Interview

with Francesco Brenti, Christian Krattenthaler and Vic Reiner

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The *Summer School on Algebraic and Enumerative Combinatorics*, sponsored by *Centro Internacional de Matemática* (http://www.cim.pt), took place in July, 2–13, 2012, at the *Centro de Estudos Camilianos*, S. Miguel de Seide, in a building of Álvaro Siza, the 1992 Laureate of the *Pritzker Architecture Prize*. It was also financially supported by the *Fundação para a Ciência e a Tecnologia* (http://www.fct.pt) by the *Centro de Estruturas Lineares e Combinatórias* (Universidade de Lisboa), the *Centro de Matemática da Universidade de Coimbra* (Universidade de Coimbra) and the *Centro de Matemática da Universidade do Porto* (Universidade do Porto).

Together with Marc Noy (Universitat Politécnica de Catalunya), Francesco Brenti (Universitá di Roma *Tor Vergata*), Christian Krattenthaler (Universität Wien) and Vic Reiner (University of Minnesota) were in S. Miguel de Seide for the Summer School, where they lectured courses on Combinatorics of Coxeter Groups, on Map Enumeration, and on Reflection Group counting and q-counting. After the school, they have kindly accepted to answer some questions we posed.

Francesco Brenti

We were pleasantly surprised by the success of the school. What was your own impression? How do your previous summer school experiences compare with this one?

F. Brenti.—The impression was very good. Students were able to solve problems that usually require a few days of thinking. Of course they worked on them all together, which certainly helped, but they were impressive just the same, also considering that many of them, though combinatorialists, came from very different areas of combinatorics, and had never worked on combinatorics of Coxeter grups before. I was also extremely pleased by how effectively the students picked up the basic and fundamental techniques that are used for research in combinatorics of Coxeter groups.

Concerning comparison to other summer schools, from the point of view of the lectures there was not much difference, but there was a huge difference for what concerns the exercises. In Luminy, for example, we were asked to assign "homework" problems to the students, and the students would work on them in the afternoons, in groups, with the teacher walking around among them and being available for questions and explanations. Then, after the afternoon coffee break, we would all gather in the lecture room and the students (or, if no one had solved the problem, the professor) would explain a solution to the others. In Guimarãaes we were given complete freedom on how to use the recitation time, so I used it as I usually do in my own courses, namely I propose a problem and then wait for input from the students, trying to follow all the threads of reasoning that they propose. I think this method is more effective for a couple of reasons. First, if the professor explains an exercise then the student thinks: "OK, he's the professor, of course he knows how to do it", but if a fellow student solves the problem then the student thinks "Gee, she could do it, why can't I?". Secondly, if I explain a solution to the problem, it is usually the simplest solution, but the students often find solutions that I would have never thought of, and which often involve many more concepts and theorems than the simplest solution, and are therefore more effective in making them learn the material, besides, false starts and mistakes are also instructive. Whenever you use this system of doing exercises, there is always a fear that we might all be staring at each other for an hour, but this never happened to me, and it did not happen also this time, in fact, quite the opposite (once, at some point one of the organizers came up to me saying that we should really all go to the restaurant for lunch, as they were waiting for us!). I like this way of doing exercises because the performance of students at exams showed to me beyond any doubt its effectiveness.

C. Krattenthaler.—I have participated now in several summer schools for PhD students and postdocs as a lecturer. This is always a pleasure — and has also been so this time — since one talks to young, motivated people who want to learn something from you and therefore are extremely interested in your lectures: they are open to absorb material which is absolutely new to them (whether they always assimilate this with ease, this is a different matter ...), and they are willing to put significant effort to master the material taught to them, with the motivation that, in order to become — and be (!) — a true researcher, one has to constantly enlarge one's own expertise and perspective. If I am to compare this experience with previous ones, then I would say that the level of enthusiasm and commitment of the young people in São Miguel de Seide has been the same as at previous schools, as it should be!

V. Reiner.—I was very pleased with the willingness of the students to ask questions during lectures, and to really grapple with exercises during the problem sessions. Perhaps I shouldn't have been surprised. My one previous experience as a summer school lecturer was at an ACE (Algebraic Combinatorics in Europe) Summer School in Vienna 2005, run by Christian Krattenthaler. Looking back, the two summer school experiences were quite similar. Except in 2005, I seem to remember eating more *schnitzel*, less *bacalhau*.

How important do you think that summer schools like this one, in S. Miguel de Seide, are for students wishing to work in mathematics and in combinatorics, in particular?

F. Brenti.—I think that they are very important, in fact almost essential, because Ph.D. programs in mathematics in Europe (except

at the very top schools) are not as comprehensive as they are in the U.S. So, it is common for a Ph.D. student in Europe to be able to take only one graduate course in combinatorics, even if this is the field in which he/she wants to work in, as opposed to the 3 or 4 that would be available in the best U.S. schools. This is certainly true in Italy.

C. Krattenthaler—I have already mentioned the important point in my answer to the first question: in order to become — and be — a true researcher, one has to constantly enlarge one's own expertise and perspective. Consequently it is particularly important for young people to attend such schools, where they are together with international senior and young scientists, where they are offered instruction in material or views which may not be presented at their home institution, and where they can profit from the expertise of the other participants of the school. Moreover, this is also an ideal place for building up scientific (and non-scientific …) contacts and collaborations.

V. Reiner.—They are very important, as an easier path into topics that might otherwise seem mysterious and forbidding. In addition, I think they give an invaluable opportunity to meet other students, postdocs, and faculty in combinatorics, in a setting that is closer and friendlier than a typical conference.

How did you start working in combinatorics? Could you tell us briefly about your mathematical beginnings, and subsequent career development? Who (or which event) influenced you most?

F. Brenti.—I've been reading math books ever since I was 12. At age 16 I stumbled upon a book that was a collection of essays, each one about some areas of modern mathematics, that had been translated into Italian by U.M.I. (the Italian Mathematical Society). I read essentially all of them but the one that definitely fascinated me the most was the one written by G. C. Rota (that was the first time I had ever heard this name) about combinatorics. The simplicity of the problems discussed and yet at the same time their extreme difficulty fascinated me. But life is often different from how we imagine it so when I was a graduate student working under the direction of Rota I did not like the mathematics that he was doing at the time, and I remember that I was studying Richard Stanley's papers in my spare time (!). After about a year of this, I decided that it couldn't go on, and I switched to Stanley. I have never regretted this. After M.I.T. I was a Hildebrandt Assistant Professor at the University of Michigan in Ann Arbor for 3 years and then I was a member of the Institut Mittag-Leffler in Sweden for a year. That was when I started working in combinatorics of Coxeter groups. I had no teaching duties in Sweden, so I had a lot of free time, and one day in the new books section of the Library I found this book written by J. E. Humphreys entitled “Reflection groups and Coxeter groups”. It was so well written that I started reading it and eventually ended up working in the subject. After Sweden I came back to Italy, where I have remained except for several years of leave spent in various places.

C. Krattenthaler.—How did I start to work in combinatorics? This began at the age of 14 or so, when I became interested in figuring out what the probability was that, if you threw — say — n dice, the sum of the scores added up to — say — S. I remember that I computed long tables in small cases (no computers yet!), discovered partial results, then, at some point, learned about factorials and binomial coefficients (which was really helpful …), and in the end (I believe roughly two years later) I figured out the formula which (now I know that!) one can easily get by inclusion-exclusion. However, at the time, I had no clue how to prove my formula …

Around that time, one of the mathematics teachers at our school started to give voluntary mathematics sessions, in which I participated, where he introduced us to material which was not covered by the standard school *curriculum*. This teacher could have had an academic career (but probably his parents did not allow him to pursue such a career …), but he was entirely inappropriate as a school teacher. The latter fact was no problem during these mathematics sessions, since the participants wanted to be there and to learn something (and we could learn a lot from him). But when he had to deal with a crowd of pupils in an ordinary class, then

Vic Reiner

things were entirely different: his strongest “weapon” against the noise coming from the various not so interested pupils consisted in standing in front of the classroom, smiling helplessly …

At age 17, these mathematics sessions turned into preparation courses for the mathematics olympiad. In the first year I participated in the “beginner’s competition” of the Austrian mathematics olympiad (earning a gold medal), and in the second year in the “competition for the advanced” (earning a silver medal). I thus qualified for the International Mathematics Olympiad (which that year, 1977, took place in Beograd), where I ended up at a place which will not be mentioned explicitly here …

It was then “clear” that, when I would enter university, I would go for mathematics (and piano, as a matter of fact), which I did at the University of Vienna (and the University for Music and Performing Arts in Vienna). (This was less clear for my father, but he did not object …)

In the first year of my mathematics studies, Johann Cigler (who later became my advisor) taught a combinatorics course, which I attended with enthusiasm. I also followed several other courses and seminars given by Cigler. These were always very fascinating since Cigler did not just present the material like that, but instead did it always in original ways, and he would always present us his own thoughts and ideas he had on the subject, including (open) questions for us students. Answers and solutions to Cigler’s questions and problems that I found became in the end (more or less) my thesis.

V. Reiner.—I figured out a bit later than many mathematicians that I really liked math. I was supposed to go to medical school! That’s the nature of my family background.

In college, I chose a math major as a pre-medical student, and quickly realized two things: (1) Math classes were more interesting, and taught better than at my high school. (2) Math people were — really — smart! I soon realized how frequently dumb I would feel if I were to go into mathematics, but I just started enjoying the material more and more.

Once in math graduate school, I feel lucky to have received excellent advice from my older office-mate, Maciej Zworski (now at UC Berkeley), who recommended Richard Stanley as an advisor. This turned out to be a great choice for me.

Combinatorics, as a systematic study of discrete configurations that encode complex structures and, in particular, the enumeration of objects according to certain restrictions, is now widely recognised as an integral area of pure mathematics. It also has an increasingly important interface with neighbouring areas such as physics, computer science and molecular biology, for example. What is your personal opinion about the impact of combinatorics on these areas, and vice-versa?

F. Brenti.—I think that, as always in mathematics and in science, connections between different areas of mathematics (and of science) are mutually beneficial. Many problems in my current area of research (combinatorics of Coxeter groups) come from algebra, and you would not have considered them if it wasn’t for this connection. Similarly for many combinatorial problems that have arisen from research in geometry, physics, or computer science. Regarding the mutual impact of combinatorics and the areas with which it interfaces, I think that in algebra and geometry the influence has been mainly on using results from these areas to solve combinatorial problems but not much on using combinatorial results to solve algebraic and geometric problems. I think this is due to the fact that combinatorics is a much younger subject than both algebra and geometry, and therefore has at its disposal tools that are not yet as advanced as in those areas. I think that this will naturally change as combinatorics matures and discovers deeper and deeper theorems, and that we have already seen some examples of this.

C. Krattenthaler.—Interplay between different disciplines is always extremely fruitful for *all* the involved disciplines, and therefore I am very excited about the interactions between combinatorics and (statistical) physics, computer science and molecular biology. The interplay between combinatorics and computer science is the one which is ongoing longest — here one has to mention the development of efficient algorithms for the solution of combinatorial problems (of daily life), and the analysis of algorithms — and it is manifest in the fact that many researchers in combinatorics are employed in computer science departments. In one part of my research work I am involved in the interplay of combinatorics and statistical physics: one can measure my excitement for this interplay if one knows that, during my studies, I had never entered a physics lecture (because I did not feel that I wanted to do learn more about physics than what I already knew from high school …). I shall say more on this interplay in my answer to Question 5. The interplay between combinatorics and molecular biology comes from the fact that computers are now strong enough to scan through data read from genomes — and extract useful information! — provided one applies sufficiently efficient algorithms for the extraction of information. Interesting combinatorial problems arise from there, but I am somewhat sceptical how useful the theoretical results which one is able to find on these problems really are for the actual biological questions. My impression is that — at least for now — computer power is more important than theoretical ideas. That may change of course.

V. Reiner.—I don't know about our impact on these other areas, but we in combinatorics owe them a lot. As my friend Mark Shimozono once modestly claimed, "I'm not smart enough to know which combinatorics is going to be interesting on its own — I need algebra as a crutch, to point me toward the right objects to study." What is true for algebra is also true for physics, computer science, biology. I would even include subjects like economics. Look at this year's Nobel Prize in economics, which was awarded essentially for matching theory.

If you had to give a synopsis of the current state-of-the-art in combinatorics, which challenging open or recently solved problems would you choose to mention?

F. Brenti.—Such a list is necessarily biased by my own preferences, taste, and expertise! Regarding recently solved problems I would definitely mention the Strong Perfect Graph Theorem of Chudnovsky, Robertson, Seymour and Thomas, the combinatorial proof of Schur-positivity of Macdonald polynomials by Assaf, the Polya-Schur Master Theorems of Borcea and Branden, and the recently announced nonnegativity of Kazhdan-Lusztig polynomials of Elias and Williamson. Regarding open problems, I would mention the combinatorial invariance conjecture for Kazhdan-Lusztig polynomials by Dyer and Lusztig, the problem of finding a combinatorial interpretation for Kazhdan-Lusztig polynomials, and the conjectured nonnegativity of the complete **cd**-index of a balanced digraph by Ehrenborg and Readdy (even just in the special case of Bruhat graphs).

C. Krattenthaler.—This is an impossible task because "Combinatorics" is a vast subject whose branches include Enumerative Combinatorics, Analytic Combinatorics, Algebraic Combinatorics, Probabilistic Combinatorics, Geometric Combinatorics, Algorithmic Combinatorics, Design Theory, Graph Theory, Extremal Combinatorics, Combinatorial Optimization (and I am sure that I forgot something). There have been exciting developments in practically in all of these branches during the past years.

So let me content myself with a corresponding commentary relating to the topic of my course, "Map Enumeration". An exciting development which has taken place in this part of combinatorics (and also related parts) is the growing interaction between (enumerative) combinatorialists, probabilists, and statistical physicists. Over a long time, these three communities worked by themselves and sometimes in parallel, (re)discovering the same things without knowing that these were also considered by the other communities (not to mention which results were known by these other communities). This has changed dramatically during the past 15 years. Researchers in these three communities have understood that the developments in the other communities are also relevant for themselves. Many interactions have now taken place, with physicists becoming interested in purely combinatorial problems, combinatorialists becoming interested in problems of probability and physics, etc. Among the recent culmination points one has to mention the solution of several notorious enumeration problems on alternating sign matrices by methods coming from statistical physics, the beautiful and deep asymptotic theory for the behaviour of large tilings (actually: perfect matchings) which contributed to the award of a Fields medal, and the proof of the so-called Razumov-Stroganov conjecture on the ground state of a certain Hamiltonian by purely combinatorial methods (carried out by two physicists).

V. Reiner.—My absolutely favorite open problem currently is this: Understand the relation between the notions of noncrossing partitions and nonnesting partitions for finite reflection groups and Weyl groups, and in particular, why they obey the same beautiful enumeration formula.

In this summer school, we have seen a number of different cultures represented within the field of combinatorics, each with its own particular set of tools. How would you describe the essence of your own research to a young student in search of a research topic?

F. Brenti.—The essence of my own research is to study combinatorial problems that arise from algebra and to use algebraic techniques to solve combinatorial problems.

C. Krattenthaler.—The search of an appropriate research topic is an extremely delicate question (in particular for advisors of PhD students …). If I am asked how I proceeded to find "my" problems then the answer is that the only criterion has always been the appeal that a problem had to me, and I did not care whether this was an "important" problem or not. Whether this is a good (or sufficient) advice for young people, I cannot tell. I believe that it is if it is interpreted in the following way: it is obvious that it makes no sense to work on problems which one actually does not like, only for the sake that somebody said that this is an "important" problem (where the notion of "importance" would require a separate discussion). A problem should somehow appeal to somebody, otherwise one will not feel the motivation to solve it. However, whether a problem appeals to somebody or not should also be implicitly guided by this question of "importance". This is something which should go together automatically: appeal and "importance" or "interest". There is no way to put this in any framework of rules: we all know that problems which have been regarded as important at some point were not regarded so at a later point, and that there were problems which were not looked at by almost anybody at some point but which turned out to be of crucial importance at a later point. One has to develop one's own feeling for this by listening to others (at such summer schools, for example) and to one's own inner voice.

V. Reiner.—I like to understand how classical theory of reflection groups and invariant theory points us toward beautiful objects to count, and how to insightfully add the q's when we q-count them.

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