**Ist Portuguese Meeting on Mathematics for Industry**
6th to 8th June 2013

Faculdade de Ciências da Universidade do Porto
Rua do Campo Alegre, 687, 4169-007 Porto
Portugal
http://cmup.fc.up.pt/cmup/apmind/meeting2013/

The Portuguese Meetings on Mathematics for Industry will give sequence to Porto Meetings on the same topic. To view previous versions please see

http://cmup.fc.up.pt/cmup/apmind/meeting2013/

The purpose of this meeting is to focus the attention on the many and varied opportunities to promote applications of mathematics to industrial problems. Its major objectives are:

- Development and encouragement of industrial and academic collaboration, facilitating contacts between academic, industrial, business and finance users of mathematics
- Through "bridging the industrial/academic barrier" these meetings will provide opportunities to present successful collaborations and to elaborate elements such as technology transfer, differing vocabularies and goals, nurturing of contacts and resolution of issues.
- To attract undergraduate students to distinctive and relevant formation profiles, motivate them during their study, and advance their personal training in Mathematics and its Applications to Industry, Finance, etc.

The meeting will be focused on short courses, of three one-hour lectures each, given by invited distinguished researchers, which are supplemented by contributed short talks by other participants and posters of case studies. This edition is especially dedicated to two main themes:

- Optimization and Financial Mathematics
- Mathematical Epidemiology

Special participation of members of the Analytics & Decision Models Area, Millennium bcp/Banco Comercial Português, SA.

The meeting is promoted by APMInd (Portuguese Association of Mathematics for Industry: http://cmup.fc.up.pt/cmup/apmind/) as one of the activities of both GEMAC (Gabinete de Estatística, Modelação e Aplicações Computacionais: http://cmup.fc.up.pt/cmup/gemac/) and Master course in Mathematical Engineering (http://www.fc.up.pt/dmat/engmat/).

An Interview

# with the Scientific Committee members of the Iberian Meeting on Numerical Semigroups — Vila Real 2012

by **Manuel Delgado** [CMUP and DM-FCUP, University of Porto] and
**Pedro García Sánchez** [Dep. Álgebra, University of Granada]

By the occasion of the Iberian Meeting on Numerical Semigroups -- Vila Real 2012, that held at the UTAD, from the 18th to the 20th of July, we posed some questions to the members of the Scientific Committee. These are Valentina Barucci, from the Università di Roma *La Sapienza*; José Carlos Rosales, from the Universidad de Granada; Ralf Fröberg, from the Stockholms Universitet; and Scott Chapman, from the Sam Houston State University.

Valentina Barucci most cited work (with over 70 cites) on numerical semigroups is her book on maximality properties on numerical semigroups with applications to one-dimensional analytically irreducible local domains, which has been for years a dictionary between these two mathematical objects. Valentina gave a talk in this meeting on differential operators on semigroup rings.

José Carlos Rosales wrote his thesis on numerical semigroups, and since then he wrote more than a hundred papers on this and related topics.

Ralf Fröberg most famous paper on numerical semigroups is *On numerical semigroups* that used numerical semigroups to solve problems on one-dimensional local rings. This paper has been cited over 65 times. In this meeting he gave a talk showing that homological tools can be used to give elegant answers to questions arising in the study of numerical semigroups.

Scott Chapman uses numerical semigroups in the study of non-unique factorization invariants, and also in the study of ideal theory on semigroup rings. He is currently editor in chief of the American Mathematical Monthly.

## Can you tell us when did your interest on numerical semigroups started? And what was the motivation?

**Valentina.—** I started to be interested in numerical semigroups after meeting Ralf Fröberg, who proposed me some problems in this subject. The problems were very concrete and at the same time they implied some consequences for geometric objects, as monomial curves, so I felt attached soon.

**José Carlos.—** I started doing mathematical research by treating problems in this field. My doctoral thesis defended in Granada in 1991 was entitled Numerical Semigroups.

**Ralf.—** I became interested in numerical semigroups in the 1980's. Mostly by chance, we and two colleagues (Gottlieb, Häggkvist) started to discuss the subject and finally we wrote a paper. Then I met Valentina Barucci and we started to work on problems concerning numerical semigroup rings. This joint work continued for many years, and later included Marco D'Anna. Our interest comes from one-dimensional rings, which has a lot to do with numerical semigroups.

**Scott.—** My main doctoral thesis problem involved studying the ideal theory of particular semigroup rings. While the actual work was in Commutative Algebra, I was fascinated by the structure of numerical monoids. As I began to work more and more on factorization problems, I began applying the ideas and techniques of the theory of non-unique factorizations to numerical monoids and semigroups.

## Would you recommend the study of numerical semigroups to start a research career?

**José Carlos.—** Over the years I have never abandoned the research on this topic. It is to me an exciting field in which many problems appear naturally and in which there are increasingly more researchers involved. This is the reason why I, undoubtedly, would recommend this field to someone interested in research in mathematics.

**Scott.—** Enthusiastically! Many of my publications involving numerical monoids and semigroups have been written with students who worked under my direction on research supported by the National Science Foundation. While problems in numerical semigroups become extremely difficult, the background for getting started is merely rational number theory. Students can quickly get basic results, but just as quickly learn how challenging mathematics can become.

## Have you had students that successfully defended their PHD thesis in numerical semigroups?

**José Carlos.—** I have supervised 5 doctoral theses and in all of them the study of numerical semigroups has played an important role.

**Ralf.—** I have had two students, who partly wrote their thesis on numerical semigroups, and I also worked informally as supervisor to two guest students from Italy, mainly on numerical semigroups.

**Scott.—** My institution does not support a doctoral program. But, many of my past students who started their careers studying numerical monoids, later went on to complete Ph.D. degrees at very high quality American institutions. Moreover, they remain interested in the subject and have even in the past attended at least one of the meetings in the IMNS series. Examples are Paul Baginski (Ph.D. University of California at Berkeley) and Nathan Kaplan (Ph.D. expected this year from Harvard).

## Do you plan to continue working on numerical semigroups? Would you like to mention some open question in the area that you feel like one of the most important?

**José Carlos.—** Absolutely. There are many issues to deepen and many open problems which, undoubtedly, attract the attention of many mathematicians. In this line I would highlight the Frobenius problem, the Wilf conjecture and the Bras-Amoros conjecture.

**Scott.—** Yes, I plan on continuing to work in this area. Here is an open problem that I think is particularly important. Let $S$ be a numerical monoid and assume its elements are listed in increasing order as $S = s_1, s_2, s_3, \ldots$. In the paper Delta sets of numerical monoids are eventually periodic, (Aequationes Math. 77 (2009), 273–279) written by Chapman, Hoyer and Kaplan, it is shown the the sequence of delta sets $\Delta(s_1), \Delta(s_2), \Delta(s_3), \ldots$ is eventually periodic. Is the same true for the sequences $c(s_1), c(s_2), c(s_3), \ldots$ and $t(s_1), t(s_2), t(s_3), \ldots$ where $c(x)$ represents the catenary degree of $x$ in $S$ and $t(x)$ the tame degree of $x$ in $S$?

## Which is from your point of view the most important question on numerical semigroups that has been solved? What was its impact and relevance in other areas of Mathematics?

**José Carlos.—** The study of numerical semigroups is a classic theme. However, from the second half of the twentieth century suffered a major boost due to its applications in many and interesting fields such as algebraic geometry, coding theory, number theory and computer algebra.

**Ralf.—** I am not able to point at the most important theorem in numerical semigroup theory. I think that several connections with other subjects will be found, so perhaps it is too early to say what is most important.

**Scott.—** My interest in numerical semigroups centers in the study of non-unique factorizations. While there are significant papers concerning the structure of numerical semigroups, I will focus on their factorization properties in answering this question. I think the solution of the question cited above (about the eventual periodicity of the sequence of delta sets) is a very important result. The wealth of results on the factorization properties of numerical monoids has led to similar investigations in other types of monoids (such as block monoids, Diophantine monoids and congruence monoids) which might have otherwise never been completed.

**Membros da Comissão Científica, organizadores e organizadores locais do IMNS 2012. Da esquerda para a direita: Luís Roçadas, UTAD, Ralf Froberg, Stockholms Universitet, Scott Chapman, Sam Houston State University, Valentina Barrucci, Univ. di Roma *La Sapienza*, Paula Catarino, UTAD, P. A. García-Sánchez, Universidad de Granada, José Carlos Rosales, Universidad de Granada, André Oliveira, UTAD, M. Delgado, Universidade de Porto, Paulo Vasco, UTAD**

## Has the study of numerical semigroups some relevance in the mathematics developed in your country? Is it difficult to get funding for doing research in numerical semigroups?

**José Carlos.—** In my opinion the importance of this line of research is comparable to the most relevant lines of mathematical research. That is why we have not had difficulties in obtaining funds through national and regional research projects.

**Ralf.—** It is generally hard to get money for research in Sweden, not particularly for semigroups.

**Scott.—** Yes, it has particular relevance in Commutative Algebra and Algebraic Geometry. It is difficult in the United States to obtain funding for almost any pure mathematical subject. I have over the past 15 years obtained such funding on three different occasions from the National Science Foundation to run Research Experiences in Mathematics Programs for undergraduate students.

This is the third edition of the Iberian Meeting on Numerical Semigroups. The authors of this interview have been in charge of the organization of all the three editions, with very important collaborations in the second and third. Its success has exceeded by far the initial expectations for a meeting that was intended to gather together mathematicians from various areas where numerical semigroups appear in a rather informal way. Please give us your opinion on the usefulness or not of this kind of meetings for the development of an area, in particular in an area that constitutes a small intersection point of several others.

**Valentina.—**Numerical semigroups is apparently a narrow subject, but it gathers people from different areas.

Thus, the most interesting talks in the meeting were for me the talks where also other subjects, e.g. from commutative algebra or from code theory, appear.

It was also interesting and useful to meet personally some mathematicians who worked on similar problems than me and that I know only though their papers.

The meeting was also pleasant because there are not people that consider themselves big stars, as sometime happens, and there was a very nice cooperative atmosphere.

**José Carlos.—**These meetings are very useful since they encourage the grouping of mathematicians interested in the study and applications of numerical semigroups coming from all over the world. This provides a contact at first hand with the latest advances in this field. It also provides discussions between different researchers that could not happen otherwise.

**Ralf.—**It is very important to have a chance to meet people in this relatively narrow area. This gives possibilities to talk about problems, but still to get views from different angles.

**Scott.—**I think it is highly useful. Most of the participants at the IMNS meetings found numerical semigroups by working in some other area. In my case, it was Commutative Algebra. In other cases, it was Computer Science, Graph Theory, Algebraic Geometry, …. The list is almost endless. I am not so sure that the intersection mentioned above is so small. In fact, I think it has grown drastically over the past 10 years and I believe that attendance at the next IMNS meetings will exceed that of any of the first three editions of this congress.

# The multivariate extremal index and tail dependence

**by Helena Ferreira***

Abstract.—If we obtain a tail dependence coefficient of the common distribution of the vectors in a multivariate stationary sequence then we do not have necessarily the correspondent coefficient of the limiting multivariate extreme value model. In opposition to sequences of independent and identically distributed random vectors, the local clustering of extremes allowed by stationarity can increase or decrease the tail dependence.

The temporal dependence at extreme levels can be summarized by the function multivariate extremal index and its effect in the tail dependence is well illustrated with Multivariate Maxima of Moving Maxima processes.

Keywords.—multivariate moving maxima, multivariate extremal index, tail dependence, multivariate extreme value distribution.

## 1. Introduction

For a random vector $\mathbf{X} = (X_1, \dots, X_d)$ with continuous marginal distributions $F_1, \dots, F_d$ and copula $C$, let the bivariate (upper) tail dependence coefficients $\lambda_{jj'}^{(\mathbf{X})} \equiv \lambda_{jj'}^{(C)}$ be defined by

$$\lim_{u\uparrow 1} P\left(F_j(X_j) > u | F_{j'}(X_{j'}) > u\right), \quad (1)$$

for $1 \leq j < j' \leq d$. Tail dependence coefficients measure the probability of extreme values occurring for one random variable given that another assumes an extreme value too. Positive values correspond to tail dependence and null values mean tail independence. These coefficients can be defined via copulas and it holds that

$$\lambda_{jj'}^{(C)} = 2 - \lim_{u\uparrow 1} \frac{\ln C_{jj'}(u,u)}{\ln u}, \quad (2)$$

where $C_{jj'}$ is the copula of the sub-vector $(X_j, X_{j'})$ ([4], [7]).

Let $F$ be a multivariate distribution function with continuous marginals, which is in the domain of attraction of a Multivariate Extreme Value (henceforth MEV) distribution $\hat{H}$ with standard Fréchet margins, that is, $F^n(nx_1, \dots, nx_d) \underset{n\to\infty}{\longrightarrow} \hat{H}(x_1, \dots, x_d)$ with marginal distributions $\hat{H}(x_j) = \exp\left(-x_j^{-1}\right)$, $x_j > 0$, $j = 1, \dots, d$. It is known that any bivariate tail dependence coefficient of $F$ is the same as the corresponding coefficient of $\hat{H}$ ([8]).

Let $\{\mathbf{Y}_n\}_{n\geq 1}$ be a multivariate stationary sequence such that $F_{\mathbf{Y}_n} = F$ and $\mathbf{M}_n = (M_{n1}, \dots, M_{nd})$ is the vector of componentwise maxima from $\mathbf{Y}_1, \dots, \mathbf{Y}_n$. If $\lim_{n\to\infty} P(M_{n1} \leq nx_1, \dots, M_{nd} \leq nx_d) = H(x_1, \dots, x_d)$, for some MEV distribution $H$, one question that naturally arises is Does dependence across the sequence affect the bivariate tail dependence of the limiting MEV? In other words, we would like to know the relation between the tail dependence coefficients of the limiting MEV $H$ and the limiting MEV

$$\begin{gathered} \hat{H}(x_1, \dots, x_d) = \\ \lim_{n\to\infty} P\left(\hat{M}_{n1} \leq nx_1, \dots, \hat{M}_{nd} \leq nx_d\right) = \\ \lim_{n\to\infty} F^n(nx_1, \dots, nx_d), \end{gathered}$$

where $\hat{\mathbf{M}}_n = (\hat{M}_{n1}, \dots, \hat{M}_{nd})$ is the vector of pointwise maxima for a sequence of i.i.d. random vectors $\{\hat{\mathbf{Y}}_n\}_n \geq 1$ associated to $\{\mathbf{Y}_n\}_n \geq 1$, that is, such that, $F_{\hat{\mathbf{Y}}_n} = F_{\mathbf{Y}_n} = F$.

Our main purpose is to compare the bivariate tail dependence coefficients for the margins of the two Multivariate Extreme Value distributions $H$ and $\hat{H}$ through the function multivariate extremal index ([6]), which resumes temporal dependence in $\{\mathbf{Y}_n\}_{n\geq 1}$.

We recall that the $d$–dimensional stationary sequence $\{\mathbf{Y}_n\}_{n\geq 1}$ is said to have a multivariate extremal index $\theta(\boldsymbol{\tau}) \in [0, 1]$, $\boldsymbol{\tau} = (\tau_1, \dots, \tau_d) \in \mathbb{R}_+^d$, if for each $\boldsymbol{\tau}$ in $\mathbb{R}_+^d$,

* Department of Mathematics — University of Beira Interior, Covilhã