interaction, and others. The common denominator of all this research is related to the use of tools of dynamic analysis and dynamic optimization.

Filipe Martins: After my undergraduate studies I had no real idea about the nature of research in mathematics, but after three years as an undergraduate, I decided to undertake a Master's degree in Mathematics, specializing in statistics and probability. Really, I only became more aware of research in mathematics when I was working on my Master's thesis. I liked it very much and noticed that to continue research in mathematics could be a good idea, and then I started thinking about taking a Ph.D. in the subject, and, happily, I got a Ph.D. scholarship. I would describe my areas of interest concisely as applied mathematics, which is what I like. What I studied for my Master's thesis was financial mathematics. Now I'm continuing on that topic, but I am also working on applications of dynamical systems to biology and economics. Again, the best way to designate it is applied mathematics. There is a wide range of topics for future work in this area. The rate at which work possibilities arise in applied mathematics is far greater than the rate at which you solve them. For each one you work on, a lot more appear as possible continuations. My favorite theorem in mathematics is possibly Banach's fixed point theorem.

Bruno Oliveira: After my degree in Astronomy, I completed a Master's degree in Applied Mathematics and, later, a Ph.D. in Applied Mathematics. My motivation has been a desire to understand how things work: from the universe to quantum mechanics, passing through humans in diverse subjects such as immune responses by T cells, price formation in random markets, firms competing with investment in R&D, children's growth, dietary patterns, or obesity treatment. And the tools that I have been using are rooted in mathematics, in particular, dynamical systems, game theory, and statistics, with links to computer modeling, and also requiring my knowledge of physics when studying interaction phenomena. In my career, I have taught subjects in astronomy, physics, and biostatistics. In particular, in these latter years I have been teaching biostatistics applied to nutrition and food sciences, which led me to do a Habilitation in Basic Sciences of Clinical Nutrition.

Alexandre Rodrigues: I really started my work in this area during my Ph.D. study, as after my M.Sc. it became clear to me that I really wanted to do research in dynamical systems. At the beginning, the motivation was the challenge of completing a Ph.D. in Mathematics. I remember quite well that I had two main concerns. (i) Could I discover something new in mathematics? (ii) Could I develop some important step towards an open problem? In fact, I do not know if I have achieved these goals. The main motivation was to complete a Ph.D. in Mathematics in a subject that I tried to pursue during my M.Sc. At the time, it seemed unattainable.

Filipe Silva: Working mostly in quantitative ecology, I became more and more interested in the use of statistical models to describe ecological phenomena. I became aware that statistical thinking evolved in close connection with biology and other sciences, and that its historical evolution had a parallel in the development of the other sciences. I also became involved in teaching biostatistics and quantitative methods to different student at levels, which further developed my interest in the area.

Luis Silva: My main motivation came from J. Sousa Ramos. He taught Introduction to Computation in my first year, and he had an uncommon point of view about that (and any other) discipline. He strongly believed that the students should be challenged from the beginning, so in the first classes he presented us with some of the most important math problems of that time: Fermat's theorem, Collatz, P/NP, and Poincaré's conjecture, etc., then he taught us Pascal and stimulated us to start exploring Julia sets, Mandelbrot sets, the Lorenz attractor, and so on. I think that he was mainly responsible for my decision of trying to be a mathematician instead of a high school teacher. Then I finished my undergraduate work and immediately got a job as assistant professor at FMH-UTL. At the same time I started a Master's study at IST-UTL and made my thesis with Sousa Ramos, then changed to the University of Évora, then finished my Ph.D. with Sousa Ramos again, and after ten years came back to Lisbon, to ISEL, where I am now.

How would you describe the essence of your own research to a young student?

Elvio Accinelli: Mathematical economics is both an intellectual challenge and an important tool for understanding the economy, for better social development.

Mário Bessa: Well, first I would describe how dynamical systems is not exactly an area but a confluence of several areas and so offers a good opportunity to study different aspects of mathematics. Then, I will emphasize that dynamical

systems problems are often easy to formulate and to understand, although they are usually hard to solve. I would also like to say that working in dynamics is quite amusing, because our objects are continuously changing when time evolves and we should be aware that intuition frequently tricks us. Finally, I really like to work with my co-authors, because we then enjoy enormous creativity. Indeed, when we try to explain to each other the questions, problems, solutions, and arguments that we are interested in, again and again we say to each other, "Imagine that . . .".

Fabio Chalub: I work in applied mathematics; therefore, I decide the problems I want to solve, but I do not decide the mathematical techniques necessary to solve them. My general interest is in population dynamics, and currently I work on two fronts: population genetics and epidemiology. In the former case, I am interested in exploring the mathematical richness of widely used models. In the latter case, we study the interaction between deterministic models and human behavior, in particular, how the course of an outbreak is affected by changes of behavior in the society. Sometimes, we find predictions in models that were not known; other times we find that some consequences do not follow from the models, contrary to the general belief; finally, we provide solid grounds for the models that appear in the literature and explicitly show their limitations. Our main goal is related to the conceptual understanding of the area, not to providing better models for specific problems.

Ana Dias: When we have interactions between units that are evolving with time, there are consequences for the dynamics that come just from the fact that there are interactions.

Orlando Gomes: I would say that economics is the field of knowledge where one can most successfully apply mathematical rigor to human decision and human action and that this is a fascinating combination independently of the type of phenomena under examination, this being of a micro or of a macroeconomic nature. Furthermore, I would say as well that my studies address dynamic processes in economics, because time is the most fundamental variable in this science; all economic issues necessarily involve a temporal dimension.

Alexandre Rodrigues: I work with dynamical systems. Roughly speaking, a dynamical system is a concept in mathematics where a rule describes how a point evolves (in time) in a geometrical space. The evolution rule may be given by the solution of a differential equation. Finding the explicit solutions of these equations is, in general, impossible. Sometimes these equations have some additional structures: algebraic symmetries which might help us to understand the qualitative behavior of the system. Heteroclinic cycles are a common feature of symmetric differential equations and persist under perturbations that preserve the symmetry. The dynamics near a heteroclinic cycle are well known and it is characterized by intermittency: a solution remaining near the cycle spends long periods of time close to a particular kind of sets and makes fast transitions among them. The rigorous analysis of the intermittent dynamics associated to the structure of the sets close to heteroclinic cycles is an exciting and challenging field of research. The characterization of the dynamics near these kinds of cycles is what I have been studying.

Filipe Silva: The fascinating idea of being able to see parts of the complexity of biological entities reflected in much more simple models, resulting from the systematic but creative activity of human mind.

Luís Silva: I work in symbolic dynamics; basically, I study the combinatorial aspects of dynamical systems.

Which would you say are the most interesting/challenging open (or recently solved) problems in your area, and what do you think the future holds in your area and in your line of research?

Elvio Accinelli: I think that understanding how the markets work could be helpful to obtain a sustainable development of mankind. The mathematical economy is a path toward that goal.

Alberto Álvarez-López: Roughly speaking, I work in elaborating mathematical models to describe some aspects of economic behavior, especially in the presence of uncertainty. Of course, there are many problems under this umbrella to be studied. I point out a very general one: we agree that the economic agent (a consumer, a firm, etc.) is not rational; well, I think a new non-rational theory describing his/her behavior is necessary — I mean a completely new theory, with a very different approach.

Mário Bessa: My preference goes to the well-known "closing lemma" problem. This is a question that dates back to seminal works of Poincaré on celestial mechanics. Like I told before, this is a good example of a problem that is easy to formulate

as you will see: if an orbit returns near to a place where it was before, is it possible to perturb the system in order to close the orbit? Of course, several aspects should be clarified; for example, what do we mean by "perturb"? Indeed, closing orbits requiring coarse approximations are well established; however, the problem is very hard when we demand finer approximations. If the requirements on the approximation increase too much, then it is known that the closing lemma has no solution!

Ana Dias: In my line of research on dynamics of coupled cell networks, I would say that it is important to have a theory for coupled cell networks like there is one for symmetric dynamical systems based on representation group theory.

Orlando Gomes: Since I study economic problems in general, I believe that although this is a very active science that has produced many meaningful results and advances in the last few decades, there are still many open questions. In macroeconomics, for instance, the permanent conflict between neoclassical and Keynesian economics and the difficulty in handling concrete aggregate problems (such as high rates of unemployment and deep recessions) reveal that much work still has to be developed in order to reach a unifying macroeconomic theory. At the micro level, a well-established theory of decision and individual behavior based on revealed preferences is now being challenged by advances in neuroscience, which indicate that one must go beyond the effective choices of economic agents and focus on the processes inside the human brain that trigger the decisions.

Alexandre Rodrigues: We do not know persistent classes of dynamical systems for which there is a set of positive measure which consists of initial points of orbits with historic behavior. For special dynamical systems, i.e., with boundary or with symmetry, historic behavior may persist. The main problem, however, remains open for dynamical systems without such constraints. In this context, R. Bowen described a system of differential equations on the plane whose flow has a heteroclinic cycle consisting of a pair of saddle equilibria connected by two trajectories. The eigenvalues of the derivative of the vector field at the two saddles are such that the cycle attracts solutions that start inside it. In this case each solution in the domain has historic behavior. Breaking the cycle, the flow loses this feature. This type of behavior may become persistent for dynamical systems in manifolds with boundary or in the presence of symmetry.

Filipe Silva: There is considerable excitement about the growing use of Bayesian statistics in different fields of biology. But, the future might bring new conceptual developments that will link or eventually merge frequentist and Bayesian statistics.

How do you see your area in terms of its importance in mathematics and in other fields of knowledge, the impact on and from other areas, and how do you expect this interplay to develop further?

Elvio Accinelli: I think that economic theory is in actuality a source of challenges for mathematics, whose resolution can achieve progress of both sciences. I would venture to say that economic theory, at present, can be as important for mathematics as it was physical in the nineteenth and early twentieth centuries.

Mário Bessa: Since the area of dynamical systems is a junction of several areas, there is intrinsically a large connection between mathematical subjects that are sometimes apparently unrelated. Moreover, its relation with other sciences (life, exact, social, computer, etc.) greatly enlarge these types of interactions. I believe that nowadays the classical nomenclature of dynamical systems is also used in other areas and turns out to be part of the language of these fields.

Fabio Chalub: The importance is growing a lot, in the world in general and in some particular countries like the USA, UK, France, the Netherlands, Spain, Germany, Sweden, and others. Fortunately, Portugal is no exception. It is still difficult to go from the theory to real applications, as this cannot be done by the same person or even the same groups. We have to talk to people with completely different backgrounds, and this is not easy. Generally, it is not difficult to get funding from government agencies, but for young Ph.D. graduates it is still difficult to find positions, as most of the mathematicians do not see "mathematical biology" as an area differently from "mathematical physics." It is seen as a topic of research, but not as a division of mathematics, like algebra, geometry, or analysis.

Ana Dias: As most real-world applications are governed by dynamics that can be interpreted as units interacting, any theory for coupled cell networks

that develops model-independent kinds of results is important and of interest for science in general.

Orlando Gomes: The nineteenth century philosopher Stanley Jevons once stated that if economics is to acquire the status of science, it needs to be a mathematical science. In fact, since then, the studies that contributed to the undeniable self-affirmation of economics as an autonomous scientific field have essentially adapted tools, concepts, and techniques from mathematics. Game theory, differential calculus, linear algebra, recursive analysis, optimal control, and other powerful instruments provided by mathematics have contributed to build what economic science is today. Moreover, some mathematical concepts were created and developed as specific tools of the economic theory and then served other fields of knowledge as well. The interplay between mathematics and economics is a fruitful one, and it will certainly be explored in more depth in the future.

Paulo Vasconcelos: Computational mathematics is crucial for applied mathematics. Bringing mathematics to solve problems is the ultimate purpose of our research. Other fields of knowledge depend on the knowledge transfer, and there is nothing like computers to help simulate natural, physical, chemical, or even human processes.

Do you have a favorite result, your own and/or from others?

Elvio Accinelli: Yes, my favorite result is the explanation of the economic crisis as the result of small perturbations on the fundamentals of so-called singular economies.

Ana Dias: My favorite result is on ODE-equivalent networks and concerns the idea of different graphs leading to the same kinds of dynamics — they just have be linearly equivalent: a nonlinear result that has a linear question. The part that I like more in my work is the fact that every time we have a problem, we have a challenge in hand that we try to address. When we have success, it is a good feeling: the feeling of contributing to science with something, even if it is a small contribution.

Orlando Gomes: There are many powerful and appealing results in economics. Personally, I am a fan of the so-called Ramsey growth model: a simple and elegant optimal control problem that indicates how a representative agent chooses, in an intertemporal perspective, how to optimally allocate resources between consumption and savings, in order to maximize expected utility.

Is it difficult to get funding for research in your area?

Ana Dias: Until now not so difficult. The amounts asked are not so much compared with other research areas, so that might help.

Orlando Gomes: In the last few years in Portugal it became, in my view, difficult to get funding for doing research in any scientific area.

Clara Grácio: Research and higher education have been maltreated in recent years, for decades, with an unacceptable government underfunding which translates into immense difficulties for both higher education institutions and research centers and institutes of state laboratories. Even with the dedication of Portuguese researchers, integrated or not, this policy did not allow the scientific development that would have been possible, resulting in wasted potential and resources. Combined with a real reduction in funds invested in vacancies for teachers or researchers in institutions of higher education, laboratories, and others, there has been a lack of coordination and a lack of transparency and programming in resources invested. Mathematics is no exception, and in this sense is not easy to get funds for the development of scientific work.

Filipe Silva: Yes, it's easier to get funding for applied research, such as the study of forest resources, than for more fundamental research, for instance that devoted to new methods. We thus try to mix both.

Luís Silva: Yes, but unfortunately that problem is not restricted to my specific area. On the contrary, in Portugal it is generalized to the majority of scientific activities.

Paulo Vasconcelos: Since part of my research depends on new computer architectures, yes, it is very difficult, especially in Portugal, where we do not have state laboratories or research centers with high-end machinery.

On research, more generally: What would you say are the most important things to keep a research group going?

Elvio Accinelli: A common interest in the research topics and the possibility for all team members to develop their lines of work.

Alberto Álvarez-López: Keeping in contact (personal if possible) for discussion, holding brain-storming sessions, a good coordination among members, deadlines to have the work done... I do not know if they are the most impotant things, but they are useful.

Fabio Chalub: All members should be engaged in the research, so it is crucial to find a topic of general interest that involves everybody in the production of results. We cannot think of our colleagues, even if we are leading the group, as a bunch of employees. Everybody should have autonomy to produce their own results, and be judged by the quality of the output produced. This is the case in mathematics and other more theoretical subjects; however, I am perfectly aware that we cannot apply this policy to run a lab.

Ana Dias: Not to stop and to have people that really like what they are doing. Another thing is that people have to respect each other's work.

Orlando Gomes: A common goal, the capacity to work with others and to accept their criticisms, and gaining the notion that one is contributing to the advancement of science.

Clara Grácio: Respect for the successes obtained by each of the elements of this group but fundamental support at certain times, less good, that each of the elements can benefit by. Transparency, quality, and consistency are important in defining the group's strategic line, making it a key element. When these features come together, the group is a team, it is a school. I had the privilege of belonging to one of those rare schools, coordinated by the very bright (in all these respects) Professor Sousa Ramos.

Bruno Oliveira: Motivation. People should like what they are doing and feel that their work is recognized within the group.

Filipe Silva: Leadership, commitment, cohesion.

Luís Silva: In the first place, people must trust and respect each other; then I think it is very important to define a leader for each task.

Paulo Vasconcelos: Focus, dynamics, and a good working environment.

How do you see the relation between traveling and research?

Elvio Accinelli: It is very important to travel and see the results that other people have obtained. Travel expenses are one of the better investments in research, even if the results are not displayed immediately.

Alberto Álvarez-López: Well, if you do not have an assistant to arrange the details . . . organizing travel consumes energy. Anyway, I find traveling a very good way to meet colleagues and interchange ideas. In

some workshops social aspects are very important: scholars are persons in the end, and they need to talk and share opinions and ideas with other persons.

Michel Benaim: Traveling is a good way to meet people and develop new research. It's often much easier to talk with someone in front of a blackboard rather than to read a math paper. However, with emails, skype, and other communication techologies things are changing rapidly, and traveling is not as important as it used to be.

Ana Dias: It is important, although now there ways to interact without having to travel that are also good, not expensive, and save time.

Orlando Gomes: Research is many times an individual and solitary effort that we make in our offices or homes, but no meaningful research contributions gain life without a discussion with others. Our colleagues can help us improve our original ideas and assist us in transforming them into relevant scientific results. The meeting between researchers in the same or in related fields is a fundamental stage of any scientific endeavor. Therefore, it is my opinion that the participation in conferences in seminars and conferences around the world is a key step for the progress of science.

Bruno Oliveira: Traveling to meet other researchers and present our results is the best way to get feedback from our research. I have made big steps in my work after speaking with others about what I have found and after hearing from others what they have found. The positive input can come from new results by others, different methodologies to apply to our work, or a simple change of perspective that will allow new insight into a problem.

Joana Pais: Even though technology for communicating with other researchers is available nowadays, so that communicating is extremely easy and virtually costless, I believe that traveling, whether to attend conferences or to visit other researchers, is essential.

Luís Silva: Particularly for young researchers, the contact with different research teams can be particularly beneficial, particularly when different skills can be developed in this way.

Filipe Silva: It is extremely important. Nowadays we have easy access to a huge quantity of information, but there are lots of things that are much easier to learn in a good conversation than by reading books or papers.

Paulo Vasconcelos: It is good in a very natural way. Research is widespread. A researcher needs to communicate with others, not only to share his research and to broadcast, but also to gather expertise from other colleagues in the field.

On teaching:
What do you think about the relation between teaching and researching?

Alberto Álvarez-López: I think there are three main aspects to our task as scholars: research, teaching, and simply studying (knowledge in itself). Every one of us shares these three aspects in some proportion. The system should allow someone with a strong proportion in one of them (any of them, with no prevalence) to feel comfortable. However, this is not always true. On the other hand, we sometimes have a fourth task: the administrative labor — and this is often the first task. Anyway, I do find that my courses are richer if there is a research related to them.

Ana Dias: Good.

Orlando Gomes: They are, undoubtedly, complements. The creation and the diffusion of knowledge are two sides of the same coin: without research, no knowledge would be available to pass to students; without any one to teach, research would be simply useless.

Joana Pais: I used to believe that research helped to improve the quality of teaching. While I still believe this can be true when we talk about teaching at the advanced/graduate level, it is certainly not the case at the undergraduate level, where we have very good teachers that do not do research. The positive effects of teaching on research are even more difficult to grasp.

Filipe Silva: It's crucial; it really is a dialectic relationship, with many ideas and skills developed in one activity, easily transferable to the other.

Luís Silva: I think that the majority of the fundamental research should be done in the universities and that all university teachers must do research and that the majority of the researchers also should teach. On the other hand, I think that the university career should be more flexible in the sense of permitting large periods for doing just one of these two things. Nowadays we feel permanently pressed to do both things simultaneously, and I don't think that this is good for either of the two activities.

Paulo Vasconcelos: A teacher without research cannot convey a message of future, of challenge.

Any thoughts on what's crucial for a university teacher and or student?

Alberto Álvarez-López: You have to find pleasure in studying. And you have to learn to say "wait a moment, and let me analyze that," instead of giving a quick "yes" or "no."

Ana Dias: A good and enthusiastic teacher and a good and enthusiastic student.

Orlando Gomes: For both, the curiosity, the will to learn, and not being afraid to make mistakes.

Filipe Silva: A never-ending curiosity and the will to continue learning.

Luis Silva: Planning.

Alexandre Rodrigues: In a few words, I would say that a teacher should view a classroom as a pool of potential researchers and honor students. Students bring enthusiasm and a fresh perspective to our research. There is always the possibility that questions that come up in class will inspire new directions for our research. I find that stimulating interaction, encouraging independent thought, and accepting criticism are crucial in a classroom. And one should have a sense of humor — students love it. Technically, I believe that a teacher should give to the student a sense of the field, its past, present, and future directions, and the origins of its ideas and concepts. He/she should present facts and concepts from related fields. Theoretically, these are achievable goals; nevertheless, I realize that combining all these points might be difficult.

Paulo Vasconcelos: The duality research/teacher is difficult to keep equilibrated. In reality, usually teaching hours may be counterproductive for academic progression.

What are your thoughts on the relation between high school and university in terms of education?

Alberto Álvarez-López: I do not find them, at least in my country, as close as they should be. In mathematics, for instance, there is a gap between the level in high school and the requirements in university, especially in some grades. This causes a delay in the correct evolution of students. The high school teacher is not necessarily the guilty party: from the university we must better connect with him/her. Anyway, a deep change in the educational scheme should be considered.

Ana Dias: So and so. There is not a smooth transition between the two.

Orlando Gomes: The university should, more than any other school level, be capable of showing to students that what they learn, how they learn, and what use they make of this learning are essentially in their own hands and dependent on their own will.

Bruno Oliveira: In Portugal, university admission is based on high school grades, and the method of evaluation places too much emphasis on memorization to the detriment of problem-solving skills. I think that the system should aim to guide the students to the degree that is more fitted to their skills.

Filipe Silva: In Portugal I presently feel a considerable gap between those two levels. It's probably not a matter of the amount of knowledge that students have, but it is the way that they face their studies. It takes them all of the first year at the university to adapt to their new habits and to eventually develop a new, more independent way of studying.

Luís Silva: Particularly in mathematics, the relation was too bad for too long. Over a long time, the high school programs changed, and the university programs for the first years took too long to adapt. At the same time students arrived at the university poorly prepared, and people from the two school systems have had great difficulty in getting together to talk about what to do.

Paulo Vasconcelos: Completely wrong. Schools tend to prepare their students to take exams so that they can enroll in good universities. But critical thinking and creativity are neither exploited nor encouraged.

Do you have any advice for students starting their research?

Elvio Accinelli: Courses must be completed within the scheduled time, and then one should begin and continue working on the thesis without interruption until it is finalized. In general, those who leave their thesis for a time will fail to finish.

Alberto Álvarez-López: Yes: prepare a question (or a list of questions) to be answered. The question should be interesting. The answer should be relevant as well as technically correct.

Mário Bessa: Be persistent, resilient, curious, patient, and especially be able to scribble through large amounts of paper with flaws, mistakes, and wrong computations. Never believe that your supervisor has a magic wand to answer your questions and solve all your problems. It is you who should make the magic wand!

Ana Dias: They should try to work in what they like.

Orlando Gomes: Enjoy it. If you plan to go into research thinking only about career or monetary rewards, do not do it. You will need to have a passion for knowledge, or else you will feel frustrated.

Bruno Oliveira: Having a degree or a Ph.D. in an area does not mean that you will do the same thing for the rest of your life. You can use the expertise you have obtained in one area and apply it to a different one. The interface you create can be extremely rich in content and very motivating to explore.

Alexandre Rodrigues: A Ph.D. in Mathematics takes several years, and unless you really want to do it for its own sake, you will probably drop out at some stage. The four years of Ph.D. work can be very frustrating — you need real determination to stick to a handful of projects and get the job done. You should be completely sure that you love doing research in that specific field. You will enjoy it sometimes, but other times it will be very frustrating. It is, in general, solitary work; you speak to a few people including your advisor, but it is still solitary. The results will be unconvincing many times; basically, you will end up with a thesis for which only a few individuals in the world can assess the exact value. If you have started your Ph.D., do not give up. Make an effort to make the difference; be really good. Even when the proof of a result is already given, try to do it by yourself.

Filipe Silva: I consider it a privilege to be able to devote our lives, or at least a part of them, to research, that is to try to better understand our world. Also, research activities have the potential to develop scientific reasoning and many other skills (e.g., persistence, creativity, statistical reasoning) that can be useful in other fields of activity.

And for the ones who are hesitating between pursuing a Ph.D. and looking for a different job?

Alberto Álvarez-López: Well, If you like to study, if you really like to work hard studying, go ahead with your Ph.D. The job of a scholar is one of the best you can choose, in the sense that almost everything you do is a direct "investment" in yourself. There are, of

course, several contras: the wages are usually low, labor promotion is sometimes difficult, you work a lot of hours, bureaucratic tasks often feed you . . . .

Ana Dias: They should try to do what they prefer.

Orlando Gomes: It is a matter of vocation. There are many appealing and well-paid jobs that do not require a Ph.D. It is all a matter of making the choices that we are most comfortable with.

Filipe Silva: I don't like to push students into academic activities, since the path to the Ph.D. is as important as the final result, so they have to be fully committed to endure (and enjoy) their own research voyage.

Have all of your research students chosen academic careers?

Alberto Álvarez-López: Most of them. I have to say that a few students were part-time students; they were also working out of the university.

Mário Bessa: Since academic jobs, in the area of mathematics, experienced a large decrease in supply in the last decade, Ph.D. students, after finishing their Ph.D. program, try to find business and finance jobs. Fortunately, my former students (Master and Ph.D.) are working as risk analysts in a bank. I point out that their employers are very satisfied with their skills and competence.

Ana Dias: I just have two and both are academics, although not yet with stable jobs.

Filipe Silva: No, several students have professions as teachers or in areas related to the environment. I think that the society, namely the private sector, should interact much more closely with researchers and they should eventually think how their skills can be put to work for the common interest, even if they are not directed to pure research. But it seems that we are still at a considerable distance from a complete integration of researchers in the society as a whole.

Luís Silva: Three out of four.

Paulo Vasconcelos: No, mainly lately they are finding jobs outside of academia.

On other issues:

Do you have hobbies?

Alberto Álvarez-López: I very much enjoy good literature, and reading and writing in general.

Ana Dias: Right now, maybe just cooking, due to the lack of time.

Orlando Gomes: I like to take long walks and to enjoy the company of my family.

Filipe Martins: I am a proud Portuguese, and enjoy my country very much. My main hobbies are reading, music, and playing the piano and watching Boavista F.C. play. I am an avid reader.

Filipe Silva: Jogging and swimming in the ocean. Fortunately, I can do it all the year round in the Azores.

Luís Silva: I am a big fan of enduro mountain biking.

Do you have a connection to Portugal? How do you see its development?

Elvio Accinelli: I have an excellent relationship with Portugal, especially with the group of applied mathematics from Porto. With my work group in México we could make many joint projects with the group led by Alberto Pinto. The development of joint work with this group is of particular interest to us.

Alberto Álvarez-López: I feel as if I had a brother in Portugal. My visits to this brother are not very frequent, but when I am with him, I always feel exactly as if I were at home.

Ana Dias: I am working at the University of Porto. I see that Portugal is progressing with many people working hard, and I hope they will not lose their enthusiasm.

Orlando Gomes: I am Portuguese. I think Portugal is a victim of a drifting European Union and of the poor quality of its own economic policies. Visible setbacks in the areas of culture and science are, for me, the most painful.

Filipe Silva: Living in the Azores islands, I am aware of the consequences that can arise from unplanned development. Development without knowledge will hardly be development at all, and surely not sustainable. That's why universities and other innovation/research institutions play a crucial role in training the new generations and in contributing to a development that will not compromise Earth's resources and future generations.